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Versatile weighting strategies for a citation-based research evaluation model

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VERSATILE WEIGHTING STRATEGIES FOR A CITATION-BASED RESEARCH EVALUATION MODEL

GIANNA M. DEL CORSO* AND F. ROMANI *

Abstract. In this paper, we first give a quick review of the most used numerical indicators for evaluating research, and then we present an integrated model for ranking scientific publications together with authors and journals. Our model relies on certain adiacency matrices obtained from the relations of citation, authorship and publication. These matrices are first normalized to obtain stochastic matrices and then are combined together by means of weights to form a suitable irreducible stochastic matrix whose dominant eigenvector provides the ranking. We discuss various strategies for choosing the weights and we show on large synthetic datasets how the choice of a weighting criteria rather than another can change the behavior of our ranking algorithm.

Key words. PageRank, Perron vector, perturbation results, impact factor

1. Introduction. The problem of research evaluation is a very important problem, in fact the number of scientific journals and the number of papers published is increasing at an almost exponential rate [21]. The size and grown of research literature places a tremendous burden on research. For example, it is becoming common to rely on search engines such as `Google Scholar` to choose what to read or what to cite. This burden doesn't only affects researchers but also funding agencies, university administrators and even reviewers who are called on to evaluate productivity of researchers and institutions. Most of the time it is however impossible to give an in-depth evaluation of the research performed by a scholar and it is becoming more and more popular to use indirect indicators of quality.

Despite the over simplification of using just a few numbers to quantify the scientific merit of a research, the entire community is relying more and more on citation analysis for assessing quality. Of course, the evaluation of research using citation analysis has weaknesses. For example, it is based on the assumption that a citation is a sort of trusting vote, but this is not always the case, since an author can add a citation to a paper to criticize its content, and many of the citations are self-citations. However, as soon as a paper is discovered to contain errors and is discredited, usually is not going to receive other citations. Also, many studies [2, 14, 13] showed that self-citations cannot inflate citation rates as one could expect since they rapidly lose their weight as time elapses, aging much faster than citation coming from other sources. Another criticism to the use of citations as the "corner stone" to asses quality of research is that many items contained in the reference list of a paper are papers written by people in the entourage of the authors. However, the peer review process of the published papers should guarantee the appropriateness of the reference list.

However, there are many pros, which make the approach based on citation analysis credible and convincing, especially as a quick, simple and objective way to fast parse a large amount of data when peer review in not practicable.

Mathematically, we represent the citation process as a direct graph and hence as a binary matrix C where the entry $c_{ij} = 1$ if paper p_i contains a reference to paper p_j. Of course one can model the problem by using weights to capture the confidence in the citation, but in this paper we will consider a simpler model.

In the literature there have been proposed many different metrics for research evaluation. In particular, different measures for evaluating papers, journals or re-

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searchers have been proposed. In fact, there are many different purposes for ranking. For instance, librarians are interested in the ranking of journal to decide subscriptions, and scholars might be interested on such measures to decide where to publish one of their papers. The ranking of papers is becoming useful for untangling the maze of the many papers published everyday and decide what to read or what to cite. Moreover, the evaluation of scholars on the basis of their scientific productivity for distributing funds or even for hiring people is becoming very common.

Among the different methods proposed in the literature for ranking journal we can distinguish between methods based on citation statistics, such as Impact Factor (IF) (see [11] and references therein for an historical review), simple Citation Count, the MCQ by the American Mathematical Society [3], and methods based on approaches similar to PageRank, such as Eigenfactor [4], SCImago [20] and many others [19, 18].

The metrics based on citation statistics are very easy to calculate but not all the scientific community agrees on the effectiveness of these measure to capture concepts such as reputation or influence. In particular, one of the major point against the use of measures such as IF is that in the same journal are published papers with very different citation rates, and that the "culture" of citations depends on the scientific areas [1]. For examples, in fields such as mathematics or economics, the process of citation gathering is much slower that in fields such as cell biology and it can take decades before the process stabilizes [21]. This reflects on the fact that the average length of the reference list significantly varies among different disciplines.

Ranking schemes based on PageRank-like techniques seem more convincing since papers and web pages share some similarities. The main idea is that not all the citations are equal and that it is not important how many citations papers in a journal are collecting but rather the "quality" of citations. For these metrics mathematical properties can be proved, as done in [18] where it is shown that a centrality measure similar to PageRank is the only ranking satisfying a number of axiomatic requirements.

Although impact factor and similar citation-based statistics can be used when ranking journals, their use becomes a misuse when techniques designed to evaluate journals are applied to the evaluation of single papers. In fact, it is becoming very popular to judge the quality of a single paper on the basis of the prestige or IF of the journal where the research is published, especially by committees responsible of the evaluation of a large number of publications. Of course, we cannot ascribe the properties of an individual journal to each article within the journal. The idea of evaluating the quality of a paper counting the citations received is not, at the same time, fair with respect to relatively recent papers. In fact, many years can elapse before a paper starts receiving citations. The same misuse of citation based metrics for journals is done when evaluating authors. Commonly, these measures designed for journals are used either implicitly or explicitly to compare individuals. Recently, many different indexes have been proposed for ranking scientists. Indexes such as the h-index [15], g-index [9], m-index [15] are based on the citations received by most cited papers of an individual scientist.

In [5, 6] we elaborate the integrated three-class model approach where papers, authors and journals mutually contribute to the attribution of a ranking score of each others.

The idea is that in order to evaluate an author we have to consider not only the quality of the journals where his/her papers have been published but also the quality of every single paper of this author. Moreover, also the quality of the co-authors must be taken into account. In fact, an important author who writes a joint paper with

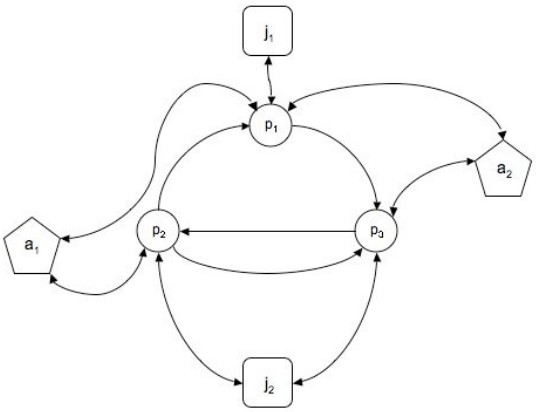


FIGURE 2.1. *A graph where we have different nodes for each category. We have three papers, two authors and two journals.*

a less important one, expresses a sort of trusting vote by conferring to that author more visibility with respect to the international community. Similarly, to evaluate the quality of a paper one has to look at the quality of the journal where the paper is published, at the citations received and at the reputation of its authors. Also when evaluating a journal we take into account not only the cross-citations among journals as done by many methods such as Impact Factor [10], Eigenfactor [4], and many others [7, 18], but also the quality of every single paper published there and the authoritativeness of the scholars writing on that journal.

