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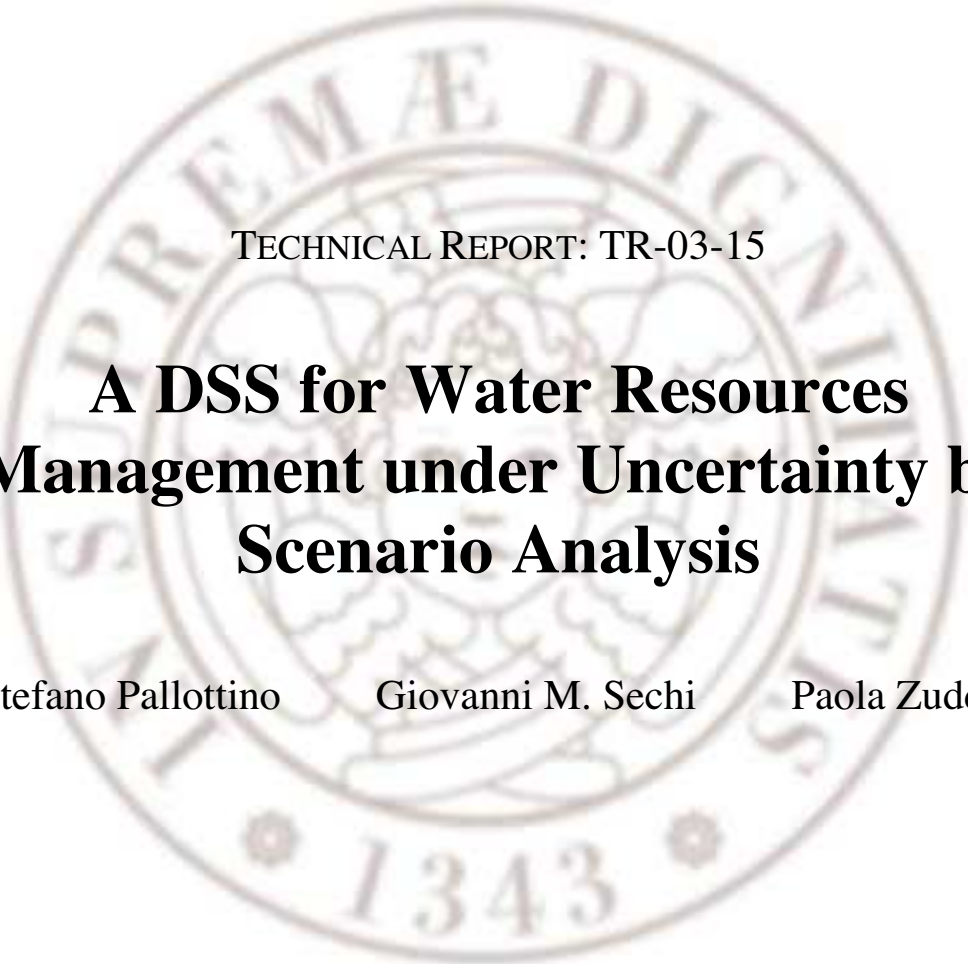

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A DSS for Water Resources Management under Uncertainty by Scenario Analysis

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A DSS for Water Resources Management under Uncertainty by Scenario Analysis

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Abstract

In this paper we present a scenario analysis approach for water system planning and management under conditions of climatic and hydrological uncertainty. The scenario analysis approach examines a set of statistically independent hydrological scenarios, and exploits the inner structure of their temporal evolution in order to obtain a "robust" decision policy, so that that the risk of wrong decisions is minimised. In this approach uncertainty is modelled by a scenario-tree in a multistage environment, which includes different possible configurations of inflows in a wide time-horizon. In this paper we propose a Decision Support System (DSS) that performs scenario analysis by identifying trends and essential features on which to base a robust decision policy. The DSS prevents obsolescence of optimiser codes, exploiting standard data format, and a graphical interface provides easy data-input and results analysis for the user. Results show that scenario analysis could be an alternative approach to stochastic optimisation when no probabilistic rules can be adopted and deterministic models are inadequate to represent uncertainty. Moreover, experimentation for a real water resources system in Sardinia, Italy, shows that practitioners and end-users can adopt the DSS with ease.

Keywords: water resources management; multi-period dynamic network; optimisation under uncertainty; scenario analysis; multi-scenario aggregation.

1. Introduction

Water Resources dynamic management (WR) problems with a multi-period feature are associated with mathematical optimisation models that handle thousands of constraints and variables depending on the level of detail required to reach a significant representation of the system (Loucks et al., 1981; Yeh, 1985).

One optimisation approach is to model the WR problem as a dynamic multi-period network flow problem, where all data are fixed and no level of uncertainty is considered (Kuczkera, 1992; Sechi and Zuddas, 1998). Efficient optimisation algorithms have been used to solve this kind of problem (Sechi and Zuddas, 2000). But, WR problems are typically characterised by a level of uncertainty regarding, among other things, the value of hydrological exogenous inflows and demand patterns. Assigning inaccurate values to them could well invalidate the results of the study. Consequently, deterministic models are inadequate for the representation of these problems where the most crucial parameters are either unknown or are based on an uncertain future.

The traditional stochastic approach gives a probabilistic description of the unknown parameters on the basis of historical data. This is a very efficient approach when a substantial statistical base is available and reliable probabilistic laws can adequately describe parameters' uncertainty and their possible outcomes (Infanger, 1994; Kall and Wallace, 1994; Ruszczynski, 1997). It is well know that stochastic optimisation approaches cannot be used when there is insufficient statistical information on data estimation to support the model, when probabilistic rules are not available, and/or when it is necessary to take into account information not derived from historical data. In these cases, the scenario analysis

technique could be an alternative approach (Dembo, 1991; Rockafellar and Wets, 1991). *Scenario analysis* can model many real problems where decisions are based on an uncertain future, whose uncertainty is described by means of a set of possible future outcomes, called "scenarios". Some examples are given in Mulvey and Vladimirou (1989) for investment and production planning, in Glockner (1996) for air traffic management and in Hoyland and Wallace (2001) for insurance policy and production planning.

The scenario analysis approach considers a set of statistically independent scenarios, and exploits the inner structure of their temporal evolution in order to obtain a "robust" decision policy, in the sense that the risk of wrong decisions is minimised. Rockafellar and Wets (1991) proposed a scenario aggregation formulation that produces a tree-structure of the scenarios, as explained in detail in section 3.

The aim of this paper is to apply the scenario analysis framework to WR problems and investigate its effectiveness with respect to traditional approaches. A WR model is usually defined in a dynamic planning horizon in which management decisions have to be made sequentially or globally decided as a decision strategy referred to a predefined scenario, where a *scenario* represents a possible realisation of some sets of uncertain data in the time horizon examined.

