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Abstract. A simple type confusion attack occurs in a security protocol, when a principal interprets data of one type as data of another. These attacks can be successfully prevented by "tagging" types of each field of a message. Complex type confusions occur instead when tags can be confused with data and when fields or sub-segments of fields may be confused with concatenations of fields of other types. Capturing these kinds of confusions is not easy in a process calculus setting, where it is generally assumed that messages are correctly interpreted. In this paper, we model in the process calculus LySa only the misinterpretation due to the confusion of a concatenation of fields with a single field, by extending the notation of one-to-one variable binding to multi-to-one binding. We further present a formal way of detecting these possible misinterpretations, based on a Control Flow Analysis for this version of the calculus. The analysis over-approximates all the possible behaviour of a protocol, including those effected by these type confusions. As an example, we considered the amended Needham-Schroeder symmetric protocol, where we succeed in detecting the type confusion that lead to a complex type flaw attacks it is subject to. Therefore, the analysis can capture potential type confusions of this kind on security protocols, besides other security properties such as confidentiality, freshness and message authentication.

1 Introduction

In the last decades, formal analyses of cryptographic protocols have been widely studied and many formal methods have been put forward. Usually, protocol specification is given at a very high level of abstraction and several implementation aspects, such as the cryptographic ones, are abstracted away. Despite the abstract working hypotheses, many attacks have been found that are independent of these aspects. There are situations in which this abstract view is not completely adequate, though. At a high level, a message in a protocol consists of fields: each represents some value, such as the name of a principal, a nonce or a key. This structure can be easily modelled by a process calculus. Nevertheless, at a more concrete level, a message is nothing but a raw sequence of bits. In this view, the recipient of a message has to decide the interpretation of the

bit string, i.e. how to decompose the string into substrings to be associated to the expected fields (of the expected length) of the message. The message comes with no indication on its arity and on the types of its components. This source of ambiguity can be exploited by an intruder that can fool the recipient into accepting as valid a message different from the expected one. A *type confusion attack* arises in this case.

A *simple* type confusion attack occurs when a field is confused with another [14]. To prevent such attacks, the current techniques [11] systematically associate message fields with tags representing their intended type. On reception of messages, honest participants check tags so that fields with different types cannot be mixed up. Nevertheless, as stated by Meadows [15], simple tags could not suffice for more complex type confusion attacks: "in which tags may be confused with data, and terms of pieces of terms of one type may be confused with concatenations of terms of several other types." Tags should also provide the length of tagged fields.

Here, we are interested in semantically capturing attacks that occur when a concatenation of fields is confused with a single field [21]. Suppose, e.g. that the message pair (A, N), where A is a principal identity and N is a fresh nonce, is interpreted as a key K, from the receiver of the message. For simplicity, we call them *complex type confusion attacks*. This level of granularity is difficult to capture with a standard process calculus. In a process algebraic framework, there is no way to confuse a term (A, N) with a term K. The term is assumed to abstractly model a message, plugging in the model the hypothesis that the message is correctly interpreted. In concrete implementation this confusion is instead possible, provided that the two strings have the same length.

As a concrete example, consider the Amended Needham Schroeder symmetric key protocol [7]. The protocol aims at distributing a new session key K between two agents, A and B, via a trusted server S. It is assumed that initially each agent shares a long term key, K_A and K_B respectively, with the server. The protocol narration is reported in Fig. 1 (a). In messages 1 and 2, Alice (A)

$$
\begin{array}{l}
1.\ A \rightarrow B : A \\
2.\ B \rightarrow A : \{A, N_B\}_{K_B} \\
3.\ A \rightarrow S : A, B, N_A, \{A, N_B\}_{K_B} \\
4.\ S \rightarrow A : \{N_A, B, K, \{K, N_B, A\}_{K_B}\}_{K_A} \\
5.\ A \rightarrow B : \{K, N_B, A\}_{K_B} \\
6.\ B \rightarrow A : \{N\}_K \\
7.\ A \rightarrow B : \{N-1\}_K
\end{array}
$$

(a)

$$
\begin{array}{lll}
1. & A \rightarrow B : & A \\
2. & B \rightarrow A : & \{A, N_B\}_{K_B} \\
3. & A \rightarrow S : & A, B, N_A, \{A, N_B\}_{K_B} \\
1'. & M \rightarrow A : & N_A, B, K' \\
2'. & A \rightarrow M : & \{N_A, B, K', N'_A\}_{K_A} \\
4. & M(S) \rightarrow A : & \{N_A, B, K', N'_A\}_{K_A} \\
5. & A \rightarrow M(B) : & N'_A \\
6. & M(B) \rightarrow A : & \{N\}_{K'} \\
7. & A \rightarrow M : & \{N-1\}_{K'}
\end{array}
$$

(b)

Fig. 1. Amended Needham-Schroeder Symmetric Protocol: Protocol Narration (a) and Type Flaw Attack (b)

initiates the protocol with Bob (B). In message 3 the server generates a new session key K, that is distributed in messages 4 and 5. Nonces created by A

and B are used to check freshness of the new key. Finally, messages 6 and 7 are for mutual authentication of A and B: a new nonce N is generated by B and exchanged with A, encrypted with the new session key K.

The protocol is vulnerable to a complex type flaw attack, discovered by B. W. Long [12] and shown in Fig. 1 (b). The attack requires two instances of the protocol, running in parallel. In one, A plays the roles of initiator and in the other that of responder. In the first instance, A initiates the protocol with B. In the meanwhile, the attacker, M, initiates the second instance with A and sends the triple N_A, B, K' to A (in step $1'$). The nonce N_A is a copy from step 3 in the first instance and K' is a faked key generated by the attacker. A will generate and send out the encryption of the received fields, N_A, B, K', and a nonce N'_A. The attacker $M(S)$ impersonates S and replays this message to A in the first instance. A decrypts this message, checks the nonce N_A and the identity B, and accepts K' as the session key, which is actually generated by the attacker. After completing the challenge and response in step 6 and 7, A will communicate with the attacker using the faked key K'.

Our idea is to explore complex type confusion attacks, by getting closer to the implementation, without leaving the comfortable borders of process calculi. To this aim, we formally model the possible misinterpretations between terms and concatenations of terms. More precisely, we extend the notation of one-to-one variable binding to multi-to-one binding in the process calculus LySa [3], that we use to model security protocols. The Control Flow Analysis soundly over-approximates the behaviour of protocols, by collecting the set of messages than can be sent over the network, and by recording which values variables may be bound to. Moreover, at each binding occurrence of a variable, the analysis checks whether there is any multi-to-one binding possible and records it as a binding violation. The approach is able to detect complex type confusions possibly leading to attacks in cryptographic protocols.

The paper is organized as follows. In Section 2, we present the syntax and semantics of the LySa calculus. In Section 3 we introduce the Control Flow Analysis and we describe the Dolev-Yao attacker we use in our setting. Moreover, we make an experiment in analysing the amended Needham-Schoreder symmetric key protocol. Section 4 concludes the paper.

2 The LySa Calculus

The LySa calculus [3] is a process calculus, designed especially for modelling cryptographic protocols. in the tradition of the π- [18] and Spi- [1] calculi. It differs from these essentially in two aspects. The first is the absence of channels: all processes have only access to a single global communication channel, the network. The second aspect concerns the inclusion of pattern matching into the language constructs where values can become bound to values, i.e. into input and into decryption.

Syntax In LySa, the basic building blocks are values, $V \in Val$, which correspond to *closed* terms, i.e. terms without free variables. Values are used to rep-

resent keys, nonces, encrypted messages, etc. Syntactically, they are described by expressions $E \in Expr$ (or terms) that may either be variables, names, or encryptions. Variables and names come from two disjoint sets Var, ranged over by x, and $Name$, ranged over by n, respectively. Finally, expressions may be encryptions of a k-tuple of other expressions, in which case, E_0 is the key used to perform the encryption. LySa expressions are, in turn, used to construct LySa processes $P \in Proc$ as shown below. Here, we assume perfect cryptography.

$$
\begin{aligned}
E &::= n \mid x \mid \{E_1, \ldots, E_k\}_{E_0} \\
P &::= \langle E_1, \ldots, E_k \rangle.P \mid (E_1, \ldots, E_j; x_{j+1}, \ldots, x_k)^l.P \\
&\quad\ \mathsf{decrypt}\ E\ \mathsf{as}\ \{E_1, \ldots, E_j; x_{j+1}, \ldots, x_k\}^l_{E_0}\ \mathsf{in}\ P \mid \\
&\quad\ (\nu\ n)P \mid P_1|P_2 \mid !P \mid 0
\end{aligned}
$$

The set of free variables, resp. free names, of a term or a process is defined in the standard way. As usual we omit the trailing 0 of processes.