The paper is organized as follows. In Section 2 we briefly recall and discuss the model presented in [5, 6]. In Section 3 a discussion about the probabilistic interpretation of the model as well as the choice of some weighting parameters is carried on. Section 4 contains experimental results on synthetic data as well as comparisons between the ranking provided by our method and that returned by other already known measures.

2. The basic model. Assume we are given n_P papers together with the bibliographic data of each paper. In particular, of each paper we know the authors, the journal where the paper is published and the list of citations contained in the paper. With this information we can construct a graph with three different kind of nodes (see Figure 2.1). We can associate with the graph three matrices, one for every kind of nodes: the matrix F accounting for the journal publishing each paper, the matrix K which stores information about authorship and the matrix H which records the citation structure among papers. In particular, let n_J be the total number of distinct journals where the n_P papers are published, and let n_A the number of distinct authors who authored the given n_P papers. We define $F = (f_{i,j})$ the $n_J \times n_P$ binary matrix such that

$$f_{i,j} = \begin{cases} 1 & \text{if paper } j \text{ is published in journal } i \\ 0 & \text{otherwise,} \end{cases}$$

$K = (k_{i,j})$ the $n_A \times n_P$ binary matrix such that

$$k_{i,j} = \begin{cases} 1 & \text{if author } i \text{ has written paper } j \\ 0 & \text{otherwise,} \end{cases}$$

and $H = (h_{i,j})$ the $n_P \times n_P$ matrix such that

$$h_{i,j} = \begin{cases} 1 & \text{if paper } i \text{ has paper } j \text{ in its reference list} \\ 0 & \text{otherwise.} \end{cases}$$

In the example of Figure 2.1 we have

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad K = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

We can combine these three matrices to obtain the following 3×3 block matrix

$$A = \begin{bmatrix} FHF^T & FK^T & F \\ KF^T & KK^T & K \\ F^T & K^T & H \end{bmatrix} \tag{2.1}$$

of size $N = n_J + n_A + n_P$. For the example in Figure 2.1 matrix A becomes

$$A = \left[\begin{array}{cc|cc|ccc} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 & 0 & 1 & 1 \\ \hline 1 & 1 & 2 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 2 & 1 & 0 & 1 \\ \hline 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right].$$

Each block of this matrix expresses the relationship between the subjects belonging to the three classes *Journals*, *Authors* and *Papers*. More specifically, the entry in position (i, j) of the block FHF^T contains the number of citations that the papers published in journal i receives from the papers published in journal j; the entry in position (i, j) of the block FK^T contains the number of papers that author j has published in journal i; the entry in position (i, j) of the block KK^T contains the number of joint papers that author i has in collaboration with author j.

We can scale the rows of A to obtain a row-stochastic matrix P, that is a matrix such that $P\boldsymbol{e} = \boldsymbol{e}$, where $\boldsymbol{e} = (1, \ldots, 1)^T$, . In this way, the entries of $P = (p_{i,j})$ can be used as weights to transfer amounts of importance from a subject to another subject. More precisely, numbering the subjects from 1 to N, the importance π_j of subject j is the weighted sum of the importances π_i of all the other subjects i which are in relation with j, where the weights are $p_{i,j}$, that is

$$\pi_j = \sum_{i=1}^{N} \pi_i p_{i,j}.$$

This condition expresses the fact that $\boldsymbol{\pi} = (\pi_i)$ is eigenvector of P corresponding to the eigenvalue 1:

$$\boldsymbol{\pi}^T = \boldsymbol{\pi}^T P.$$

The row stochasticity of P implies that the overall amount of importance that a subject i transfers to the other subjects coincides with the importance of i. In other words, the amount of importance in the system is neither created nor destroyed.

To guarantee the existence and uniqueness of a solution we need A to be irreducible. Under this condition, it is always possible to find a scaling technique such that the matrix P can be constructed, and the Perron Frobenius theorem [16] guarantees the existence of a unique vector $\boldsymbol{\pi}$, such that $\pi_i > 0$ and $\sum_i \pi_i = 1$. We refer to $\boldsymbol{\pi}$ as the *Perron vector* of P. Moreover, in order to have nice convergence properties of iterative algorithms for the computation of $\boldsymbol{\pi}$ we need A to be aperiodic.

Note that working with the stochastic matrix P rather than computing the dominant eigenvector of A has advantages also from a numerical point of view. In fact, the approximation of the dominant eigenvector is done using iterative procedure, and working with a stochastic matrix guarantees that we don't need to perform a normalization at each step to limit the growth of the entries of the intermediate vectors.

There are many ways to enforce irreducibility, for example we can apply the ideas used by the Google Page-Rank model [8]. A way to obtain an irreducible and aperiodic matrix which fits better in our model is to introduce a dummy paper, a dummy author, and a dummy journal, similarly to what done for the one-class model [5]. The dummy paper is cited by every paper and it cites back all the papers except itself. The dummy paper is written by the dummy author and is published in the dummy journal. Mathematically, this corresponds to consider the matrices $\widehat{H}, \widehat{K}$ and $\widehat{F}$ obtained from H, K and F as follows,

$$\widehat{H} = \left[\begin{array}{c|c} H & \boldsymbol{e} \\ \hline \boldsymbol{e}^T & 0 \end{array}\right], \quad \widehat{K} = \left[\begin{array}{c|c} K & \boldsymbol{0} \\ \hline \boldsymbol{0}^T & 1 \end{array}\right], \quad \widehat{F} = \left[\begin{array}{c|c} F & \boldsymbol{0} \\ \hline \boldsymbol{0}^T & 1 \end{array}\right],$$

and to replace H, K and F in (2.1) with $\widehat{H}, \widehat{K}$ and $\widehat{F}$, respectively. It is easy to prove the following theorem [6].