One common approach is to carry out a set of experiments on a number of generated series (parallel scenarios) followed by a simulation-testing phase of each scenario in order to validate the solutions under investigation. All the solutions (each one is a sequence of decisions) are completely independent one from the other because they are obtained from scenarios analysed separately. As a consequence, the decisions adopted are closely related to the scenario selected at the end of the simulation and the study must start all over again if a different scenario comes true. To overcome the above difficulties, in this paper we analyse the scenario approach for WR offering some general rules for organising a predefined set of scenarios into the *scenario tree* and for identifying a complete set of decision variables relative to all the scenarios under investigation. The scenario tree is obtained by aggregate common portions of scenarios; the *aggregation condition* guarantees that the solution (that is, the decisions) in any given period is independent of the information not yet available, as detailed in section 3.

Scenario analysis approach for WR was proposed in (Escudero, 2000) and in Wam-Me EU project (Sechi and Zuddas, 2002), and tested on some real physical systems. Our proposal is to embed an evolution of the above WR scenario analysis tool into a DSS that allows in depth investigation of the robustness of the solution and, if necessary, refinement of decisions. In fact, the decision policy found in the scenario analysis can be considered "implementable" in the sense that the sequence of decisions is congruent, but it may not correspond to any solution obtainable by single scenario optimisation. In the proposed DSS, the availability of an efficient computer graphical interface, that is designed to facilitate the use of models and database, helps end-users to evaluate with ease the best choice and reach a robust solution starting from the physical system. The tool is a greatly improved version of the DSS WARGI (Sechi and Zuddas, 2000). Moreover, the proposed DSS allows "weight tuning phases" that the WR manager can use to refine the "relative importance" that he should assign to each single scenario.

2. Water Resources Dynamic Model

A successfully applied approach is to model the problem by an optimisation network flow model supported by a multi-period dynamic graph where all data are fixed and no level of uncertainty is considered (Kuczkera, 1992; Sechi and Zuddas, 1998). Network flow models allow adopting highly efficient computational algorithms even when thousands of variables and constraints require management (Ahuja et al., 1993). In WR management problems, the authors explored the possibility of maintaining network flow structure even if non-network constraints are present in the model (Manca et al., 2002). In subsection 2.1 a detailed description of a dynamic network flow model for water resources management

is provided. In this section we illustrate the way in which the associated multi-period dynamic graph can be generated and describe the main components of the final mathematical deterministic model.

We formulate a WR management model in a deterministic framework, i.e., having previous knowledge of the time sequence of inflows and demand. We extend the analysis to a sufficiently wide time horizon and assume a time step (period), *t*. The scale and number of time-steps considered must be adequate to reach a significant representation of the variability of hydrological inflows and water demands in the system.

Referring to a "static" or single-period situation, we can represent the physical system by a direct network (basic graph), derived from the physical sketch. Figure 1a shows a physical sketch of a simple water system. In the figure, nodes maintain the shape of the common hydraulic notation in order to recall the different function of system components. Nodes could represent sources, demands, reservoirs, groundwater, diversion canal site, a hydropower station site, etc. Arcs represent the activity connections between them. Physical components corresponding to nodes and arcs can be in the project stage (work planned) and/or operational (existing works with a known dimension). Nodes corresponding to reservoirs represent the system memory since they can store the resource in one period and transfer it in a successive period. A dynamic multi-period network is generated by replicating the basic graph for each period *t*. We then connect the corresponding reservoir nodes for different consecutive periods by additional arcs carrying water stored at the end of each period. We call these *inter-period arcs*. Figure 1b shows a segment of a dynamic network generated by the simple basic graph of figure 1a. Reservoir-nodes are supply-nodes that store and supply the resource. Demand-nodes use and consume the resource. Junction-nodes allow resource passage without consumption.

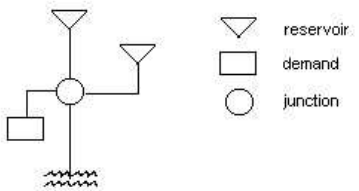

Figure 1a. Water system.

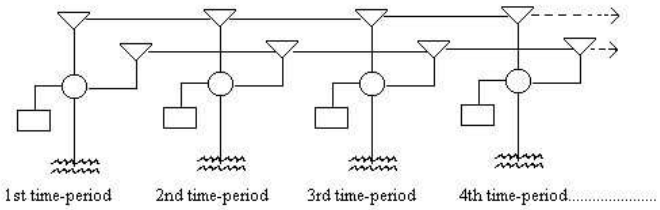

Figure 1b. Segment of a dynamic network.

It may be convenient to add "dummy" nodes and arcs to represent not only physical components but also events that may occur in the system. Figure 2 shows the dynamic multi-period network, corresponding to that of figure 1b, including dummy nodes and arcs marked with a dot.

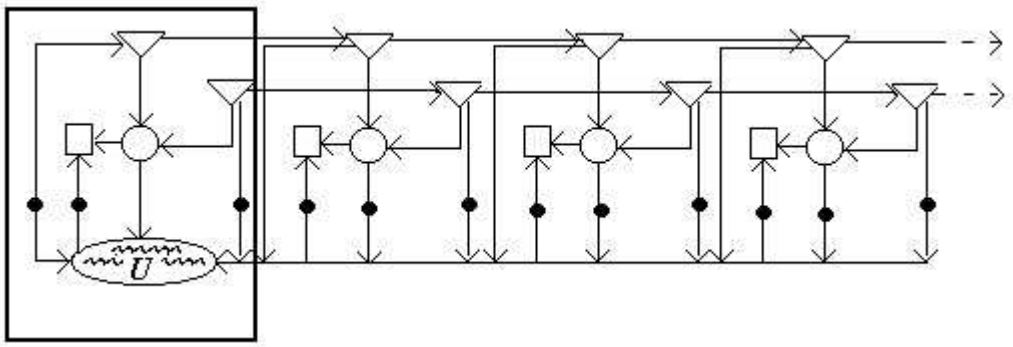

Figure 2. Dynamic multi-period network with dummy nodes and arcs.

The basic graph is in the frame. The dummy node, *U*, represents a possible "external system" acting as a supposed source or demand of flow. In this way each arc (*i*,*U*) represents a spillway from reservoir-node *i*, each arc (*U*,*i*) represents a supposed additional flow in case of shortage in order to meet request in the

demand-node i and prevent solutions which are not feasible. Flow on arcs (U,i) highlights possible system deficits and the need to modify the dimensions of the works or, alternatively, to make recourse to external water resources.

The aim of this paper is not to identify the components of the deterministic mathematical model but rather to provide formulation for a reduced model that can be adopted to formalize uncertainty in water resources management. To illustrate our approach, we adopt deterministic Linear Programming (LP) as described in the following subsection.