In addition to the classical constructs for composing processes, LySa contains an input and a decryption construct with matching. The label l from a numerable set Lab ($l \in Lab$) in the input and in the decryption constructs uniquely identifies each input and decryption point, resp., and is mechanically attached. Patterns are in the form $(E_1, \cdots, E_j; x_{j+1}, \cdots, x_k)$ and are matched against k-tuples of values $(E'_1, \cdots, E'_k)$. The intuition is that the matching succeeds when the first $1 \leq i \leq j$ values E'_i pairwise correspond to the values E_i, and the effect is to bind the remaining $k - j$ values to the variables $x_{j+1}, \cdots, x_k$. Syntactically, this is indicated by a semi-colon that separates the components where matching is performed from those where only binding takes place. For example, let $P = \mathsf{decrypt}\ \{y\}_K\ \mathsf{as}\ \{x;\}^l_K\ \mathsf{in}\ P'$ and $Q = \mathsf{decrypt}\ \{y\}_K\ \mathsf{as}\ \{;x\}^l_K\ \mathsf{in}\ Q'$. While the decryption in P succeeds only if $x = y$, the one in Q always does,binding x to y.

Extended LySa As seen above, in LySa, values are passed around among processes through pattern matching and variable binding. This is the way for modelling how principals acquire knowledge from the network, by reading messages (or performing decryptions), provided they have certain format forms. A requirement for pattern matching is that patterns and expressions are of the same length: processes only receive (or decrypt) messages, whose length is exactly as expected and each variable is binding to one single value, later on as *one-to-one* binding. This constraint, however, implicitly removes the possibility of modelling complex type confusions, i.e. the possibility to accept a concatenation of fields as a single one. It has to be relaxed. Consider the complex type flaw attack on the amended Needham-Schroder protocol, shown in the Introduction. The principal A, in the role of responder, is fooled by accepting N_A, B, K' as the identity of the initiator and generates the encryption $\{N_A, B, K', N'_A\}_{K_A}$, which will be replayed by the attacker later on in the first instance. In LySa, A's input can be roughly expressed as $(; x_b)$, as she is expecting a single field representing the identity of the initiator of the protocol. Because of the length requirement, though, x_b can only be binding to a single value and not to a concatenation of values, such as (N_A, B, K') object of the output of the attacker.

To model complex type confusions, we need to allow a pattern matching to succeed also in the cases in which the length of lists is different. The extension of the notation of pattern matching and variable binding, will be referred as *multi-to-one* binding. Patterns are then allowed to be matched against expressions with *at least* the same number of elements. A single variable can then be bound also to a concatenation of values. Since there may be more values than variables, we partition the values into groups (or lists) such that there are the same number of value groups and variables. Now, each group of values is bound to the corresponding variable. In this new setting, the pattern in A's input $(;x_b)$ can instead successfully match the expression in the faked output of the attacker $\langle N_A, B, K' \rangle$ and result in the binding of x_b to the value (N_A, B, K').

We need some auxiliary definitions first. The domain of *single values* is built from the following grammar and represents closed expressions (i.e. without free variables), where each value is single, i.e. it is not a list of values. In other words, no multi-to-one binding has affected the expression. These are the values used in the original LySa semantics.

$val \ni v ::= n \mid \{v_1, \ldots, v_k\}_{v_0}$.

General values are closed expressions, where each value V can be a list of values $(V_1, .., V_n)$. These values are used to represent expressions closed after at least one multi-to-one-binding and are the values our semantics handles.

$Val \ni V ::= v \mid (V_1, .., V_n) \mid \{V_1, .., V_n\}_{V_0}$

To perform meaningful matching operations between lists of general values, we first flatten them, thus obtaining *flattened values* that can be either single values v or encryptions of general values.

$Flat \ni T := v \mid \{V_1, .., V_n\}_{V_0}$

Flattening is obtained by using the following *Flatten* function $Fl : Val \rightarrow Flat$

- $Fl(v) = v$;
- $Fl((V_1, .., V_n)) = Fl(V_1), ..., Fl(V_n)$;
- $Fl(\{V_1, .., V_n\}_{V_0}) = \{V_1, .., V_n\}_{V_0}$.

Example 1. $Fl(((n_1, n_2), (\{m_1, (m_2, m_3)\}_{m_0}))) = n_1, n_2, \{m_1, (m_2, m_3)\}_{m_0}$

The idea is that encryptions cannot be directly flattened when belonging to a list of general values. Their contents are instead flattened when received and analysed in the decryption phase.

To perform multi-to-one bindings, we resort to a partition operator $\prod_k$ that, given a list of flattened values $(T_1, \ldots, T_n)$, returns all the possible partitions composed by k *non-empty* groups (or lists) of flattened values. For simplicity, we use $\widetilde{T}$ to represent a list of flattened values $(T_1, \ldots, T_j)$

$$\prod_k (T_1, \ldots, T_n) = \begin{cases} \{(\widetilde{T_1}, ..., \widetilde{T_k}) \mid \forall i : \widetilde{T_i} \neq \emptyset \wedge Fl((\widetilde{T_1}, ..., \widetilde{T_k})) = (T_1, \ldots, T_n)\} & \text{if } n \geq k \\ undefined & \text{if } n < k \end{cases}$$

Note that in case $n \geq k$, the function returns a set of results satisfying the condition. Now binding k variables $x_1, ..., x_k$ to n flattened values amounts to

partitioning the values into k (the number of variables) non-empty lists of flattened values, $(\widetilde{T_1}, ..., \widetilde{T_k}) \in \prod_k(T_1, \ldots, T_n)$, and binding variables x_i to the corresponding list $\widetilde{T_i}$.

Example 2. Consider the successful matching of $(m; x_1, x_2)$ against (m, n_1, n_2, n_3). Since $\prod_2(n_1, n_2, n_3) = \{((n_1), (n_2, n_3)), ((n_1, n_2), (n_3))\}$, it results in two possible effects (recall that for each i, $\widetilde{T_i}$ must be non-empty),

- binding variable x_1 to (n_1) and binding variable x_2 to (n_2, n_3), or
- binding variable x_1 to (n_1, n_2) and binding variable x_2 to (n_3).

Finally, we define the relation $=^F$ as the least equivalence over Val and (by overloading the symbol) and over $Flat$ that includes:
- $v =^F v'$ iff $v = v'$;
- $(V_1, ..., V_k) =^F (V'_1, ..., V'_n)$ iff $Fl(V_1, ..., V_k) = Fl(V'_1, ..., V'_n)$;
- $\{V_1, ..., V_k\}_{V_0} =^F \{V'_1, ..., V'_n\}_{V'_0}$ iff $Fl(V_1, ..., V_k) = Fl(V'_1, ..., V'_n)$ and $Fl(V_0) = Fl(V'_0)$;

Semantics LySa has a reduction semantics, based on a standard structural equivalence. The *reduction relation* $\rightarrow_{\mathcal{R}}$ is the least relation on closed processes that satisfies the rules in Tab. 1.

At run time, the complex type confusions are checked by a reference monitor, which aborts when there is a possibility that a concatenation of values is bound to a single variable. We consider indeed two variants of the *reduction relation* $\rightarrow_{\mathcal{R}}$, graphically identified by a different instantiation of the relation $\mathcal{R}$, which decorates the transition relation. The first variant takes advantage of checks on type confusions, while the other one discards them: essentially, the first semantics checks for the presence of complex type confusions. More precisely, the reference monitor performs its checks at each binding occurrence, i.e. when the pattern $V_1, \ldots, V_k$ is matched against $V'_1, \ldots, V'_k; x_j, ..., x_t$. Both the lists of values are flattened and result in $T_1, \ldots, T_s$ and $T'_1, \ldots, T'_{len}$, respectively. The reference monitor checks whether the length of the list $len + (t - j)$ of the flattened values of the pattern, corresponds to the length s of the list of the general values to match against it. If $(len + t - j) = s$ then there is a correspondence one-to-one between variables and flattened values. Otherwise, then there exists at least a variable x_i, which may bind to a list of more than one value. Formally:
- the reference monitor semantics, $P \rightarrow_{\mathsf{RM}} P'$, takes $\mathcal{R} = \mathsf{RM}(s, len + t - j)$ true when $s = len + t - j$, where s and $len + t - j$ are defined as above;
- the standard semantics, $P \rightarrow P'$ takes $\mathcal{R}$ to be universally true.