THEOREM 2.1. *The matrix $\widehat{A}$ obtained by replacing the blocks H, K and F in* (2.1) *with the blocks $\widehat{H}$, $\widehat{K}$, and $\widehat{F}$, respectively, is irreducible and aperiodic.*

2.1. Row and column scaling. In the previous section we simply propose to scale the rows of A in order to obtain a row-stochastic matrix. A possibility is that of dividing each row of A by the sum of the entries in the row. A more flexible way, introduced in [5] and completed in [6], consists in performing a separate normalization of each block of A. That is, each block of A is normalized to yield nine row-stochastic matrices; then these matrices are compounded with weights $\Gamma = (\gamma_{i,j})_{i,j=1,3}$, where Γ is row stochastic, into a new stochastic matrix. The entries of this new matrix are used to weight the amount of importance that each class (*Journal*, *Authors*, and *Papers*) gives to the other classes. In [6] an in-depth discussion about the different possible normalization of the single blocks is presented and a proposal is done.

Denote by

$$Q = \begin{bmatrix} J_J & J_A & J_P \\ A_J & A_A & A_P \\ P_J & P_A & P_P \end{bmatrix}, \tag{2.2}$$

where each block is row-stochastic and is obtained from the corresponding block in the matrix $\widehat{A}$ of Theorem 2.1, so for example J_J is the stochastic matrix obtained by the row-normalization of $\widehat{F}\widehat{H}\widehat{F}^T$.

Here, the notation used in (2.2) points out the role of each block with respect to the classes *Journals*, *Authors* and *Papers*. For instance, the entries of the block J_A weight the amount of importance that *Journals* transfer to *Authors*.

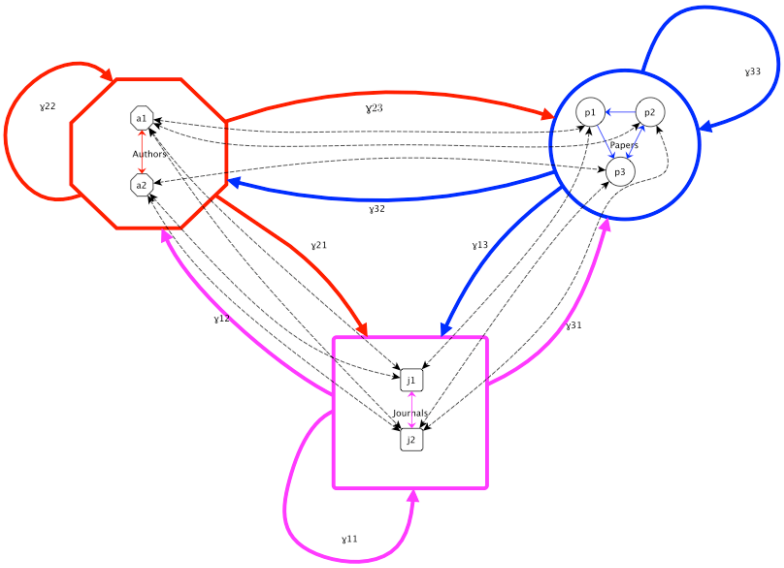


FIGURE 2.2. *A two-level graph, representing the matrix P. Tick solid lines represent direct arcs between the three different classes and are labeled with the weights expressed by γ. Following the dashed arcs we can recover informations about the authors of a given paper and the journal where the paper has been published. Thin solid directed arcs between subjects in the same class represent the link described by the diagonal blocks of Q.*

Let $\Gamma = (\gamma_{i,j})$ be a 3×3 row-stochastic matrix, then the matrix

$$P = \begin{bmatrix} \gamma_{1,1}\, J_J & \gamma_{1,2}\, J_A & \gamma_{1,3}\, J_P \\ \gamma_{2,1}\, A_J & \gamma_{2,2}\, A_A & \gamma_{2,3}\, A_P \\ \gamma_{3,1}\, P_J & \gamma_{3,2}\, P_A & \gamma_{3,3}\, P_P \end{bmatrix}. \tag{2.3}$$

is row-stochastic and its entries $p_{i,j} \geq 0$ express the amount of importance that subject i transfers to subject j. The parameters $\gamma_{i,j}$ can be used to tune the role that each class has with respect to the other classes. For instance, choosing $\gamma_{3,3}$ greater than $\gamma_{2,3}$ and $\gamma_{1,3}$ means to base the importance of papers more on the citations that they receive rather than on the importance of their authors or of the journals where they are published. In Figure 2.2 we can see a representation of the graph represented by matrix P.

3. Probabilistic interpretation and how to choose the weights. Similarly to what is done in the Google PageRank model, we can give a probabilistic interpretation of our model in terms of a "random reader" or "random evaluator". Accordingly with this interpretation, the dummy journal represent the library, the dummy author is the librarian and the dummy paper is the catalog of the library. So, we should expect the rank of the dummy subjects to be higher than that of the subjects belonging to the same class, since the random reader consults more frequently the library or the catalog than a single paper or journal.

The random reader after entering the library and asking the librarian for the catalog, picks a paper $\mathcal{P}$ and then she can perform one of the following three actions. She can keep reading papers choosing among the papers in the reference list of $\mathcal{P}$, or

jump to one of the coauthors of $\mathcal{P}$ or she can look at the journal where $\mathcal{P}$ is published. Each of these actions happens with probability $\gamma_{3,i}, i = 1, 2, 3$. While examining an author $\mathcal{A}$ the random reader with probability $\gamma_{2,2}$ chooses one of the coauthors, with probability $\gamma_{2,1}$ she browses the journals where $\mathcal{A}$ has published or with probability $\gamma_{2,3}$, she starts reading one of the papers written by $\mathcal{A}$. While examining a journal $\mathcal{J}$, the reader can move to another journal cited by papers in $\mathcal{J}$, can pick a paper published in $\mathcal{J}$ or can start examining an author who has published papers in journal $\mathcal{J}$. The random reader jumps from a class to another with a probability described by the 3×3 Markov chain Γ. The probability of picking in a particular class is, on the other hand, ruled by the underlining Markov chain described by the nine matrices $J_J, J_A, J_P, A_j, A_A, A_P, P_J, P_A$ and P_P.

The choice of modeling the problem with stochastic matrices, combining them with weights allows one to tune how much of the importance of a class we want to transfer to another class. In particular, denoting by

$$\mu_J = \sum_{i=1}^{n_J} \pi_i, \quad \mu_A = \sum_{i=n_J+1}^{n_J+n_A} \pi_i, \quad \mu_P = \sum_{i=n_J+n_A+1}^{n_J+n_A+n_P} \pi_i, \tag{3.1}$$

it turns out that the vector (μ_J, μ_A, μ_P) is the left Perron eigenvector of Γ, corresponding to the eigenvalue 1.

In fact, the following Theorem holds [17].