2.1 Definition of Water Resources Optimisation Model Components

Though it is almost impossible to define a general mathematical model for water resources planning and management problems, our DSS makes it possible to take into account all possible general system components based on the most typical characterisation of these types of models. Different components can be added or deleted updating constraints and objectives. In this paper, we describe only a few of them. A more detailed description of this approach can be found in (Onnis et al., 1999; Sechi and Zuddas, 2000). Hereafter, we refer to the dynamic network $R = (N, A)$, where N is the set of nodes and A is the set of arcs. T represents the set of time-steps t. Sets of nodes (subsets of N) can represent reservoir nodes, demand nodes (such as civil, industrial, irrigation, etc.), hydroelectric nodes associated with hydroelectric plants, confluence nodes (such as river confluence, withdraw connections for demands satisfaction), etc. Sets of arcs (subsets of A) can represent conveyance work arcs, artificial channels, transfer arcs, spilling arcs, etc.

Variables considered in the LP model can be divided into operation and project variables. Operation variables can refer to different types of water transfer (flow on arcs) such as water-transfer in space along an arc connecting different nodes at the same time, water transfer in an arc connecting similar nodes at different times and so on. Project variables are associated to the dimension of future works: reservoir capacities, pipe dimensions, irrigation areas, etc. Constraints in the LP model can represent mass balance equations, demands for the centres of water consumption, evaporation at reservoirs, relations between flow variables and project works, upper and lower bounds on decision variables.

The DSS allows the introduction of required data taking into account the different state of the hydraulic components, that is if they are to be considered operational or in the project stage. To illustrate this concept, we provide hereunder, some details of possible variables, constraints and related data referred to a reservoir-node and a demand node.

For a reservoir-node, y^t, represents the portion of water stored in the reservoir at the end of period t that can be used in subsequent periods. A corresponding constraint, for each time period t is:

$$\rho^t_{min}\, Y_{max} \leq y^t \leq \rho^t_{max}\, Y_{max},$$

where Y_{max} represents the max storage volume for inter-period transfer arcs and ρ^t_{max} (ρ^t_{min}) represents the ratio between max (min) stored volume in each period t and reservoir capacity. These constraints ensure that, in each period, used volume y^t of the reservoir is in the prescribed range. In an operational state Y_{max} is known while in a project state it is a decision variable. In the latter case it is bounded by:

$$m \leq Y_{max} \leq M,$$

where M (m) represents the max (min) allowed capacity. For a demand-node, e.g. a civil demand, p^t represents the water demand at the civil demand centre in period t. A corresponding constraint, for each time period t is:

$$p^t = \pi^t\, d^t P,$$

where P represents the size of the population whose demand can be fulfilled and π^t is the request program in each period t. These constraints ensure the fulfilment of the demand in each period, no matter if it comes from the system or from dummy resources. In an operational state, P is known while in a project state it is a decision variable. In the latter case it is bounded by:

$$P_{min} \leq P \leq P_{max},$$

where P_{max} (P_{min}) represents the max (min) estimated population.

Moreover, mass balance constraints are introduced, involving all flows that are going in or out the reservoir or demand node, including hydrological input to the reservoir in each period.

The objective function considers costs, benefits, and penalties associated with flow and project variables as well as dummy costs or benefits associated with the dummy components of the multi-period dynamic network.

2.2 Compact Deterministic Linear Programming Model

As is well known, an LP problem can be expressed in a compact standard form where all data are classified as:

- a "cost" vector c, in the objective function, whose components c_j can represents cost, benefit, penalisation or a specific weight assigned by the manager to the variable x_j;
- a RHS (Right Hand Side) vector b, in the constraints system, whose component b_i can represent a supply or a demand associated to a node i, i.e., to the activity represented by node i as described in subsection 2.1;
- lower and upper bound vectors u and l, whose components l_j and u_j represent lower and upper limits (possible zero and infinity, respectively) imposed on the variable x_j by physical, technological, environmental and/or political requirements;
- the matrix A represents the coefficient matrix of the constraints system.

In the deterministic approach the hydrological database is derived from available historical data submitted to statistical validation on the basis of a forecast and adopted as reference scenario. As said previously, in the deterministic optimisation model we assume that the manager has previous knowledge of the time sequence of inflows and demands. As a consequence, the solution obtained is strictly connected to the adopted scenario. Given a set G of predefined scenarios, for a specific scenario g the corresponding LP model, (P_g) can be expressed as:

$$\begin{aligned} (P_g) \quad \min \quad & cT_g\, x_g \\ & A\, x_g = b_g \\ & l_g \leq x_g \leq u_g \end{aligned}$$

The index g identifies vectors c, b, l and u of data related to the predefined scenario g. Moreover, x_g represents the vector comprehensive of all operation and project variables in scenario $\boldsymbol{g}$ and all constraints are represented as lower and upper bounds or in equation form. In this way, the compact LP model includes data, variables and constraints, such as those described in subsection 2.1.

3. Water Resources Chance Dynamic Model

The deterministic models described in the previous section are not adequate to describe the variability of some crucial parameters, and small differences in some data in two different scenarios can produce different solutions which differ significantly. In the deterministic approach, a single scenario is taken as the one-time data base at the beginning of the optimisation process and the dynamic problem is converted

into a static multi-period problem. Typically, most of data in model (P_g) can be affected by uncertainty but a high level of uncertainty in WR problems is referred to exogenous inflows and demand patterns.

The last few years have shown just how unpredictable meteorological events can be, especially in the Mediterranean area and, almost always, it is impossible to represent these events by a probabilistic law. The stochastic optimisation approach cannot be adopted since in WR it is unreliable to match a valid occurrence probability to each scenario. Moreover, some experimental tests using historical data, to simulate future planning horizons, have shown just how risky the stochastic decisions on the use of reservoir water in periods of high variation in water availability have been.

The simulation approach studies a number of outcomes obtained by solving an optimisation problem (P_g) for each scenario g. During the optimisation process, different scenarios, corresponding to different dynamic multi-period WR models, proceed independently of each other obtaining a different water management policy for each scenario. Simulation verifies the performance of all policies selecting one for future decisions. Usually, to reach a viable water management policy, a large number of scenarios must be considered. The simulation approach can prove very demanding from computational point of view, especially if continuously replicated when the hydrological events occurring are very different from those foreseen in the selected scenario.

The scenario analysis approach attempts to face the uncertainty factor by taking into account a set, G, of different supposed scenarios corresponding to the different possible time evolution of some crucial data, such as inflows and demands, represented by vector b_g in problem (P_g), for each $g \in G$. Unlike simulation, the different scenarios are considered together to obtain a global set of decision variables on the whole set of scenarios. More precisely, two scenarios sharing a common initial portion of data must be considered together and partially aggregated with the same decision variables for the aggregated part, in order to take into account the two possible evolutions in the subsequent non-common part. In this way, the set of parallel scenarios is aggregated by producing a tree structure, called *scenario-tree*. The aggregation rules guarantee that the solution in any given period is independent of the information not yet available. In other words, model evolution is only based on the information available at the moment and, if necessary, scenario modification is allowed. We give detailed rules for the generation of the *scenario-tree* in subsection 3.1.