The rule (Com) in the Tab. 1 states when an output $\langle V_1, \ldots, V_k \rangle.P$ is matched by an input $(V'_1, \ldots, V'_j; x_{j+1}, \ldots, x_t)^l.P'$. It requires that: (i) the first j general values of the input pattern $V'_1, \ldots, V'_j$ are flattened into len flattened values $T'_1, \ldots, T'_{len}$; (ii) the general values $V_1, \ldots, V_k$ in the output tuple are flattened into s flattened values $T_1, \ldots, T_s$; (iii) if $s \geq (len+t-j)$ and the first len values of $T_1, \ldots, T_s$ pairwise match with $T'_1, \ldots, T'_{len}$ then the matching succeeds; (iv) in this case, the remaining values $T_{len+1}, \ldots, T_s$ are partitioned into a sequence of

(Com)

$$\frac{\wedge_{i=1}^{len} T_i =^F T'_i \ \wedge \ \mathcal{R}(s, len+t-j)}{\langle V_1,\ldots,V_k\rangle.P \mid (V'_1,\ldots,V'_j; x_{j+1},\ldots,x_t)^l.P' \rightarrow_{\mathcal{R}} P \mid P'[\widetilde{T}_{j+1}/x_{j+1},\ldots,\widetilde{T}_t/x_t]}$$

where (A) holds

(Dec)

$$\frac{V_0 =^F V'_0 \ \wedge \ \wedge_{i=1}^{len} T_i =^F T'_i \ \wedge \ \mathcal{R}(s, len+t-j)}{\mathsf{decrypt}\ \{V_1,\ldots,V_k\}_{V_0}\ \mathsf{as}\ \{V'_1,\ldots,V'_j; x_{j+1},\ldots,x_t\}^l_{V'_0}\ \mathsf{in}\ P \rightarrow_{\mathcal{R}} P[\widetilde{T}_{j+1}/x_{j+1},\ldots,\widetilde{T}_t/x_t]}$$

where (A) holds

(New)

$$\frac{P \rightarrow_{\mathcal{R}} P'}{(\nu\ n)P \rightarrow_{\mathcal{R}} (\nu\ n)P'}$$

(Par)

$$\frac{P_1 \rightarrow_{\mathcal{R}} P'_1}{P_1 \mid P_2 \rightarrow_{\mathcal{R}} P'_1 \mid P_2}$$

(Congr)

$$\frac{P \equiv P' \ \wedge \ P' \rightarrow_{\mathcal{R}} P'' \wedge P'' \equiv P'''}{P \rightarrow_{\mathcal{R}} P'''}$$

$$A = \left\{ \begin{array}{ll} Fl((V_1,\ldots,V_k)) & = T_1,\ldots,T_{len},T_{len+1},\ldots,T_s \\ Fl(V'_1,\ldots,V'_j)) & = T'_1,\ldots,T'_{len} \\ (\widetilde{T}_{j+1},\ldots,\widetilde{T}_t) & \in \prod_{t-j}(T_{len+1},\ldots,T_s) \end{array} \right\}$$

Table 1. Operational Semantics; $P \rightarrow_{\mathcal{R}} P'$, parameterised on $\mathcal{R}$.

non-empty lists $\widetilde{T}_i$, whose number is equal to the one of the variables (i.e. $t-j$), computed by the operator $\prod_{t-j}$. Furthermore, the reference monitor checks for the possibility of multi-to-one binding, i.e. checks whether $s \geq (len+t-j)$. If this is the case, it aborts the execution. Note that, if instead $s = (len+t-j)$, then $Fl(V_1,\ldots,V_k) = V_1,\ldots,V_k$, $Fl(V'_1,\ldots,V'_j) = V'_1,\ldots,V'_j$, $k = s$, and $j = len$.

The rule (Dec) performs pattern matching and variable binding in the same way as in (Com), with the following additional requirement: the keys for encryption and decryption have to correspond, i.e. $V_0 =^F V'_0$. Similarly, the reference monitor aborts the execution if multi-to-one binding occurs.

The rules (New), (Par) and (Congr) are standard.

As for the dynamic property of the process, we say that a process is *complex type coherent*, when there is no complex type confusions, i.e. there is no multi-to-one binding in any of its executions. Consequently, the reference monitor will never stop any execution step.

Definition 1 (Complex Type Coherence). *A process P is* complex type coherent *if for all the executions $P \rightarrow^* P' \rightarrow P''$ whenever $P' \rightarrow P''$ is derived using as axiom*
- (Com) on $\langle V_1,\ldots,V_k\rangle.Q \mid (V'_1,\ldots,V'_j; x_{j+1},\ldots,x_t)^l.Q'$ or using
- (Dec) on $\mathsf{decrypt}\ \{V_1,\ldots,V_k\}_{V_0}\ \mathsf{as}\ \{V'_1,\ldots,V'_j; x_{j+1},\ldots,x_t\}^l_{V'_0}\ \mathsf{in}\ Q$
it is always the case that $s = len+t-j$, where $Fl(V_p,\ldots,V_k) = T_p,...,T_s$ and $Fl(V'_p,\ldots,V'_j) = T_p,...,T_{len}$ with $p = 1$ ($p = 0$) in the case of (Com), (Dec), respectively.

3 The Control Flow Analysis

Our analysis aims at giving a safe over-approximation of the protocol behaviour and safely approximating when the reference monitor may abort the computation. The Control Flow Analysis describes a protocol behaviour by collecting all the communications that a process may participate in. In particular, the analysis records which value tuples may flow over the network (see the analysis component κ below) and which value variables may be bound to (component ρ). This gives information on bindings due to pattern matching. Moreover, at each binding occurrence, the Control Flow Analysis checks whether there is any multi-to-one binding possible, and records it as a binding violation (component ψ). Formally, the approximation, or *estimate*, is a triple (ρ, κ, ψ) (respectively, a pair (ρ, θ) when analysing an expression E) that satisfies the judgements defined by the axioms and rules in Tab. 3.

Canonical Names Both the analysis components κ and ρ have to do with recording values in some format. However, a LySa process may generate infinitely many values during an execution because of the restriction and replication constructs, e.g. $!(\nu\ n)\langle n\rangle$, which means that the analysis components have to be able to record infinitely many names. For keeping the estimates finite, we partition all the names used by a process into finitely many equivalence classes, and we use a representative, called *canonical name*, of each of them, instead of the actual names. Consequently there are only finitely many canonical names in any execution of a given process. This is enforced by using in the structural congruence a *disciplined* α-equivalence that allows for substituting names only within the same equivalence class. Notationally, the canonical name $\lfloor n \rfloor$ is for a name n. Similarly, $\lfloor x \rfloor$ is the canonical variable of a variable x. Hereafter, when unambiguous, we shall simply write n (resp. x) for $\lfloor n \rfloor$ (resp. $\lfloor x \rfloor$).

Analysis of Expressions For each expression E, our analysis will determine a superset of the possible values it may evaluate to. For this, the analysis keeps track of the potential values of variables, by recording them into the global *abstract environment*:

• $\rho : \mathcal{X} \rightarrow \mathcal{P}(Val)$ that maps the variables to the sets of general values that they may be bound to, i.e. if $a \in \rho(x)$ then x may take the value a.

The judgement for expressions takes the form $\rho \models E : \vartheta$ where $\vartheta \subseteq Val^*$ is an acceptable *estimate* (i.e. a sound over-approximation) of the set of general value lists that E may evaluate to in the environment ρ. The judgement is defined by the axioms and rules in the upper part of Tab. 3. Basically, the rules demand that ϑ contains all the value lists associated with the components of a term, e.g. a name n evaluates to the set ϑ, provided that n belongs to ϑ; similarly for a variable x, provided that ϑ includes the set of value lists $\rho(x)$ to which x is associated with.