THEOREM 3.1 (Coupling Theorem). *Let P be an $n \times n$ irreducible stochastic matrix partitioned as*

$$P = \begin{bmatrix} P_{1,1} & P_{1,2} & P_{1,3} \\ P_{2,1} & P_{2,2} & P_{2,3} \\ P_{3,1} & P_{3,2} & P_{3,3} \end{bmatrix},$$

with square diagonal blocks. Then the stationary distribution vector for P is given by

$$\pi^T = (\xi_1 \boldsymbol{s}_1^T, \xi_2 \boldsymbol{s}_2^T, \xi_3 \boldsymbol{s}_3^T)$$

where $\boldsymbol{s}_i$ is the unique stationary distribution vector for the stochastic Shur complement S_{ii}. The vector

$$\xi^T = (\xi_1, \xi_2, \xi_3)$$

is the unique stationary distribution vector for the 3×3 irreducible stochastic matrix C whose entries are defined by

$$c_{ij} = \boldsymbol{s}_i^T P_{ij} \boldsymbol{e}. \tag{3.2}$$

The matrix C is referred as the coupling matrix and the scalars ξ_i are called the coupling factors. □

The Coupling Theorem applied to matrix P of equation (2.3), proves that Γ is the *coupling matrix*. In fact, from (3.2), since the nine matrices $J_J, J_A, J_P, \ldots$ are stochastics

$$\begin{aligned} c_{ij} &= \boldsymbol{s}_i^T P_{ij} \boldsymbol{e} \\ &= \gamma_{ij} \boldsymbol{s}_i^T \boldsymbol{e} = \gamma_{ij} \end{aligned}$$

where the last equality holds because $\boldsymbol{s}_i$ are distribution vectors and $\boldsymbol{s}_i^T \boldsymbol{e} = 1$. Moreover, the scalars μ_J, μ_A, μ_P, introduced in (3.1), are proportional to the coupling

factors since $\xi_1 = \|\xi_1 \boldsymbol{s}_1\|_1 = \sum_{i=1}^{n_J} \pi_i = \mu_J$ and similarly for μ_A and μ_P. This means that the vector $(\mu_J, \mu_A, \mu_P)^T$ is the Perron eigenvector of the 3×3 matrix Γ, or equivalently it corresponds the unique stationary distribution vector of the coupling matrix Γ.

In [6] it was suggested to use uniform weights, which corresponds to have $\Gamma = 1/3\boldsymbol{e}\boldsymbol{e}^T$. The dominant (left) eigenvector of Γ, or equivalently the stationary distribution of the coupling matrix, is the vector $1/3\,\boldsymbol{e}$. In this way, each class has the same role in determining the importance of each subject since $\mu_J = \mu_A = \mu_P = 1/3$. This means that the mean value of a journal is $1/(3\,n_J)$ while, the mean value of an author and a paper are respectively $1/(3\,n_A)$ and $1/(3\,n_P)$. In practical situation, we have however that the number of journals n_J, of authors n_A and the number of papers n_P differ in order of magnitude. Typical values [3] are $n_J = O(10^3)$, $n_A = O(10^5)$, $n_P = O(10^6)$, making the mean value of a journal $O(10^{-3})$, that of authors $O(10^{-5})$ and of papers $O(10^{-6})$. This means that journals play a bigger role in the determination of the ranking of the other subjects while papers and authors have a smaller role. Of course, citations are still important because they influence the rank of journals in block J_J.

To correct this situation, we can think to a different weighting criteria. Which is the best weighting strategy if we want the average paper to hold as the average journal or author? Since the average value of each class is μ_i/n_i, with $i \in \{J, A, P\}$, the solution to this problem relies on solving an inverse problem, where the Perron eigenvector is given and is $(n_J/N, n_A/N, n_P/N)^T$ with $N = n_J + n_A + n_P$, and we are seeking a stochastic 3 matrix Γ. By direct substitution we see that a solution is given by

$$\Gamma = \frac{1}{N} \begin{bmatrix} n_J & n_A & n_P \\ n_J & n_A & n_P \\ n_J & n_A & n_P \end{bmatrix}. \tag{3.3}$$

In Section 4 we present experimental results to show the differences of these weighting strategies.

In view of the considerations just done, we can observe that working with a symmetric stochastic Γ will always produce an unbalanced average importance for each class. In fact, if

$$\Gamma = \begin{bmatrix} 1-a-b & a & b \\ a & 1-a-c & c \\ b & c & 1-b-c \end{bmatrix}, \quad a, b, c \in [0, 1], \tag{3.4}$$

we will obtain a Perron vector equal to $1/3(1, 1, 1)^T$ and the average value of each class will be the same as in the uniform model. Of course, the actual value of each subject will change even if the average value remains the same for each choice of the parameters in (3.4). Note moreover, that for small values of the parameters, we obtain a diagonally dominant matrix, and hence, the rank of each class will depend mostly on the values within the class but the problem will become close to reducibility and we will have problem from a numerical point of view.

It is now clear that we can influence the average outcome values for each class, playing with the coefficients Γ. However, not in a direct way as one can expect, but rather solving an inverse eigenvector problem and looking for a possible 3×3 stochastic matrix Γ, with the prescribed eigenvector corresponding to the eigenvalue 1.

A question then arises spontaneous. How to force our method to rely more on citations rather then on authorship or importance of journals? More in general, which is a possible choice of Γ such that the average importance of a paper is k times that of a journal, and that of an author is h times that of a journal? To address these questions we have to look for a possible stochastic Γ with Perron vector equal to $(n_J/C, hn_A/C, kn_P/C)$ where $C = n_j + hn_A + kn_P$. One of the possible Γ is

$$\Gamma = \frac{1}{C} \begin{bmatrix} n_J & h\,n_A & k\,n_P \\ n_J & h\,n_A & k\,n_P \\ n_J & h\,n_A & k\,n_P \end{bmatrix}. \tag{3.5}$$

It is nice to look at the probabilistic interpretation, since the value γ_{ij} represent the probability to jump from class i to class j, here $i, j \in \{J, A, P\}$, we have that choosing Γ as in (3.5), with $k > h > 1$, the random evaluator will spend more time reading papers than browsing into the library examining authors or journals.