The problem supported by the *scenario tree*, is described by a mathematical model that includes all single-scenario problems (P_g), $\forall g \in G$, plus some inter-scenario linking constraints representing the requirement that if two scenarios $g1$ and $g2$ are identical up to time t on the basis of information available at that time, then the corresponding set of decision variables, x_1 and x_2, must be identical up to time t. This means that the subsets of decision variables corresponding to the indistinguishable part of different scenarios must be equal among themselves. Moreover a weight can be assigned to each scenario representing the "importance" assigned by the manager to the running configuration. At times the weights can be viewed as the probability of occurrence of the examined scenario. More often they are determined on the basis of background knowledge about the system.

The resulting mathematical model is named *chance-model* to indicate that it is not stochastically based but, due to the impossibility of adopting probabilistic rules and/or to the necessity of inserting information that cannot be deduced from historical data, it attempts to represent the set of possible performances of the system, as uncertain parameters vary. In subsection 2.2 we describe the *chance mathematical model* produced by the scenario analysis approach.

3.1 Scenario-tree Generation

In order to generalize the scenario approach for WR, we give some general rules to organise a predefined set of scenarios in a *scenario-tree*, and to manage the huge amount of data needed to perform scenario analysis in this field. These *scenario aggregation* rules are independent of the extension of the examined

time horizon, the number of time periods and stages, the number of predefined scenarios and the adopted optimisation techniques.

Figure 3a shows a predefined set G of nine parallel scenarios before aggregation. Each scenario corresponds to a dynamic multi-period network, associated to a synthetic hydrological sequence, as shown in figure 2. In figure 3a each dynamic multi-period network is represented by a sequence of dots where a dot represents the system in a time-period, i.e., a basic graph as in figure 1a. Figure 3b shows an example of the scenario-tree derived from the parallel sequences.

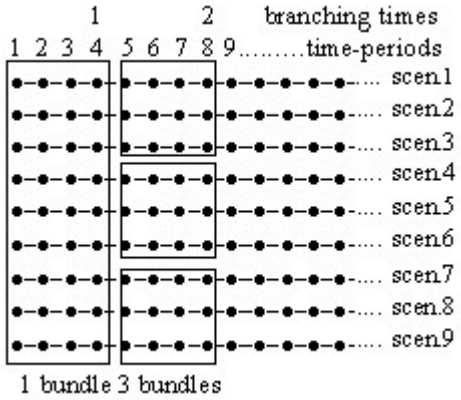

Figure 3a. Set G of nine parallel scenarios.

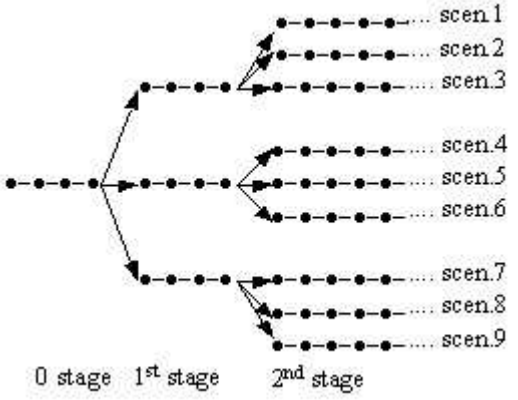

Figure 3b. Scenario-tree aggregation.

To perform scenario aggregation, *branching-times* and *stages* are defined. A *branching-time* τ identifies the time-period in which some scenarios, that are identical up to that time-period, begin to differ. A *stage* corresponds to the sequence of time-periods between two branching-times. Stage 0 corresponds to the initial hydrological characterisation of the system up to the first branch time-period. This represents the root of the scenario-tree. In stage 1 a number β_1 (3 in the figure) of different possible hydrological configurations can occur, in stage 2 a number $\beta_1 * \beta_2$ (9 in the figure) can occur, and so on and so forth.

Figure 3b represents a tree with two branches: the first branching-time is the 4th time-period, the second is the 8th period. In time periods that precede the first branch, all scenarios are gathered in a single bundle and three bundles are operated at second branch. This implies that the zero bundle includes a unique group of all scenarios; in the 1st stage 3 bundles are generated identifying the groups Γ_τ: {(scen1, scen2, scen3); (scen4, scen5, scen6); (scen7, scen8, scen9)} to include in each bundle, while in the 2nd stage the 9 scenarios run until they reach the end of the time-horizon.

Finally, the main rules adopted to organise the set of scenarios are:

Branching: to identify branching-times τ as time-periods at which to bundle parallel sequences, while identifying the stages at which to divide the scenario horizon.

Bundling: to identify the number, β_τ, of bundles at each branching-time.

Grouping: to identify groups, Γ_τ, of scenarios to include in each bundle.

The root of the scenario-tree corresponds to the time at which decisions have been taken (common to all scenarios) and the leaves of the scenario-tree represent the performance of the system in the last stage. Each back path from a leaf to the root identifies a possible scenario.

3.2 The Chance Mathematical Model

The chance model can be expressed as the collection of one deterministic model for each scenario $g \in G$ plus a set of congruity constraints representing the requirement that the subsets of decision variables, corresponding to the indistinguishable part of different scenarios, must be equal among themselves. In this case, the chance mathematical model (P_C) has the following structure:

$$
\begin{aligned}
(P_C) \quad \min \quad & \sum_g w_g \, c_g \, x_g \\
& A_g \, x_g = b_g \qquad \forall \, g \in G \\
& l_g \le x_g \le u_g \qquad \forall \, g \in G \\
& x^* \in S
\end{aligned}
$$

where:

w_g represents the weight assigned to a scenario $g \in G$;

x^* represents the vector of variables submitted to congruity constraints;

$x^* \in S$, the set of congruity constraints.

Congruity constraints require that the flows (decision variables) in those scenarios that are indistinguishable up to branching-time τ are the same up to τ. Specifically, in these scenarios, the water stored in a reservoir at the end of the time τ, to transfer in the period $\tau+1$, must be the same. Of course, these constraints make the model redundant as, thanks to scenario aggregation, some scenarios are overlapped in some time-periods. As a consequence, the components of the model (variables and constraints), that are associated to overlapped scenarios, can be reported only once. To illustrate this, fig. 4 shows a branch corresponding to a reservoir j for two scenarios, $g1$ and $g2$, identical up to the branching-time τ. In particular, congruity constraint requires that the flow (stored volume in a reservoir) from reservoir j at time τ to reservoir j at time $\tau+1$ in scenario $g1$ is equal to the flow from reservoir j at time τ to reservoir j at time $\tau+1$ in scenario $g2$.

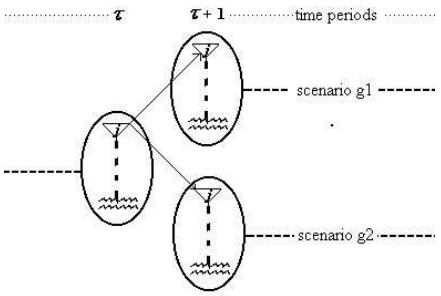

Figure 4. Segment of branch at branching-time τ referred to a reservoir j.