The rule (Enc) (i) checks the validity of estimates θ_i for each expression E_i; (ii) requires that all the values $T_1, ..., T_s$ obtained by flattening the k-tuples $V_1, ..., V_k$, such that $V_i \in \theta_i$, are collected into values of the form $(\{T_1, \cdots, T_s\}^l_{V_0})$, (iii) requires these values to belong to ϑ.

$$\text{(Name)}\ \frac{(n) \in \vartheta}{\rho \models n : \vartheta} \qquad \text{(Var)}\ \frac{\rho(x) \subseteq \vartheta}{\rho \models x : \vartheta}$$

$$\text{(Enc)}\ \frac{\begin{array}{c} \wedge_{i=0}^{k}\ \rho \models E_i : \vartheta_i\ \wedge \\ \forall V_0, \ldots, V_k : \wedge_{i=0}^{k}\ V_i \in \vartheta_i\ \wedge\ Fl(V_1, ..., V_k) = T_1, ..., T_s \Rightarrow \\ (\{T_1, ..., T_s\}_{V_0}) \in \vartheta \end{array}}{\rho \models \{E_1, \ldots, E_k\}_{E_0} : \vartheta}$$

$$\text{(Out)}\ \frac{\begin{array}{c} \wedge_{i=1}^{k}\ \rho \models E_i : \vartheta_i\ \wedge \\ \forall V_1, \ldots, V_k : \wedge_{i=1}^{k} V_i \in \vartheta_i\ \wedge\ Fl(V_1, ..., V_k) = T_1, ..., T_s \Rightarrow \\ \langle T_1, ..., T_s \rangle \in \kappa\ \wedge\ (\rho, \kappa) \models P : \psi \end{array}}{(\rho, \kappa) \models \langle E_1, \ldots, E_k \rangle.P : \psi}$$

$$\text{(In)}\ \frac{\begin{array}{c} \wedge_{i=1}^{j}\ \rho \models E_i : \vartheta_1\ \wedge \\ \forall V'_1, \ldots, V'_j : \wedge_{i=1}^{j} V'_i \in \vartheta_i\ \wedge\ Fl(V'_1, ..., V'_j) = T'_1, ..., T'_{len} \Rightarrow \\ \forall \langle T_1, ..., T_s \rangle \in \kappa : T_1, ..., T_{len} =^F T'_1, ..., T'_{len} \Rightarrow \\ \forall (\widetilde{T}_{j+1}, \ldots, \widetilde{T}_t) \in \prod_{t-j}(T_{len+1}, \ldots, T_k) \Rightarrow \\ (\wedge_{i=j+1}^{t}\ \widetilde{T}_i \in \rho(x_i)\ \wedge (s > len + t - j) \Rightarrow l \in \psi\ \wedge\ (\rho, \kappa) \models P : \psi) \end{array}}{\begin{array}{c} (\rho, \kappa) \models (E_1, \ldots, E_j; x_{j+1}, \ldots, x_t)^l.P : \psi \\ \text{where } s \geq len + t - j \end{array}}$$

$$\text{(Dec)}\ \frac{\begin{array}{c} \rho \models E : \vartheta\ \wedge\ \wedge_{i=0}^{j}\ \rho \models E_i : \vartheta_i \wedge \\ \forall V'_0, \ldots, V'_j : \wedge_{i=0}^{j} V'_i \in \vartheta_i\ \wedge\ Fl(V'_1, ..., V'_j) = T'_1, ..., T'_{len} \Rightarrow \\ \forall \{T_1, \ldots, T_s\}_{V_0} \in \vartheta : T_1, ..., T_{len} =^F T'_1, ..., T'_{len} \Rightarrow \\ \forall (\widetilde{T}'_{j+1}, \ldots, \widetilde{T}'_t) \in \prod_{t-j}(T_{len+1}, \ldots, T_k) \Rightarrow \\ (\wedge_{i=j+1}^{t}\ \widetilde{T}'_i \in \rho(x_i)\ \wedge (s > len + t - j) \Rightarrow l \in \psi\ \wedge\ (\rho, \kappa) \models P : \psi) \end{array}}{\begin{array}{c} (\rho, \kappa) \models \mathsf{decrypt}\ E\ \mathsf{as}\ \{E_1, \ldots, E_j; x_{j+1}, \ldots, x_t\}_{E_0}^{l}\ \mathsf{in}\ P : \psi \\ \text{where } s \geq len + t - j \end{array}}$$

$$\text{(New)}\ \frac{(\rho, \kappa) \models P : \psi}{(\rho, \kappa) \models (\nu\ n)P : \psi} \qquad \text{(Par)}\ \frac{(\rho, \kappa) \models P_1 : \psi\ \wedge\ (\rho, \kappa) \models P_2 : \psi}{(\rho, \kappa) \models P_1 | P_2 : \psi}$$

$$\text{(Rep)}\ \frac{(\rho, \kappa) \models P : \psi}{(\rho, \kappa) \models !P : \psi} \qquad \text{(Nil)}\ (\rho, \kappa) \models 0 : \psi$$

Table 2. Analysis of terms; $\rho \models E : \vartheta$, and processes: $(\rho, \kappa) \models P : \psi$

Analysis of Processes In the analysis of processes, we focus on which tuples of values can flow on the network:

• $\kappa \subseteq \mathcal{P}(Val^*)$, the *abstract network environment*, includes all the tuples forming a message that may flow on the network, e.g. if the tuple $\langle a, b \rangle$ belongs to κ then it can be sent on the network.

The judgement for processes has the form: $(\rho, \kappa) \models P : \psi$, where ψ is the possibly empty set of "error messages" of the form l, indicating a binding violation at the point labelled l. We prove in Theorem 2 below that when $\psi = \emptyset$ we may do without the reference monitor. The judgement is defined by the axioms

and rules in the lower part of Tab. 3 (where $A \Rightarrow B$ means that B is analysed only when A is evaluated to be *true*) and are explained below.

We now briefly comment on the rules. The rule for *output* (Out), computes all the messages that can be obtained by flattening all the general values to which sub-expressions may be evaluated to. The use of the flatten function makes sure that each message is plain-structured, i.e. redundant parentheses are dropped. More precisely, it (i) checks the validity of estimates θ_i for each expression E_i; (ii) requires that all the values obtained by flattening the k-tuples $V_1, ..., V_k$, such that $V_i \in \theta_i$, can flow on the network, i.e. that they are in the component ρ; (iii) requires that the estimate (ρ, κ, ψ) is valid also for the continuation process P. Suppose e.g. to analyse $\langle A, N_A \rangle.0$. In this case, we have that $\rho \models A : \{(A)\}$, $\rho \models N_A : \{(N_A)\}$, $Fl((A), (N_A)) = A, N_A$ and $\langle A, N_A \rangle \in \kappa$. Suppose instead to have $\langle A, x_A \rangle.P$ and $\rho(x_A) = \{(N_A), (N'_A)\}$. In this case we have $Fl((A), (N_A)) = A, N_A$, $Fl((A), (N'_A)) = A, N'_A$, $\langle A, N_A \rangle \in \kappa$ and also $\langle A, N'_A \rangle \in \kappa$.

The rule for *input* (In) basically looks up in κ for matched tuples and performs variable binding before analysing the continuation process. This is done in the following steps: the rule (i) evaluates the first j expressions, whose results are general values, V'_i. These are flattened into a list of values $T'_1, ..., T'_{len}$ in order to perform the pattern matching. Then, the rule (ii) checks whether the first len values of any message $\langle T_1, ..., T_s \rangle$ in κ (i.e. any message predicted to flow on the network) matches the values from previous step, i.e. $T'_1, ..., T'_{len}$. Also, the rule (iii) partitions the remaining $T_{len+1}, ..., T_s$ values of the tuple $\langle T_1, ..., T_s \rangle$ in all the possible ways to obtain $t - j$ lists of flattened values $\widetilde{T_i}$ and requires each list is bound to the corresponding variable $\widetilde{T_i} \in \rho(x_i)$. The rule (iv) checks whether the flattened pattern and the flattened value are of the same length. If this is not the case, the final step should be in putting l in the error component ψ. Finally, the rule (v) analyses the continuation process. Suppose to analyse the process $(A, x_A; x, x_B).0$, where $\langle A, N_A, B, N_B \rangle \in \kappa$ and $(N_A) \in \rho(x_A)$. Concretising the rule (Inp) gives $j = 2, t = 2$ and the followings,

$$
\begin{array}{ll}
\rho \models A : \vartheta_1 \wedge \rho \models x_A : \vartheta_2 & \text{yielding } \vartheta_1 \ni (A) \text{ and } \vartheta_2 \ni (N_A) \\
\forall V'_1, V'_2 : V'_1 \in \vartheta_1 \wedge V'_2 \in \vartheta_2 \ \wedge & \text{taking } V'_1 = (A) \text{ and } V'_2 = (N_A) \ \wedge \\
Fl(V'_1, V'_2) = T'_1, ..., T'_{len} & len = 2 \text{ and } T'_1, ..., T'_{len} = A, N_A \\
\forall \langle T_1, \ldots, T_s \rangle \in \kappa : & \text{if } \langle A, N_A, B, N_B \rangle \in \kappa \text{ and } s = 4 \\
 & \text{i.e. } T_1 = A, T_2 = N_A, T_3 = B, T_4 = N_B \\
T_1, \ldots, T_{len} =^F T'_1, \ldots, T'_{len} \Rightarrow & T_1, T_2 =^F T'_1, T'_2 = A, N_A \\
\forall (\widetilde{T_3}, \widetilde{T_4}) \in \prod_2(T_3, T_4) \Rightarrow & \prod_2(T_3, T_4) = \prod_2(B, N_B) = \{((B), (N_B))\} \\
\ (\widetilde{T_3} \in \rho(x) \wedge \widetilde{T_4} \in \rho(x_A) \wedge & \text{gives } (B) \in \rho(x) \wedge (N_B) \in \rho(x_B) \\
\ (s > len + t - j) \Rightarrow l \in \psi \wedge & \text{and } 4 = 4 \text{ does not require } l \notin \psi \\
\ (\rho, \kappa) \models 0 : \psi) & true \\
\hline
\multicolumn{2}{c}{(\rho, \kappa) \models (A, x_A; x, x_B)^l.0 : \psi}
\end{array}
$$