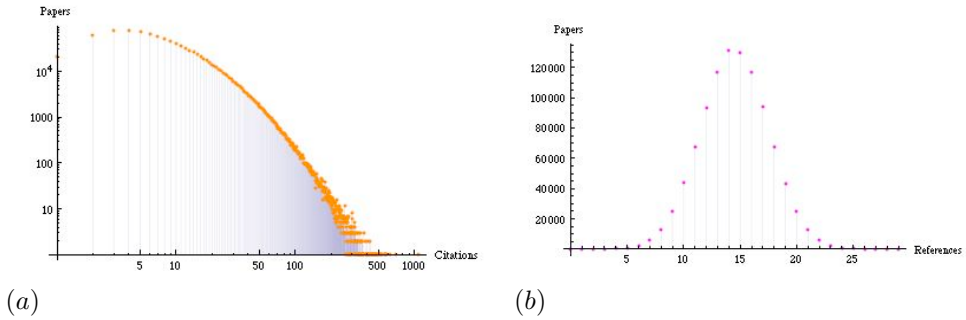
It should be clear that although it is possible to know in advance the average value of a particular class, by looking at the weight matrix Γ, we cannot predict or influence the outcome of the algorithm. In fact, the rank value of each subject is influenced by too many factors, and in particular by the citation structure, by authorship and the importance of journals.

4. Numerical experiments. In [5, 6] many examples with real and synthetic data are presented, using an uniform weighting matrix. The experiments reported in this section address two questions which are however related one to the other. The first is associated with the validation of the model on reliable data. In fact, as stressed in [6], real dataset are either not publicly available and usable, or so incomplete that the characteristics of the bibliographic items do not correspond to those recognized in real cases. In this respect, a generative model for building up synthetic matrices describing the subjects journals, authors and papers is proposed. The synthetic data produced agree with the properties monitored on real datasets, allowing us to test the algorithm on a larger set of data, where we can evaluate the robustness of our ideas on special critical situations. For example, we plan to use synthetic data to discover malicious situation where a set of papers cites each other to increase their citations count.

The second question addressed in this section is the dependence of the rank on the choice of the weight matrix Γ as discussed in Section 3. Since the tests have been performed on synthetic data, let us first discuss the generative model. The problem consists of generating the three matrices H, K and F when the parameters of the problems, that is the number of papers n_P, the number of authors n_A and the number of journals n_J variates. Of course, these parameters are strictly related one to each by a proportionality dependence. For example, when more papers are published, one should expect an increase also of n_A and n_J.

The matrix H is a $n_P \times n_P$ boolean matrix where $H_{i,j} = 1$ if paper i contains j in its reference list. To make the model more realistic we assume that, for each paper i, we know the publication year $y(i)$ as well as the incoming $\mathcal{I}(i)$ and outgoing citations $\mathcal{O}(i)$. The matrix H has to satisfy some basic requirements as well as some statistical properties observed on real data. In particular, we can recognize the following requisites

- A paper i can cite a paper j only if $y(i) \geq y(j)$, that means that i can cite only already published papers at the time i was issued.



(a) (b)

FIGURE 4.1. *Figure (a) represents s log-Log scale histogram showing the distribution of papers versus citations. There are few papers with many citations, while the big majority receives less than 5 citations each. In Figure (b) the histogram showing the distribution of references on papers.*

- The distribution of the outgoing citations $\mathcal{O}(i)$ follows a normal distribution with mean 15 and variance 3. This is motivated by the fact that it has been observed that the length of the reference list for the majority of papers in mathematics is between 10 and 20. A few papers have less than 5 and more then 25 bibliographic items.
- The distribution of incoming citations $\mathcal{I}(i)$ follows a Zipf law. In fact, a few papers receive many citations while the majority are cited seldom.

The publication year is chosen randomly in a preset interval, so that the papers are distributed uniformly over the years. To implement the Zipf distribution we used a method proposed in [21]. We assign to each paper a "quality index" $Q(i)$ which follows a normal distribution, $Q(i) \in N(\mu, \sigma)$. The index of quality will drive the citation process, so that the number of citation $c(i)$ that a paper i is going to receive is such that $c(i) \leq \lfloor 10^{Q(i)} - \gamma \rfloor$, where γ is a minimum standard of quality we ask to a paper for assuming it will receive at least a citation. For our experiments the values of $\mu = 1$ and $\sigma = 0.4$ have been chosen on the ground of the experimental results presented in [21]. It is possible to show that, in this way, the incoming citations follow a Zipf law.

The informations about the incoming and outgoing citations extracted from a randomly generated H with $n_P = 10^6$ are depicted in Figures 4.1. In particular, Figure 4.1 (a) represents an histogram in a log-log scale of papers versus citations. We see that there are many papers receiving few citations, while less than 10 papers receive hundreds of citations. In Figure 4.1(b) we see the distribution of the outgoing citations $\mathcal{O}(i)$ with a shape resembling a normal distribution (gaussian) with mean 15.

Matrix K stores information about authorship. K is a boolean matrix $n_A \times n_P$ and $K(a, p) = 1$ iff a is an author of paper p. From the analysis of real datasets [3, 12] we can see that an author has in general peaks where she is more productive and periods in her career where she writes a minor number of papers. This leads to a model where the distribution of the publications of each author follows a normal distribution over the time with a normal standard deviation which is a proper characteristic of an author. The distribution of papers for each author follows a Zipf law, in fact we can hypothesize that a few authors publish a larger amount of papers, while the majority publish a restricted number of papers. To enforce into the model the presence of

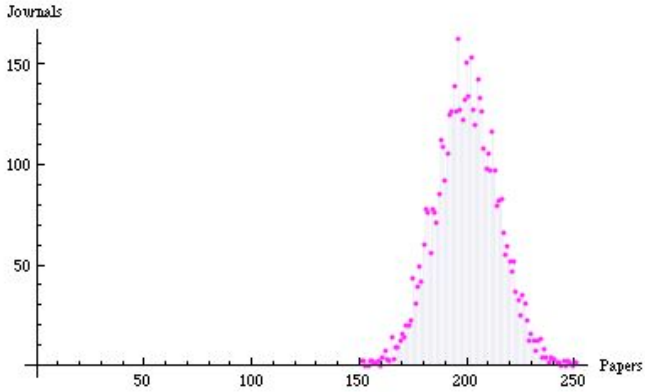


FIGURE 4.2. *Histogram of the distribution of papers on journals. It emerges the gaussian shape.*

coauthors, we have to assume that the sum of papers written by all the authors is greater than the number of distinct articles published. This guarantees that at the time of the matching between authors and papers, we can assign the same paper to more than an author.

Matrix F keeps track of the journal where a paper is published. Of course, this boolean matrix has only an entry equal to one for each column, since each paper cannot be published twice. The construction of F is done assigning uniformly papers to journals and forcing each journal to publish at least a paper. The histogram showing the resulting distribution is visualized in Figure 4.2.