Finally they require that for each τ, decisions taken up to that time must be the same for all scenarios that have a common past and present in such a way that the choices corresponding to different scenarios can be considered truly applied by the manager.

Regarding weight definitions, if the water manager were able to evaluate the weight w_g as the probability that scenario g will occur, he could estimate it by some stochastic technique or statistical test. More often the manager has no, or few, possibilities to do this due to the difficulty in deriving a probabilistic rule from conceptual considerations. Instead, in scenario analysis, a weight w_g assigned to a scenario g can be interpreted as the "relative importance" of that scenario in the uncertain environment. In other words, in scenario analysis, weights are interpreted as subjective parameters assigned on the basis of the experience of the water management board. Different weights can also be assigned to different stages. Then, the definitive weight in the objective function will be calculated considering the contribution of scenarios and stages. A good compromise in weight settlement might be to assign scenario importance on the basis of subjective considerations, and assign weights to stages on the basis of statistical tests.

This kind of model can be solved by decomposition methods such as Benders decomposition or Lagrangian relaxation techniques, which exploit the special structure of constraints. When the problem is huge, it is possible to resort to parallel computing (Glockner and Nemhauser, 2000).

4. Using Decision Support System for Water Resources Management

The DSS has been developed in order to:

- guarantee simplicity of use in the input phase, in scenario setting and in processing output results;
- simplify modification of system configuration to perform sensitivity analysis and to process uncertainty;
- prevent obsolescence of the optimiser exploiting the standard input format in optimisation codes.

A graphical interface allows scenario analysis, starting from a physical system following the main steps:

- time period definition and scenario settlement;
- system element characterisation;
- connections topology and transfer constraints;
- links to hydrological data and demand requirement files;
- planning and management rule definition;
- benefits and costs attribution;
- call to optimisers;
- output processing.

The DSS has been developed and tested within a HP-Unix and PC-Linux environment. The various software components have been coded in C++ and TCL-TK graphic language. The graphical components are introduced on an empty canvas by means of a tool palette. A time range window provides data covering time range and time steps. Once the graph has been completed, the necessary data are inserted in correspondence to nodes and arcs in operation or project mode as required by the physical system.

Different templates are associated with different components. Data can be provided in scalar, cyclic or vector modality. Hydrological input can be directly linked to the appropriate files containing the generated synthetic series to be used by the DSS to generate the scenario-tree. Input data of scenario components are introduced with the appropriate tools defining number of scenarios and setting branches and stages as in subsection 2.1. Branch periods and multiplicity are then defined.

Once the graph has been completed and all necessary data have been inserted, it is possible to generate the MPS (Mathematical Programming System) file to feed to the solver. MPS is now the standard format supported by most efficient commercial and non-commercial Mathematical Programming computer codes. Standard data format allows the use of any solver, possibly the most efficient or the most suitable or the most readily available in the work environment. Moreover, it prevents obsolescence of the used solvers and ensures portability of the DSS. The solver can be selected from a prefixed list (or configured if not in the list) and launched.

A graphical post-processor manages the great quantity of information provided by the DSS. The user can select what is of interest in the environment under examination. Indeed, it is possible to show any flow in any scenarios in any period such as volume transferred to reservoir in a specific scenario, period by period, as well as in different scenarios in the same graph.

4.1 Test Case I: A Sample System

To illustrate the scenario analysis approach we present, initially, a sample case derived from a simple water system with a reservoir and a demand centre (e.g. civil demand). We assume that dimensions of the reservoirs and the demand centre are known, that is the system is in an operational state. Variables of the optimisation problem are referred to stored water y^t_g, flows transferred from reservoir to demand centre x^t_g, deficits u^t_g in each period t and in each scenario g. Water demand p^t_g is assigned as population P is known (see subsection 2.1). We adopt, as historical data, a hydrological series of 48 monthly time-periods (4 years) reproducing typical behaviour of the Mediterranean system: a wide range of inflow variability

between humid and drought periods. Inflows average is adopted as civil demand by the demand centre assuming that the water system is balanced. Moreover, in order to facilitate result interpretation, we assume that the volume, Y_{max}, of the reservoir is large enough to prevent spillage and that evaporation losses are negligible. We generate two scenarios, $g1$ and $g2$, assuming that uncertain parameters correspond to hydrological inflows, inp^t_g, i.e., supplies in reservoir node in period t in scenario g. Costs and bounds associated to variables are considered without uncertainty and, as a consequence, are the same in both scenarios. We generate a scenario-tree with two stages and one branching-time τ. Figure 5 shows the scenario-tree for this simple example.

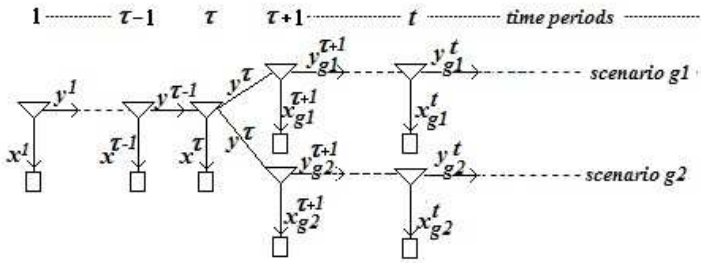

Figure 5. Scenario-tree for the sample system: two scenarios, two stages, one branching-time.

In the figure, variables up to branching time τ are reported without scenario subscribe because they are the same in the two scenarios, as required by congruity constraints. Dummy node U and dummy arcs, corresponding to deficit u^t_g, are not reported in the figure. The two scenarios are both identical to the historical data up to $\tau = 12^{th}$ time-period. Scenario $g2$ follows the historical data from $\tau+1$ to the last time-period, while scenario $g1$ has the hydrological inflows reduced by 50% with respect to it. The scenario-tree is then generated following the aggregation rules in subsection 3.1. Starting from a set of two parallel scenarios, stage 0 represents the initial hydrological configuration from the 1st to the 12th time-period. In stage 1, a number $\beta_1 = 2$ of bundles (different possible hydrological configurations) can occur so that $\Gamma_\tau = \{(\text{scen1}); (\text{scen2})\}$. Then the two scenarios run until they reach the end of the time-horizon. We assume that $w_{g1} = w_{g2} = 1$ and that c^t and α^t represent costs per unit of flow x^t_g and u^t_g respectively, for $g = g1, g2$.