In particular, $((B), (N_B)) \in \prod_2(B, N_B)$ implies that $(B) \in \rho(x)$ and $(N_B) \in \rho(x_B)$. Suppose to have also that $\langle A, N_A, B, N_B, K \rangle \in \kappa$. In this case, $((B), (N_B, K)) \in \prod_2(B, N_B, K)$ and therefore $(B) \in \rho(x)$ and $(N_B, K) \in \rho(x_B)$ and also $((B, N_B), K) \in$

$\prod_2(B, N_B, K)$ and therefore $(B, N_B) \in \rho(x)$ and $(K) \in \rho(x_B)$. More precisely:

$\rho \models A : \vartheta_1 \wedge \rho \models x_A : \vartheta_2$	yielding $\vartheta_1 \ni (A)$ and $\vartheta_2 \ni (N_A)$
$\forall V_1', V_2' : V_1' \in \vartheta_1 \wedge V_2' \in \vartheta_2 \wedge$	taking $V_1' = (A)$ and $V_2' = (N_A) \wedge$
$Fl(V_1', V_2') = T_1', ..., T_{len}'$	$len = 2$ and $T_1', ..., T_{len}' = A, N_A$
$\forall \langle T_1, \ldots, T_s \rangle \in \kappa :$	if $\langle A, N_A, B, N_B, K \rangle \in \kappa$ and $s = 5$
	i.e. $T_1 = A, T_2 = N_A, T_3 = B, T_4 = N_B, T_5 = K$
$T_1, \ldots, T_{len} =^F T_1', \ldots, T_{len}' \Rightarrow$	$T_1, T_2 =^F T_1', T_2' = A, N_A$
$\forall (\widetilde{T_3}, \widetilde{T_4}) \in \prod_2(T_3, T_4, T_5) \Rightarrow$	$\prod_2(T_3, T_4, T_5) = \prod_2(B, N_B, K) =$ $\{((B), (N_B, K)), ((B, N_B), (K))\}$
$(\widetilde{T_3} \in \rho(x) \wedge \widetilde{T_4} \in \rho(x_A)$	gives $(B), (B, N_B) \in \rho(x)$ and $(K), (N_B, K) \in \rho(x_B)$
$(s > len + t - j) \Rightarrow l \in \psi \wedge$	$5 > 2 + 4 - 2$ requires $l \in \psi$
$(\rho, \kappa) \models 0 : \psi$	*true*

$$(\rho, \kappa) \models (A, x_A; x, x_B)^l.0 : \psi$$

The rule for *decryption* (Dec) is similar to (In): the values to be matched are those obtained by evaluating the expression E; while the matching ones are the terms inside decryption. If the check succeeds then variables are bound and the continuation process P is analysed. Moreover, the rule checks the possibility of multi-to-one binding: the component ψ must contain the label l corresponding to the decryption. Suppose e.g. to have $\mathsf{decrypt}\ E\ \mathsf{as}\ \{E_1, \ldots, E_2; x_3, \ldots, x_4\}^l_{E_0}\ \mathsf{in}\ P$, with $E = \{A, N_A, B, N_B\}_K$, $E_0 = K, E_1 = A$, $E_2 = x_A$ and $\rho(x_A) = \{(N_A)\}$. Then we have that $\rho \models A : \{(A)\}$, $\rho \models x_A : \{(N_A)\}$ and $Fl((A), (N_A)) = A, N_A$. Then $((B), (N_B)) \in \prod_2(B, N_B)$ implies that $(B) \in \rho(x_3)$ and $(N_B) \in \rho(x_4)$. Suppose to have instead $E = \{A, N_A, B, N_B, K_0\}_K$, then $((B), (N_B, K_0)) \in \prod_2(B, N_B, K_0)$ and therefore $(B) \in \rho(x_3)$ and $(N_B, K_0) \in \rho(x_4)$ and also $((B, N_B), (K_0)) \in \prod_2(B, N_B, K_0)$ and therefore $(B, N_B) \in \rho(x_3)$ and $(K_0) \in \rho(x_4)$. Furthermore $l \in \psi$.

The rule (Nil) does not restrict the estimate, while the rules (New), (Par) and (Rep) ensure that the estimate also holds for the immediate sub-processes.

Semantics Properties Our analysis is correct with respects to the operational semantics of LySa. The detailed proofs are reported in the Appendix.

We have the following results. The first states that estimates are resistant to substitution of closed terms for variables, and it holds for both extended terms and processes. The second one says that estimate respect $\equiv$.

Lemma 1. *1. (a)* $\rho \models E : \vartheta \wedge (T_1, \ldots, T_k) \in \rho(x)$ *imply* $\rho \models E[T_1, \ldots, T_k/x] : \vartheta$
(b) $(\rho, \kappa) \models P : \psi \wedge (T_1, \ldots, T_k) \in \rho(x)$ *imply* $(\rho, \kappa) \models P[T_1, \ldots, T_k/x] : \psi$
2. If $P \equiv Q$ *and* $(\rho, \kappa) \models P$ *then* $(\rho, \kappa) \models Q$

Our analysis is semantically correct regardless of the way the semantics is parameterised, furthermore the reference monitor semantics cannot stop the execution of P when ψ is empty. The proof is by induction on the inference of $P \to Q$.

Theorem 1. ***(Subject reduction)*** *If* $P \to Q$ *and* $\rho, \kappa, \psi \models P$ *then* $\rho, \kappa, \psi \models Q$.

The next theorem shows that our analysis correctly predicts when we can safely do without the reference monitor. We shall say that the reference monitor

RM *cannot abort* a process P when there exist no Q, Q' such that $P \rightarrow^* Q \rightarrow Q'$ and $P \rightarrow^*_{\mathsf{RM}} Q \not\rightarrow_{\mathsf{RM}}$. (As usual, * stands for the transitive and reflexive closure of the relation in question, and we omit the string of labels in this case; while $Q \not\rightarrow_{\mathsf{RM}}$ stands for $\nexists Q' : Q \rightarrow_{\mathsf{RM}} Q'$.) We then have:

Theorem 2. ***(Static check for reference monitor)*** *If $\rho, \kappa, \psi \models P$ and $\psi = \emptyset$ then RM cannot abort P.*

Proof. Suppose *ad absurdum* that such Q and Q' exist. A straightforward induction extends the subject reduction result to $P \rightarrow^* Q$ giving $\rho, \kappa, \psi \models Q$ and $\psi = \emptyset$. The part 2 of the subject reduction result applied to $Q \rightarrow Q'$ gives $Q \rightarrow_{\mathsf{RM}} Q'$ which is a contradiction.

Modelling the Attackers In a protocol execution, several principals exchange messages over an open network, which is accessible to the attackers and therefore vulnerable to malicious behaviour. We assume an active Dolev-Yao attacker [8]: it can eavesdrop, and replay, encrypt, decrypt, generate messages providing that the necessary information is within his knowledge, that it increases while interacting with the network.

This scenario can be modelled in LySa as a process running in parallel with the protocol process. Formally, we shall have $P_{sys} \mid P_\bullet$, where P_{sys} represents the protocol process and $P_\bullet$ is some *arbitrary* attacker. The attacker acquires its knowledge by interacting with P_{sys}, starting from the public knowledge. Note that the secret messages and keys, e.g. K_{ab}, are restricted to their scope in P_{sys} and thus they are not immediately accessible to the attacker. Instead of considering only one attacker, we want to consider how P behaves under the attack of arbitrary attackers $P_\bullet$. To get an account of the infinitely many attackers, the overall idea is to find a formula (for a similar treatment see [3]) that characterizes all $P_\bullet$: this means that whenever an estimate (ρ, κ, ψ) satisfies it, then $(\rho, \kappa) \models P_\bullet : \psi$ for all attackers $P_\bullet$. The estimates we consider hereafter will satisfy the formula. Intuitively, the formula has to mimic how all the $P_\bullet$ are analysed.