We used the above generative algorithms to produce a dataset with one million of papers, half a million of authors and 5,000 journals, which respects the proportion of the cardinality of the classes in real databases [3]. We tested our methods with different choices of the weighting matrix Γ. In particular, in the results appearing below we denoted by $G1$ the uniform choice of the parameters γ as in equation (3.3) and by $G2, G3$ and $G4$ the choice of the weights in accordance with equation (3.5) for different choices of h and k. More precisely, for $h = 1, k = 1$ we have $G2$, for $h = 5, k = 1$, $G3$, and finally when $h = 1, k = 10$ we get $G4$. From the discussion carried on in Section 3, choosing as weighting technique $G1$, which corresponds to using uniform weights, the rank will depend essentially on journals. With $G2$, the mean value of a generic subject is the same, independently of the class the subject belongs to. Hence, for determining the rank of a subject we are gathering, on the average, the same amount of importance from the citing papers, from authors and from the journals. The choice of weighting techniques in accordance with $G3$ or $G4$ depicts more extreme solutions, where we give more importance to authors, and more importance to citations, respectively.

Since we are interested in the relative rank rather than on the absolute value of the rank of subjects, the plot in this section are obtained by normalizing the value to span from 0 to 1.

In Figure 4.3 we see, for the four different choices of the weights Γ, the behavior of the rank respect to the number of citations received. As expected and desired, we notice a linear dependence on the number of citations, however, this dependence is less evident in the last three plots, where for the same value of rank, there are papers receiving a number of citations belonging to a quite large interval. For example, the

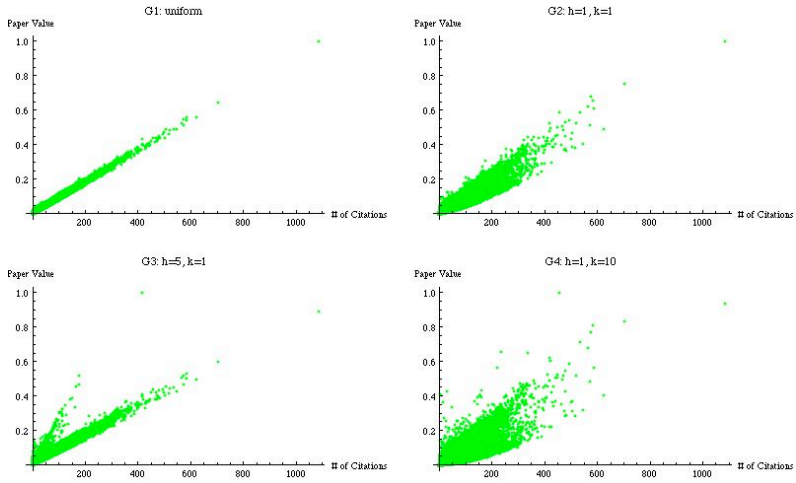


FIGURE 4.3. *Dependence of the rank of papers from the number of citation received for the 4 models.*

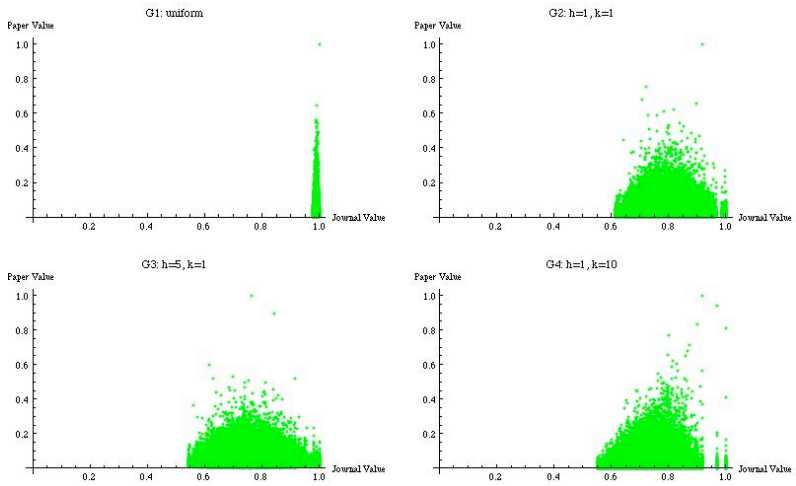


FIGURE 4.4. *Dependence of the rank of papers from the Journal value for the four models.*

plot corresponding to $G3$, where authors contribute more to the value of papers, the linear dependence is less strong. In particular we can identify two clusters of papers whose linear dependence on iterations is different. This behavior might depend on the fact that the coauthor-ship matrix has different connected components, since there are independent sets of authors working together which don't have strong connections with another group of authors. With this weighting strategy, where authors have more importance than journals or papers, the reducibility of the diagonal block A_A in (2.2) starts to emerge and, as experimentally observed, poses also problems of convergence for higher values of the parameter h.

On the other hand, using $G1$, we see that the dependence on the citations is much strong, since the rank of papers depends on the rank of journals, and the rank of journals is ruled by cross citations between journals. Plots in Figure 4.4 show the

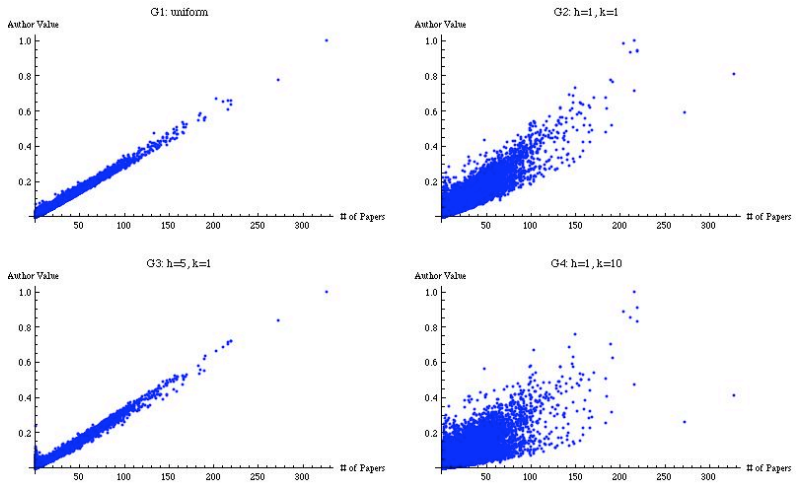


FIGURE 4.5. *For the four models it is shown the dependence of the rank of authors on the number of papers written.*

dependence of the rank of papers on the rank of the journals where they are published. Again, we see that with weights $G2$ and $G4$, it is more evident that good papers are published in good journals while it is not true the contrary, that is, on good journals, also low ranking papers can appear. When we give more importance to citations using $G4$, we see that it completely disappears the situation of good ranking papers that are published in low ranking journals. When using weight matrix $G3$, we loose the "triangle shape", because the rank depends more on the quality of the authors than on citations. In Figure 4.5 the dependence of the rank of authors from the number of papers written is shown. The relationship between authors and number of papers is similar to the one occurring between rank of papers and number of citations depicted in 4.3. Again, using as weight strategy $G2$ or $G4$, we get more interesting results, where we still have that authors publishing more papers get more chances to become important, but the importance of an author cannot be determined by a mere counting the number of papers published, but it concurs in the attribution of a ranking score to authors. Note that using $G3$, we have a clear linear dependence. It is reasonable, in fact that giving more importance to the class "Authors" we have that single authors can be compared on the basis of the number of papers written.