The objective is to minimise operative total costs, that is transfer costs from reservoir to demand centre plus deficit costs. The chance model, for this simple example, following notation in subsection 2.1, can be written as follows:

$$\sum_t (c^t x^t_{g1} + c^t x^t_{g2} + \alpha^t u^t_{g1} + \alpha^t u^t_{g2}) \qquad \textit{objective function}$$

$$\left.\begin{array}{ll} \rho^t_{min} Y_{max} \le y^t_g \le \rho^t_{max} Y_{max} & \textit{bounds on stored volumes} \\ y^{t-1}_g - y^t_g - x^t_g = inp^t_g & \textit{mass balance in reservoir-node} \\ x^t_g + u^t_g = p^t_g & \textit{mass balance in demand-node} \end{array}\right\} \quad t = 2,\ldots,48, \quad g = g1, g2$$

$$\left.\begin{array}{l} y^{t-1}_{g1} = y^t_{g2} \\ x^{t-1}_{g1} = x^t_{g2} \\ u^{t-1}_{g1} = u^t_{g2} \end{array}\right\} \quad \textit{congruity constraints (set S in the chance model)} \quad t = 2,\ldots,\tau$$

Hereunder we illustrate some results concerning stored volumes in reservoir, y^t_g, and transferred water to demand centre, x^t_g, obtained by scenario analysis, solving the above optimisation chance model.

Figure 6 shows stored volumes with $\tau = 12^{th}$ time-period. Moreover the figure shows stored volumes obtained by an optimisation deterministic model when the scarce scenario $g1$ is assumed as database. We call $s1$ this independent scenario. The graph referred to deterministic results with scenario $s1$ represents decisions, in water transfer, in a deterministic optimisation process The zone (band) between the two

graphics of the aggregated scenarios, $g1$ and $g2$, represents the possible decisions that can be taken for stored volumes. Then we can say that the part of $s1$ that does not stay between $g1$ and $g2$ represents the error that the manager would have made if he had adopted decision $s1$.

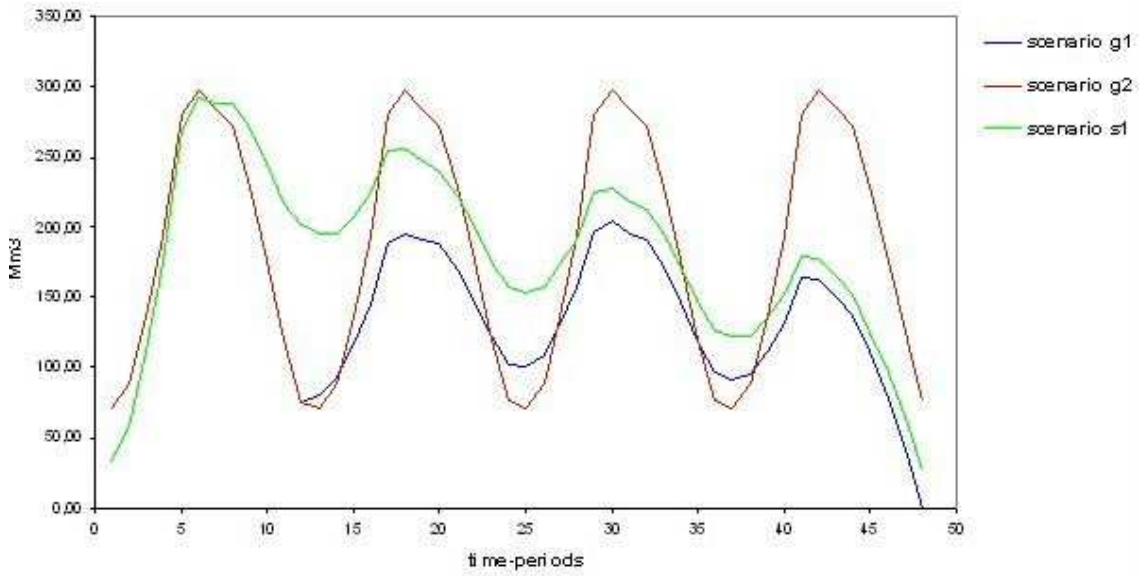

Figure 6. Sample system: stored volumes in the reservoir

Figure 7 shows the transferred water, x'_g, from reservoir to the demand centre in aggregated scenarios, and in the independent scenario. The behaviour of these flows shows that in the scenario $g2$ demand is fulfilled while in scenario $g1$ deficits are present after branching time. But, comparing with results in deterministic optimisation under scenario $s1$, we can see that as regards the scarce resource conditions, scenario optimisation gives a smoother distribution, i.e., with a lower variance of flow distribution in scenario $g1$ even if the average is almost the same as scenario $s1$. Then, the management policy suggested by scenario analysis results less dramatic and more implementable with respect to that adopted in deterministic optimisation in the case of scarce resources. These effects are even more evident in a real wider system as described in the following subsection.

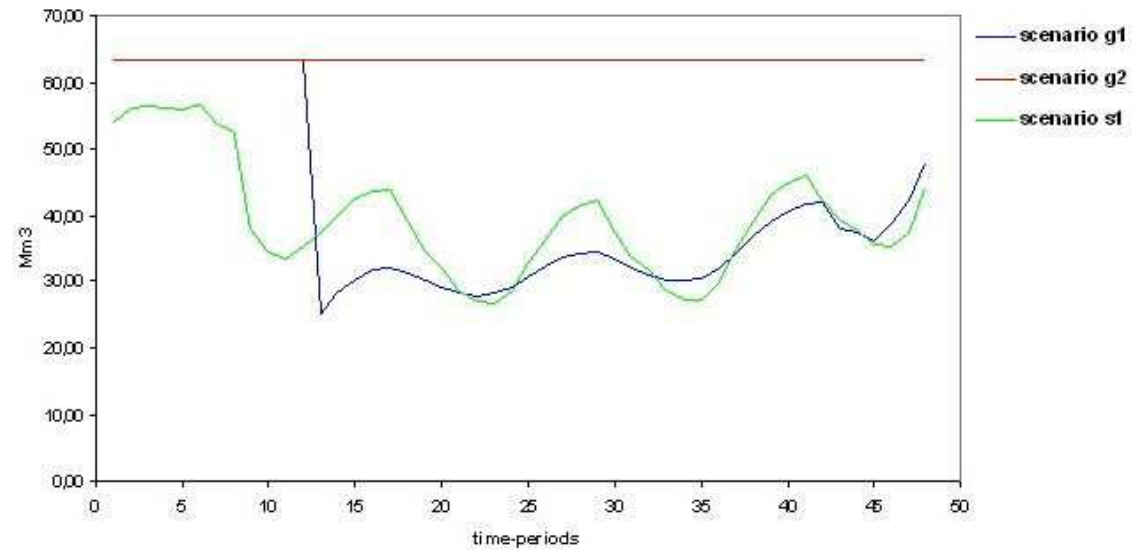

Figure 7. Water transfer from reservoir to the demand centre.

4.2 Test Case II: A Real Physical System

Scenario analysis was performed on the Flumendosa-Campidano-Cixerri (FCC) system in Sardinia, Italy. Figure 8 shows a sketch of the system.