3.1 Constructing Attacker Process Formula

Our aim here is to define a general formula, able to characterise all the attackers and then to prove its correctness.

One concern regarding the analysis of the attacker process is about the names and variables the attacker uses, which have to be apart from the ones used by P_{sys}. Let all the names used by P_{sys} to be in a finite set $\mathcal{N}_\mathsf{c}$ and all the variables in a finite set $\mathcal{X}_\mathsf{c}$; we can then postulate a new name $n_\bullet$ not in $\mathcal{N}_\mathsf{c}$ and a new variable $z_\bullet$ not in $\mathcal{X}_\mathsf{c}$. This means that all the names and variables of the attacker are coalesced in $n_\bullet$ and $z_\bullet$. Therefore $n_\bullet$ may represent any name generated by the attacker, while $\rho(z_\bullet)$ represents the attacker knowledge.

In order for the attacker process to interact with the protocol process, some basic information of the protocol process has to be known in advance. We shall

(1) $\wedge_{k \in \mathcal{A}_\kappa} \ \forall \langle V_1, \ldots, V_k \rangle \in \kappa : \ \wedge_{i=1}^{k} V_i \in \rho(z_\bullet)$ the attacker may learn by eavesdropping
(2) $\wedge_{k \in \mathcal{A}_{\mathsf{Enc}}} \ \forall \{V_1, \ldots, V_k\}_{V_0} \in \rho(z_\bullet) : V_0 \in \rho(z_\bullet) \Rightarrow \wedge_{i=1}^{k} V_i \in \rho(z_\bullet)$ the attacker may learn by decrypting messages with keys already known
(3) $\forall \ V_0, \ldots, V_k : \wedge_{i=0}^{k} V_i \in \rho(z_\bullet) \ \wedge \ BLen(\{V_1, \ldots, V_k\}_{V_0}) \in \mathcal{B}_{\mathsf{Enc}} \Rightarrow \{V_1, \ldots, V_k\}_{V_0} \in \rho(z_\bullet)$ the attacker may construct new encryptions using the keys known
(4) $\forall V_1, \ldots, V_k : \wedge_{i=1}^{k} V_i \in \rho(z_\bullet) \ \wedge \ BLen(V_1) + \ldots + BLen(V_k) \in \mathcal{B}_\kappa \Rightarrow \langle V_1, \ldots, V_k \rangle \in \kappa$ the attacker may actively forge new communications
(5) $\{n_\bullet\} \cup \mathcal{N}_f \subseteq \rho(z_\bullet)$ the attacker initially has some knowledge

Table 3. The Attacker's Capabilities

say that a process P_{sys} has the type $(\mathcal{N}_\mathsf{f}, \mathcal{A}_\kappa, \mathcal{A}_{\mathsf{Enc}})$ whenever: (1) it is close, (2) all the free names of P_{sys} are in $\mathcal{N}_\mathsf{f}$. (3) all the arities used for sending or receiving are in $\mathcal{A}_\kappa$ and (4) all the arities used for encryption or decryption are in $\mathcal{A}_{\mathsf{Enc}}$. Obviously, $\mathcal{N}_\mathsf{f}$, $\mathcal{A}_\kappa$ and $\mathcal{A}_{\mathsf{Enc}}$ are all finite and can be computed by inspecting the process P_{sys}.

In the complex type flaw analysis, each message sent to or received by principals is viewed as a sequence of bits. Protocol participants then parse the bit sequences into a number of fields as specified by the protocol. It is reasonable to assume that each protocol participant knows the bit length of the expected message beforehand and hence refuse to accept messages of different length. Under this assumption, we say that the attacker can only send out messages, whose bit length equals to one of the messages exchanged during the protocol execution. We claim that the attacker will not gain anything more by sending out a message of a different bit length. Similarly for encryptions, the attacker only generates encryptions, whose bit length equals to one of the encryptions generated during the protocol execution. Formally, we define a function, $BLen$, to represent the bit length of names and encryptions:

$BLen(V)$: the bit length of the value V
$BLen(\{V_1, \ldots, V_k\}_K)$: the bit length of the encryption $\{V_1, \ldots, V_k\}_K$

Furthermore, we require that

- the bit lengths of all the sending or receiving messages are in $\mathcal{B}_\kappa$, and
- the bit lengths of all the encryptions are in $\mathcal{B}_{\mathsf{Enc}}$

Obviously, $\mathcal{B}_\kappa$ and $\mathcal{B}_{\mathsf{Enc}}$ are all finite and can be computed by inspecting the process P_{sys}.

Given the assumptions as above, the extended Dolev-Yao condition for the LySa calculus can be expressed as the conjunction of the five components in the Table 3.

Implementation Following [3], the implementation can be obtained along the lines that first transform the analysis into a logically equivalent formulation written in Alternation free Least Fixed Point logic (ALFP) [19], and then followed by using the Succinct Solver [19], which computes the least interpretation of the predicate symbols in a given ALFP formula.

3.2 Validate the Amended Needham-Schroeder Protocol

Here, we will show that the analysis applied to the Amended Needham-Schroeder protocol, successfully captures the complex type confusion leading to the attack, presented in the Introduction.

In LySa, each instance of the protocol is modelled as three processes, A, B and S, running in parallel within the scope of the shared keys. To allow the complex type confusion arise, we put two instances together, and add indices to names and variables used in each instance in order to tell them apart, namely

$$P_{NS} \;=\; (\nu\, K_A)(\nu\, K_B)(A_1 \mid A_2 \mid B_1 \mid B_2 \mid S)$$

To save space, processes without indices are shown in Tab. 4. For clarity, each message begins with the pair of principals involved in the exchange. In LySa we do not have other data constructors than encryption, but the predecessor operation can be modelled by an encryption with the key PRED that is also known to the attacker. For the sake of readability, we directly use $N-1$. We

Initiator A :	Responder B :
$/*1*/\ \langle A, B, A\rangle.$	$/*1*/\ (A, B; y_a)^{l_6}.$
$/*2*/\ (B, A; x_{enc})^{l_1}.$	$/*2*/\ (\nu\ N_B)\ \ \langle B, A, \{y_a, N_B\}_{K_B}\rangle.$
$/*3*/\ (\nu\ N_A)\langle A, S, A, B, N_A, x_{enc}\rangle.$	
$/*4*/\ (S, A; x_z)^{l_2}.$	
$\quad$ decrypt x_z as $\{N_A, B; x_k, x_y\}^{l_3}_{K_A}$ in	$/*5*/\ (A, B; y_{enc})^{l_7}.$
$/*5*/\ \langle A, B, x_y\rangle.$	$\quad$ decrypt y_{enc} as $\{N_B, A; y_k\}^{l_8}_{K_B}$ in
$/*6*/\ (B, A; x_{no})^{l_4}.$	$/*6*/\ (\nu\ N_0)\langle B, A, \{N_0\}_{y_k}\rangle.$
$\quad$ decrypt x_{n0} as $\{; x_n\}^{l_5}_{x_k}$ in	$/*7*/\ (A, B; y_{n0})^{l_9}.$
$/*7*/\ \langle A, B, \{x_n - 1\}_{x_k}\rangle.0$	$\quad$ decrypt y_{n0} as $\{N_0 - 1\}^{l_{10}}_{y_k}$ in 0
Server S :	
$/*3*/\ (A, S, A, B; z_{na}, z_{enc})^{l_{11}}.$	
$\quad$ decrypt z_{enc} as $\{A; z_{nb}\}^{l_{12}}_{K_B}$ in	
$/*4*/\ (\nu\ K)$	
$\quad \langle S, A, \{z_{na}, B, K, \{z_{nb}, A, K\}_{K_B}\}_{K_A}\rangle.0$	

$(A) \in \rho(y_a^1)$	$(B) \in \rho(z_\bullet)$	$(n_\bullet) \in \rho(x_k^1)$
$(\{A, N_B^1\}_{K_B}) \in \rho(x_{enc}^1)$	$(N_A^1) \in \rho(z_\bullet)$	$\langle A, B, N_A^1, B, n_\bullet, N_A^2\rangle \in \kappa$
$(N_A^2) \in \rho(x_y^1)$	$\langle A, B, N_A^1, B, n_\bullet\rangle \in \kappa$	$(N_A^1, B, n_\bullet, N_A^2) \in \rho(x_z^1)$
$\langle A, B, N_A^1, \{A, N_B^1\}_{K_B}\rangle \in \kappa$	$(N_A^1, B, n_\bullet) \in \rho(y_a^2)$	$l_6 \in \psi$

Table 4. Amended Needham-Schroeder protocol: specification (above); some analysis results (below).

can apply our analysis and check that $(\rho, \kappa) \models P_{NS} : \psi$, where ρ, κ and ψ have the non-empty entries (only the interesting ones) listed in Tab. 4.