The rank of a Journal does not depend on the number of papers it publishes, since this was especially requested (see [6]) when designing the normalization techniques of the block (1,1) in matrix A (2.1). The dependence on the number of citations received is linear but the plot is a sort of cloud. In this case we don't have much differences among the four models.

It is interesting to comment on Figure 4.6 where the dependence on the mean value of the papers published on the journals is shown. We see that when using $G1$ and $G4$, we have a linear dependence, that is, the rank of a journal is related to the value of the average paper published therein. This is expected in the model using $G4$, since with this weighting method we are giving 10 times more importance to papers. For models based on $G2$ and $G3$, the results show that the influence of the quality of the average paper on the quality of papers is minimal. In models $G2$ and $G3$, the ranking of journals is however dominated by the sum of the importances of the

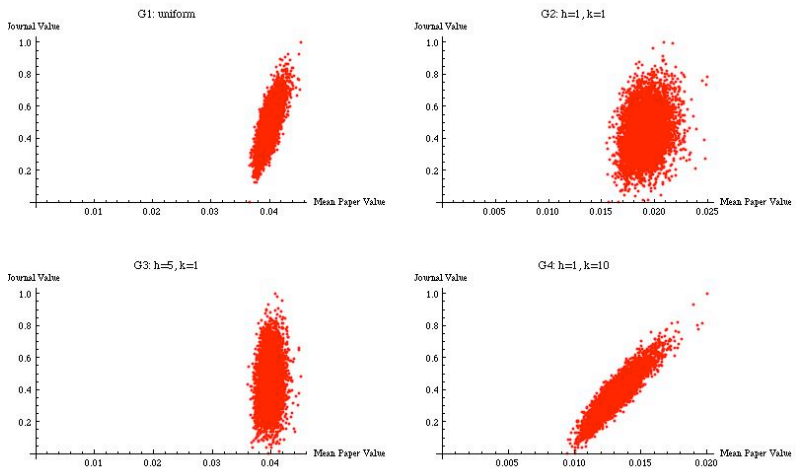


FIGURE 4.6. *For the four models it is shown the dependence of the rank of journals on the mean value of the papers published therein.*

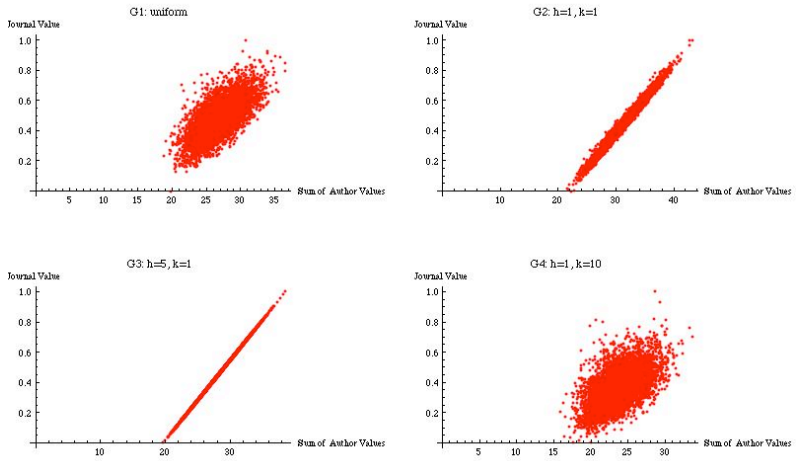


FIGURE 4.7. *For the four models it is shown the dependence of the rank of journals on the sum of the values of the authors publishing therein.*

authors publishing in those journals. In the author-centedred model $G3$ we have a very neat linear dependence.

Finally, in Figure 4.8 the journal rank is plotted against the Impact Factor over a two-year period showing that independently from the weighting strategy used, the results returned by our method are profoundly different.

A more complete set of plots of the models is available at the address `http://www.di.unipi.it/~romani/JAP4/JAP.html`.

Of course, the choice of a weight matrix rather than another, should be ruled by the particular problem one is addressing. For example, if one is interested in ranking papers on the basis of citations, model $G4$ is the more adequate. On the other hand, it can be interesting to value more coauthor-ship, because for example,

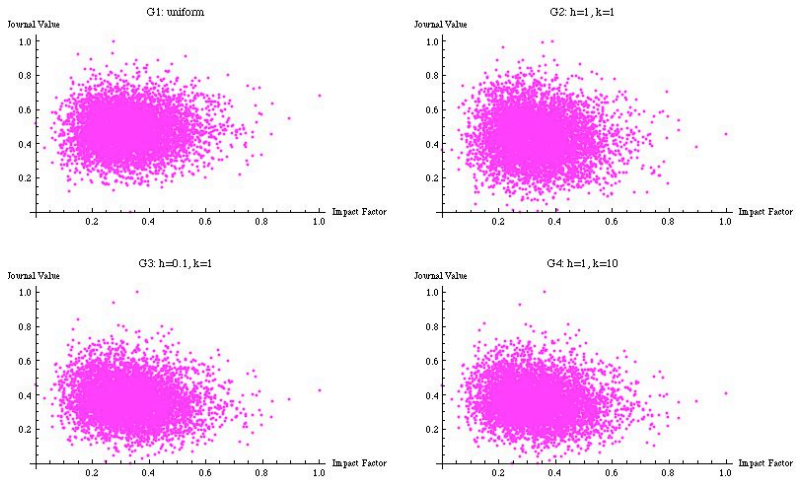


FIGURE 4.8. *For the four models it is shown that there is not a relation between The rank of journals obtained using our method and that provided by applying the two-year Impact Factor algorithm.*

we want to form a research team and we are looking at good researchers who have the ability of working with collaborators. In this case, we can pick the free parameters in accordance with $G3$, or something similar, eventually lowering the value of h to leave more space to journals and papers. Model $G2$ keeps balanced the average influence of the various classes, while with the uniform weight distribution $G1$ we provide a ranking dominated by the importance of journals.