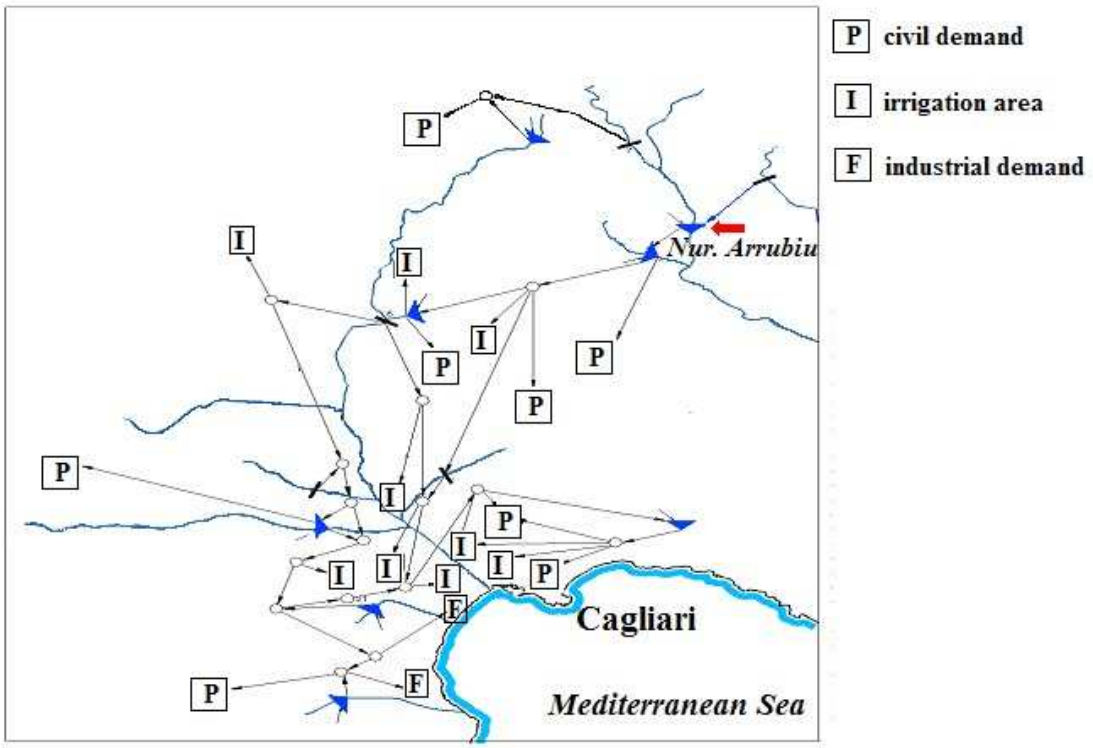

Figure 8. Flumendosa-Campidano-Cixerri system.

Since 1987, the Sardinia-Water-Plan has highlighted the necessity of defining an optimal water works assessment and the urgency of defining optimal management rules for the water system. Correct evaluation of system performances and requirements became increasingly urgent, as system managers were obliged to face serious resource deficits caused by the drought events of the past decade accompanied by an almost total uncertainty in hydrological inflows. The main infrastructures were built in the mid 50's and supply most of southern Sardinia. The main water supply source of the system is represented by the three reservoirs with a total storage capacity of 666.4 million cubic meters (Mm3). Gravity galleries connect the reservoirs. Total yearly average distributed volume in the period examined is 225 Mm3 for civil, industrial and agricultural demands. No significant aquifers are present in the system. The principal works can be identified in 13 water supply sources (dams and weirs), 3 water diversion and communication galleries, 5 main water supply and distribution networks, 2 hydroelectric power stations, 1 irrigation distribution network, 11 pumping stations, and 2 drinking water plants.

A number of synthetic series is generated and adopted as the set of predefined scenarios. The basic hydrological data is derived from the report in RAS (2003) and different scenario generation techniques have been compared.

The graphical interface of the DSS make it possible to design the system following the network component identification like those described in subsection 2.1 and the simple rules reported in subsection 3.1. Figure 9 shows the input window of the system examined.

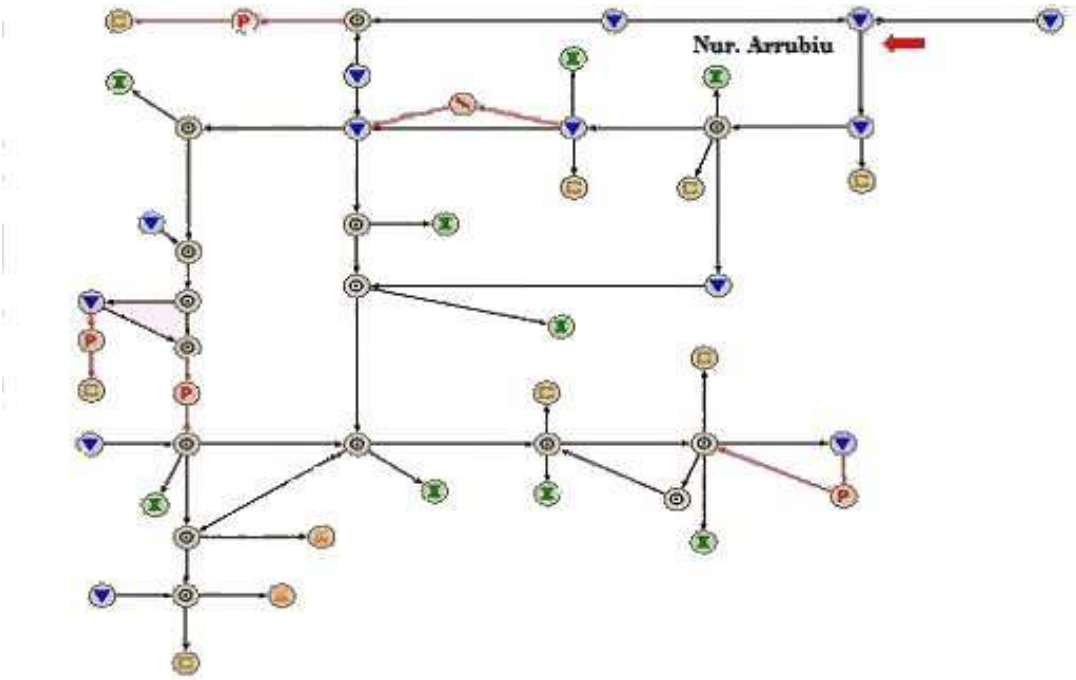

Figure 9. Input window of Flumendosa-Campidano-Cixerri system.

Starting from a data base with a time-horizon up to 75 years, corresponding to 900 monthly time-periods, a set of 30 scenarios was submitted to statistical validation and selected. Scenario analysis was performed on a scenario-tree of 2 and 3 stages up to 30 leaves. Since each scenario involves about 3,000 variables, the chance model supports several thousands of variables and constraints.