The message exchanges of the regular run (the first instance) performed by A and B are correctly reflected by the analysis. In step 1, B receives the tuple sent by A and binds variable y_a^1 to the value (A), as predicted by $(A) \in \rho(y_a^1)$. In step 2, B generates a nonce N_B^1, encrypts it together to the value of y_a^1 and sends it out to the network. A reads this message, binds the variable x_{enc}^1 to the value $(\{A, N_B^1\}_{K_B})$, as reflected by $(\{A, N_B^1\}_{K_B}) \in \rho(x_{enc}^1)$; then, in step 3, it generates N_A^1 and sends it to S as a plain-text, together with x_{enc}^1 as predicted by $\langle A, S, N_A^1, \{A, N_B^1\}_{K_B}\rangle \in \kappa$, and so on.

Moreover, the non-empty error component ψ shows that a multi-to-one binding may happen in the decryption with label l_6 and therefore suggests a possible complex type confusion leading to a complex type flaw attack.

By studying the contents of the analysis components ρ and κ, we can rebuild the attack sequence. Since $\langle A, S, N_A^1, \{A, N_B^1\}_{K_B}\rangle \in \kappa$, then $(N_A^1) \in \rho(z_\bullet)$. This corresponds to the fact that the attacker, able to intercept messages on the net, can learn N_A^1. The entry $\langle A, B, N_A^1, B, n_\bullet\rangle \in \kappa$ reflects that the attacker is able to constructs and sends to A a new message $(N_A^1, B, n_\bullet)$ to initiate the second instance, where $(n_\bullet)$ is within its knowledge. The entry $(N_A^1, B, n_\bullet)$ in $\rho(y_a^2)$ corresponds to the fact A receives this message, by binding y_a^2 to the value $(N_A^1, B, n_\bullet)$. This is a multi-to-one binding, detected by the analysis, as reported by the error component: $l_6 \in \psi$. Afterwards, A encrypts what she has received with a new nonce N_A^2 and sends it out, as indicated by $\langle A, B, N_A^1, B, n_\bullet, N_A^2\rangle \in \kappa$. The attacker replays this to A, who takes it as the message from S in the step 4 of the first instance $((N_A^1, B, n_\bullet, N_A^2) \in \rho(x_z^1))$. The entry $(n_\bullet) \in \rho(x_k^1)$ reflects that in decrypting message 4, A binds x_k^1 to the concatenation of values $(n_\bullet)$ to be used as the session key. After completing the challenge and response in step 6 and 7, A then believes she is talking to B using the session key K, but indeed she is talking to the attacker using $(n_\bullet)$ as the new key. This exactly corresponds to the complex type flaw attack shown before.

The protocol can be modified such that each principal use different keys for different roles, i.e. all the principals taking the initiator's role A_i share a master key K_A^i with the server and all the principals taking the responder's role B_j share K_B^j with the server. In this case, the analysis holds for $\psi = \emptyset$ and thereby it guarantees absence of complex type confusions attacks.

Here, only two sessions are taken into account. However, as in [3], the protocol can be modelled in a way that multiple principals are participating in the protocol at the same time and therefore mimic the scenario that several sessions are running together. Due to space limitation, further details are skipped here.

4 Conclusion

We say that a complex type confusion attack happens when a concatenation of fields in a message is interpreted as a single field. This kind of attacks is not easy to deal with in a process algebraic setting, because message specifications are

given at a high level: the focus is on their contents and not on their structure. In this paper, we extended the notation of variable binding in the process calculus LySa from one-to-one to multi-to-one binding, thus making it easier modelling the scenario where a list of fields is confused with a single field. The semantics of the extended LySa makes use of a reference monitor to capture the possible multi-to-one bindings at run time. We mechanise the search for complex type confusions by defining a Control Flow Analysis for the extended LySa calculus. It checks at each input and decryption place whether a multi-to-one binding may happen. The analysis ensures that, if no such binding is possible, then the process is not subject to complex type flaw attacks at run time. As far as the attacker is concerned, we adopted the standard notion from Dolev-Yao threat model [8], and we enriched it to deal with the new kind of variable binding.

We applied our Control Flow Analysis to the Amended Needham-Schroeder Protocol (as shown in Section 3), to Otway-Rees [20], Yahalom [6] (not reported, because of lack of space). It has confirmed that we can successfully detect the complex type confusions leading to type flaw attacks on those protocols. This detection is done in a purely mechanical and static way. The analysis also confirms the complex type flaw attacks on a version of the Neuman-Stubblebine protocol, found in [22].

The technique presented here is for detecting *complex* type flaw attacks only. *Simple* type flaw attacks, i.e. two single fields of different types are confused with each other, not considered here, have been addressed instead in [4]. There, under a framework similar to the present one, a type annotation (type tags) is developed, to be attached to LySa processes, for checking simple type confusions. Besides the type tags, several kinds of annotations for LySa has been developed for validating various security properties, e.g. confidentiality [10], freshness [9] and message authentication [3]. They can be easily combined with the annotations introduced here, thus giving more comprehensive results.

Usual formal frameworks for the verification of security protocols need to be suitably extended for modelling complex type flaw confusions. Extensions include the possibility to decompose and rebuild message components, that we obtain by playing with single, general and flattened values. In [5], for instance, the author uses a concatenation operator to glue together different components in messages. His approach is based on linear logic and it is capable of finding the complex type flaw attack on the Otway-Rees protocol. Meadows [15, 16] approach is more general and can address also even more complex type confusions, e.g. those due to the confusion between pieces of fields of one type with pieces of another. The author, using the GDOI protocol as running example, develops a model of types that assumes differing capacities for checking types by principals. Moreover, she presents a procedure to determine whether the types of two messages can be confused, that also evaluates the likelihood of the misinterpretations in terms of probability. In [13], using the AVISPA [2] model checking tool, type flaw attacks of the GDOI protocol are captured. Furthermore, by using the Object-Z schema calculus [23], as in some previous work, e.g. [12] the authors verify the attacks at a lower level and find which are the low-level assumptions that leads to the

attacks and which are the requirements that prevent them. Type confusions are captured also in [17], by using an efficient Prolog based constraint solver. The above settings, especially the ones in [15, 16, 13], are more general than our. They make it possible to capture more involved kind of type confusions in a complex setting like the one of the GDOI protocol. Furthermore, we cannot deal with probabilities. Our work represents a first step in modelling lower level features of protocol specifications in a process algebraic setting, like the ones that lead to type confusions. The idea is to only perform the refinement of the high-level specifications necessary to capture the low-level feature of interest. Our control flow analysis procedure always guarantees termination, even though it only offers an approximation of protocols behaviour and of their dynamic properties. Here we focussed on a particular kind of confusions, leaving other kind of type confusions for future work. We also would like to move to the multi-protocol setting, where the assumptions adopted in each protocol could be different, but messages could be easily confused, e.g. because of the re-use of keys.

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5 Appendix

We present the complete list of lemmata and theorems concerning the semantics correctness, endowed with the corresponding proofs.

The first lemma shows that the analysisonly distinguish processes up to assignment of canonical names.

Lemma 2. ***(Invariance of canonical names)*** *If* $(\rho, \kappa) \models P$ *and* $\lfloor n \rfloor = \lfloor n' \rfloor$ *then* $(\rho, \kappa) \models P[n \mapsto n']$.

Definition 2. ***(Disciplined*** $\alpha-$***equivalence)*** *Two process* P_1 *and* P_2 *are disciplined* $\alpha-$*equivalence whenever* $P_1 \overset{\alpha}{\equiv} P_2$ *with the extra requirement that* $\lfloor n_1 \rfloor = \lfloor n_2 \rfloor$.

Since the disciplined α-equivalence cannot modify canonical names, then the analysis results for two α-equivalent processes are the same.