To better understand the different choices of parameters also from a numerical viewpoint, an experimental sensitivity analysis has been performed. In particular, denote by $\bar{\pi} = (\bar{\pi}_j; \bar{\pi}_A, \bar{\pi}_P)^T$ the Perron vector of matrix P. Of course, different choices of Γ will provide with different stationary vectors $\bar{\pi}$. Let $\mathcal{S}$ be the sorting operator, which applied to a vector, returns the vector sorted in a non-increasing order. To compute an approximation of the stationary distribution of P, we use the power method combined with a stopping criterion on the infinity norm of the difference between two successive iterations. Let $\pi^{(*)}$ be the vector obtained at convergence of our method with a stopping criterion of 10^{-15}, and let $\boldsymbol{r} = (\boldsymbol{r}_J; \boldsymbol{r}_A; \boldsymbol{r}_P)^T = (\mathcal{S}(\pi_J^{(*)}); \mathcal{S}(\pi_A^{(*)}); \mathcal{S}(\pi_P^{(*)}))^T$ the rank vector sorted. Denote by $\mathcal{P}_J, \mathcal{P}_A$ and $\mathcal{P}_P$ the permutation induced by the reordering, that is $\pi_J^{(*)}(\mathcal{P}_J) = \boldsymbol{r}_J$ and similarly for the class of authors and papers. Since we are interested in the rank position rather than in the numerical value of the subjects, we analyzed experimentally, for the four models, the sensitivity to the stopping criterion. Our analysis shares the same flavor of a rigorous and theoretical analysis for the Google PageRank model where a proposal of an alternative stopping condition is carried on [22]. Let $\pi^{(i)}$ be the approximation of $\bar{\pi}$ obtained after the $i-$th step of the power method, and let $\boldsymbol{r}^{(i)}$ the vector reordered in accordance with the permutations $\mathcal{P}_J, \mathcal{P}_\mathcal{A}$ and $\mathcal{P}_P$. In Figure 4.9 the behavior of the method separately on the three classes is depicted for the different choices of the weight matrix. In particular, at each iteration is computed the number of mismatches for each class, that is, the percentage of entries of $\boldsymbol{r}^{(i)}$ which are not sorted in a non-increasing order. On the x-axis is reported the number of iterations and on the y-axis the percentages of mismatches, for example, a value of

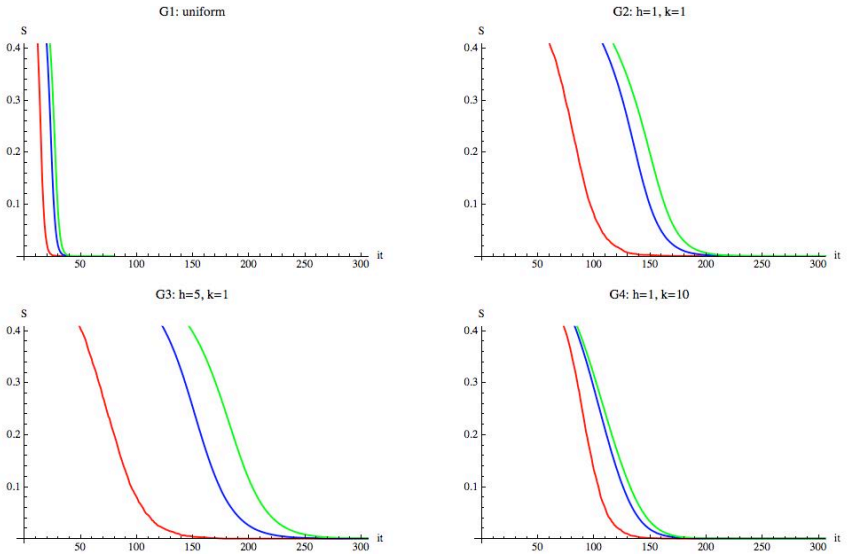


FIGURE 4.9. *For the four models, and for each class the number of mismatches for iteration is plotted. From left to right it is represented the convergence behavior for the class journals, authors and papers.*

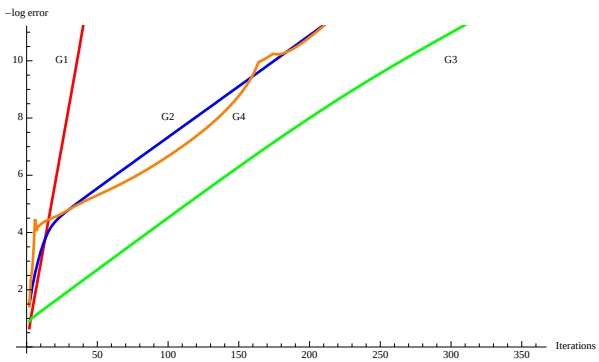


FIGURE 4.10. *Convergence behavior of the four models. On the y-axis the negative logarithm of the error expressed as the distance from the computed solution in the infinity norm.*

$S = 0.3$ at iteration i denotes that 30% of the entries are placed in the wrong position in vector $\pi^{(i)}$. From the four plots, we see that using $G1$ we get a faster convergence, respect to the other models and that the convergence of the class journals is always faster than for the other classes. Moreover, using $G3$, we see that only when we have achieved convergence on journals we start converging also on the other entries of the iteration vector, while using $G4$ the convergence is simultaneous on the three classes. In Figure 4.10 the negative logarithm of the error at each step is plotted, where the error represents the distance of iterate $i-$th from $\pi^{(*)}$ in the infinity norm. We see that all the methods have a linear convergence, and the method obtained using $G1$ as weight matrix, achieves a better convergence. The difference in the slope of the curves depends on the closeness to one of the second dominant eigenvalue. As expected, method based on $G3$ has a slower convergence, in fact with that choice of Γ, it is given more importance to authors, and the co-authorship matrix is highly reducible.

We notice that, although we used a stopping criterion with a tolerance of 10^{-15}, we get a precision of around 12 significant digits in the computed approximation $\pi^{(*)}$.

5. Conclusions. In this paper we have analyzed the performance of a method for evaluating scientific literature [5, 6] on a synthetic large dataset. In particular, we performed an experimental comparison of the dependence of the ranking provided with different choices of the nine weighting parameters presented in the model. We showed that the model analyzed in [5, 6] is tunable and that versatile weighting strategies can be applied to meet the different needs of different users. In the final part of the paper, a sensitivity analysis has been performed.

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