In this paper, we report some results selected among a wide set of output information that the DSS provides. In particular, we report some results obtained adopting a time-horizon of 48 time-periods and a branching time in the 12th time period as in the sample system. Illustrated results are referred to the "Nuraghe Arrubiu" reservoir (the red arrow in figures 8 and 9) that is considered one of the main pivots of the system as it can control water transfers to the principal demand centres. Two scenarios are deduced from the last 4 years of hydrological inflows reported in RAS (2003). We adopt these data as scenario $g2$ while scenario $g1$ is derived from it assuming that a reduction of 50% will occur after the branching-time. Figure 10 shows the behaviour of stored volumes obtained by scenario analysis (aggregation of $g1$ and $g2$) and the behaviour obtained by deterministic optimisation using the reduced independent scenario $s1$.

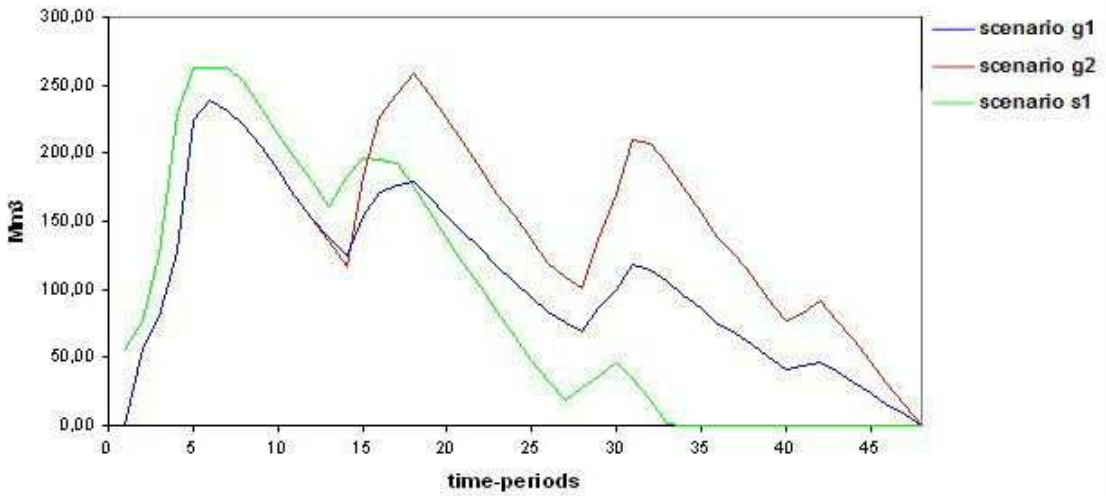

Figure 10. Stored volumes in Nuraghe Arrubiu reservoir.

The figure shows that decision policy, corresponding to deterministic optimisation, induces an early empty reservoir with respect to the decision policy given by scenario analysis in the equivalent scenario $g1$. This corresponds to the behaviour of transferred water to demand centres as illustrated in figure 11. As in the sample system, decision policy $g1$, resulting from scenario analysis, exhibits smoother resources distribution and a lower variance with respect to deterministic policy $s1$.

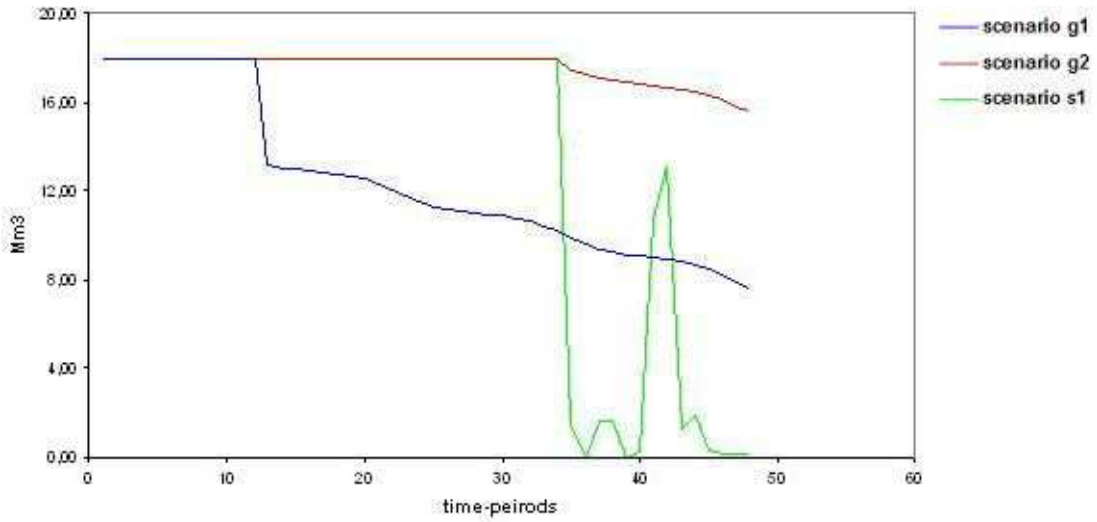

Figure 11. Transfer water from Nuraghe Arrubiu reservoir to demand centres.

5. Conclusion and Perspectives

We presented a general-purpose scenario-modelling framework to solve water system optimisation problems when input data is uncertain, as an alternative to the traditional stochastic approach, in order to achieve a "robust" decision policy that should minimise the risk of wrong decisions. The proposed tool can perform scenario analysis by generating a scenario-tree structure. It allows the exploitation of state-of-the-art efficient computer codes for general-purpose Mathematical Programming supporting some thousands of variables and constraints.

The aim of this paper is to contribute to the mathematical optimisation of water resource systems, when the role of uncertainty is particularly important. In such a problem, which involves social, economical, political, and physical events, there is often no probabilistic description of the unknown elements available, either due to the lack of a substantial statistical base or because it is impossible to derive a probabilistic law from conceptual considerations. The high level of uncertainty regarding hydrological inflows has a dramatic impact on decisions adopted for assigning water resources to demand centres, making long-term management planning difficult. The unusually significant reduction of resources in this last decade has caused onerous deficits in demand fulfilment.

Experimentation has been performed on a real water resource system in Sardinia, Italy, in collaboration with regional water managers showing that practitioners and end-users can adopt the DSS as a useful aid to decision making. Moreover, the application of our approach to a real system demonstrates that it is simple and practical to examine a number of different scenarios and to aggregate them in a scenario tree, allowing accurate sensitivity analysis.

Another primary aim in planning this approach to WR analysis was to create an easy-to-use tool for assisting water managers in a DSS context, incorporating the improvements made in the field of computer science and operation research. Thanks to new methodologies and developments in Computer Science, state-of-the-art Mathematical Programming codes evolve continuously producing algorithms that improve computational efficiency, see CPLEX (2002). The standard input format, MPS, allows inserting the best

state-of-the-art codes in the DSS. Thus, the proposed DSS is a practicable instrument which can be used by WR managers to define programmed reduction of theoretical resource demand, in order to achieve optimal system management, both to fulfil consumers' needs and minimise the cost deficit. In fact, as WR managers know well, the programming of deficits makes it possible to set up adequate preventive measures, which permit a notable reduction of management costs in the event of resource scarcity.

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