Lemma 3. ***(Invariance of*** α***-equivalence)*** *If* $(\rho, \kappa) \models P$ *and* P *is disciplined* α*-equivalent to* P'*, then* $(\rho, \kappa) \models P'$.

We need to prove two further lemmata. The first states that estimates are resistant to substitution of closed terms for variables, and it holds for both extended terms and processes.

Lemma 4. ***(Substitution results)***

1. $\rho \models E : \vartheta$ *and* $(T_1, \ldots, T_k) \in \rho(x)$ *imply* $\rho \models E[T_1, \ldots, T_k/x] : \vartheta$
2. $(\rho, \kappa) \models P : \psi$ *and* $(T_1, \ldots, T_k) \in \rho(x)$ *imply* $(\rho, \kappa) \models P[T_1, \ldots, T_k/x] : \psi$

Proof. Part 1 The proof proceeds by structural induction over expression by regarding each of the rules in the analysis.

Case (Name). Assume that $E = n$ and $\rho \models n : \vartheta$. For arbitrary choices of x and $T_1, \ldots, T_k$, it holds that $n[T_1, \ldots, T_k/x] = n$ so it is immediate that also $\rho \models n[T_1, \ldots, T_k/x] : \vartheta$.

Case (Variable). Assume that $E = x'$ and $\rho \models x' : \vartheta$, i.e. that $\rho(x') \subseteq \vartheta$. Then there are two cases. Either $x' \neq x$ in which case $x'[T_1, \ldots, T_k/x] = x'$ so clearly $\rho \models x'[T_1, \ldots, T_k/x] : \vartheta$. Alternatively, $x' = x$ in which case $x'[T_1, \ldots, T_k/x] = (T_1, \ldots, T_k)$. Furthermore assume that $(T_1, \ldots, T_k) \in \rho(x)$ and because $\rho(x') \subseteq \vartheta$, it holds that $\rho \models (T_1, \ldots, T_k) : \vartheta$ in which case $\rho \models x'[T_1, \ldots, T_k/x] : \vartheta$ by the analysis.

Case (Symmetric Encryption). Assume that $E = \{E_1, \ldots, E_k\}_{E_0}$, i.e. $\rho \models \{E_1, \ldots, E_k\}_{E_0} : \vartheta$. The result holds by applying the induction hypothesis on each individual E_i.

Proof. Part 2 The proof is done by straightforward induction applying the induction hypothesis on any sub-process and lemma 3.1 on any sub-terms.

The second lemma says that an estimate for an extended processes P is valid for every process congruent to P, as well.

Lemma 5. ***(Invariance of Structural Congruence)*** *If* $P \equiv Q$ *and* $(\rho, \kappa) \models P$ *then* $(\rho, \kappa) \models Q$

Proof. The proof amounts to a straightforward inspection of each of the clauses defining $P \equiv Q$.

The analysis of a value simply evaluates to the canonical value.

Lemma 6. ***(Evaluation of values)*** *The analysis* $\rho \models V : \vartheta$ *holds if and only if* $V \in \vartheta$.

Proof. The proof is by induction in the structure of values. Remember that values, V, are expressions without variables, the proof is straightforward.

We are now ready to state the subject reduction result. It expresses that our analysis is semantically correct regardless of the way the semantics is parameterised, furthermore the reference monitor semantics cannot stop the execution of P when ψ is empty.

Theorem 3. ***(Subject reduction)*** *If $P \to Q$ and $\rho, \kappa, \psi \models P$ then also $\rho, \kappa, \psi \models Q$. Furthermore, if $\psi = \emptyset$ then $P \to_{\mathsf{RM}} Q$*

Proof. By induction on the inference of $P \to Q$.

In case (Com) we assume $(\rho, \kappa) \models \langle V_1, \ldots, V_k \rangle.P \mid (V'_1, \ldots, V'_j; x_{j+1}, \ldots, x_t)^l.Q : \psi$ which amounts to:

$(a) \wedge_{i=1}^{k} \rho \models V_i : \vartheta_i$
$(b) \forall V_1, \ldots, V_k : \wedge_{i=1}^{k} V_i \in \vartheta_i \Rightarrow \langle T_1, ..., T_s \rangle \in \kappa$ where $T_1, ..., T_s = Fl((V_1, ...V_k))$
$(c) (\rho, \kappa) \models P : \psi$
$(d) \wedge_{i=1}^{j} \rho \models V'_i : \vartheta'_i$
$(e) \forall V'_1, \ldots, V'_j : \wedge_{i=1}^{j} V'_i \in \vartheta_i \quad \wedge \quad Fl(V'_1, ..., V'_j) = T'_1, ..., T'_{len} \Rightarrow$
$\forall \langle T_1, \ldots, T_s \rangle \in \kappa : T_1, \ldots, T_{len} = T'_1, \ldots, T'_{len} \Rightarrow$
$\forall (\widetilde{T}_{j+1}, \ldots, \widetilde{T}_t) \in \prod_{t-j} (T_{len+1}, \ldots, T_s) \Rightarrow$
$\wedge_{i=j+1}^{t} \widetilde{T}_i \in \rho(x_i) \wedge (\rho, \kappa) \models P : \psi$

Moreover we assume that $\wedge_{i=1}^{j} T_i = T'_i$ and $k \geq t$ because $\langle V_1, \ldots, V_k \rangle.P \mid (V'_1, \ldots, V'_j; x_{j+1}, \ldots, x_t).Q \to P \mid Q[\widetilde{T}_{j+1}/x_{j+1}, \ldots, \widetilde{T}_k/x_k]$ where $(\widetilde{T}_{j+1}, \ldots, \widetilde{T}_t) \in \prod_{t-j} (T_{j+1}, \ldots, T_k)$, and we have to prove that $\rho, \kappa, \psi \models P \mid Q[\widetilde{T}_{j+1}/x_{j+1}, \ldots, \widetilde{T}_k/x_k]$.

From (a) we have $\wedge_{i=1}^{k} V_i \in \vartheta_i$ since $\wedge_{i=1}^{k} \mathsf{fv}(V_i) = \emptyset$ and then (b) gives $\langle T_1, \ldots, T_k \rangle \in \kappa$ since $T_1, \ldots, T_k = Fl(V_1, \ldots, V_k)$. From (d) and the assumption $\wedge_{i=1}^{j} T_i = T'_i$ we get $\wedge_{i=1}^{j} V_i \in \vartheta'_i$. Apparently, $(V_1), \ldots, (V_j) = Fl(V_1, \ldots, V_j)$ and $len = j$. (e) gives $\forall (\widetilde{T}'_{j+1}, \ldots, \widetilde{T}'_t) \in \prod_{t-j} (T_{k-j}, \ldots, T_k) \Rightarrow \wedge_{i=j+1}^{t} \widetilde{T}'_i \in \rho(x_i))$ and $\rho, \kappa, \psi \models Q$.

The substitution result then gives $\rho, \kappa, \psi \models Q[\widetilde{T}'_{j+1}/x_{j+1}, \ldots, \widetilde{T}'_t/x_t]$ and together with (c) this gives the required result.

For the second part of the result we observe that $\neg \mathsf{RM}(s, len + t - j) \Rightarrow l \in \psi$ follows from (e) and since $\psi = \emptyset$ and $len = j$ it must the case that $t = k$. Thus the condition of the rule (Com) are fulfilled for $\to_{RM}$.

The **case (Dec)** is analogous.

Cases (New), (Par) and (Rep) follow directly from the induction hypothesis.

The case (Congr) also uses the congruence result.

The next result shows that our analysis correctly predicts when we can safely do without the reference monitor. We shall say that the reference monitor RM *cannot abort* a process P when there exist no Q, Q' such that $P \to^* Q \to Q'$ and $P \to^*_{\mathsf{RM}} Q \not\to_{\mathsf{RM}}$. As usual, "*" stands for the transitive and reflexive closure of the relation in question (we omit the string of labels in this case), and $Q \not\to_{\mathsf{RM}}$ stands for $\nexists Q' : Q \to_{\mathsf{RM}} Q'$. We then have:

Theorem 4. ***(Static check for reference monitor)*** *If $\rho, \kappa, \psi \models P$ and $\psi = \emptyset$ then RM cannot abort P.*

Proof. Suppose *per absurdum* that such Q and Q' exist. A straightforward induction extends the subject reduction result to $P \to^* Q$ giving $\rho, \kappa, \psi \models Q$ and $\psi = \emptyset$. The part 2 of the subject reduction result applied to $Q \to Q'$ gives $Q \to_{\mathsf{RM}} Q'$ which is a contradiction.