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Proving Translation and Reduction Semantics Equivalent for Java Simple Closures (Extended Version)

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Abstract

FGCJ is a minimal core calculus that extends Featherweight (generic) Java, FGJ, with lambda expressions. It has been used to study properties of Simple Closure in Java, including type safety and the abstraction property. Its formalization is based on a reduction semantics and a typing system that extend those of FGJ. $\mathcal{F}$ is a source-to-source, translation rule system from Java 1.5 extended with lambda expressions back to ordinary Java 1.5. It has been introduced to study implementation features of closures in Java, including assignment of non local variables and relations with anonymous class objects. In this paper we prove that the two semantics commute.

1 Introduction

In [BO10], we extend Featherweight Java [IPW01] with simple closures [BGR10,Rei10] (S-closure, for short). In that paper we define a minimal core calculus FGCJ to study properties of S-closures in Java and we provide a reduction semantics, $\rightarrow$, and prove type safety and abstraction property for S-closures. In [BO09], we extend Java 1.5 with S-closures. In that paper we provide a translation semantics, $\mathcal{F}$, which translates S-closures into objects of anonymous classes, built from single method interfaces. Based on the translation semantics, we obtain an implementation of Java 1.5 with S-closures by mapping, possibly through a preprocessor, programs of the extended language into programs of ordinary Java 1.5.

In this paper, we prove that these two semantics commute. As a consequence, we have that: (a) S-closures modeled in FGCJ are those considered in $\mathcal{F}$; (b) S-closures implemented in [BO09] satisfy the properties proved in [BO10]; (c) FGCJ and $\mathcal{F}$ are a framework to study and implement closures in Java in the form of S-closures and possibly, variants of them [Goe07]. To prove equivalence, we extend Featherweight Java, FGJ, to cope with interfaces and anonymous classes, obtaining FGAJ as a minimal core calculus for Java 1.5. We do the same for FGCJ obtaining FGACJ as a minimal core calculus for Java 1.5 extended with S-closures, Section 2. We extend the reduction semantics $\rightarrow$ on the new constructs and prove type safety for such extended calculi, Section 3. We restrict the translation semantics $\mathcal{F}$ to translate from FGACJ onto FGAJ and prove that the diagram in Fig. 1, commutes, Section 4.

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$$
\begin{tikzcd}
\mathrm{e} \in \mathrm{FGACJ} \arrow[r, "\rightarrow"] \arrow[d, "\mathcal{F}"'] & \mathrm{e'} \in \mathrm{FGACJ} \arrow[d, "\mathcal{F}"] \\
\mathcal{F}[\![\mathrm{e}]\!] \in \mathrm{FGAJ} \arrow[r, "\rightarrow"'] & \mathcal{F}[\![\mathrm{e'}]\!] \in \mathrm{FGAJ}
\end{tikzcd}
$$

Fig. 1. Commutation diagram

2 Featherweight GACJ

2.1 Notation and General Conventions

In this paper we adopt the notation used in [IPW01], accordingly $\overline{\mathtt{f}}$ is a shorthand for a possibly empty sequence $\mathtt{f}_1, \ldots, \mathtt{f}_n$ (and similarly for $\overline{\mathtt{T}}, \overline{\mathtt{x}}$, etc.) and $\overline{\mathtt{M}}$ is a shorthand for $\mathtt{M}_1 \ldots \mathtt{M}_n$ (with no commas) where n is the size $|\overline{\mathtt{f}}|$, respectively $|\overline{\mathtt{M}}|$, i.e. the number of terms of the sequence. The empty sequence is $\circ$ and symbol "," denotes concatenation of sequences. Operations on pairs of sequences are abbreviated in the obvious way $\overline{\mathtt{C}}\ \overline{\mathtt{f}}$ is $\mathtt{C}_1\ \mathtt{f}_1, \ldots, \mathtt{C}_n\ \mathtt{f}_n$ and similarly $\overline{\mathtt{C}}\ \overline{\mathtt{f}};$ is $\mathtt{C}_1\ \mathtt{f}_1; \ldots \mathtt{C}_n\ \mathtt{f}_n;$ and $\mathtt{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};$ is a shorthand for $\mathtt{this.f}_1 = \mathtt{f}_1; \ldots \mathtt{this.f}_n = \mathtt{f}_n;$ Sequences of field declarations, parameters and method declaration cannot contain duplications. Cast, $(\_)\_$, and closure definition, $\#\_\_$, have lower precedence than other operators, and cast precedes closure definition. Hence $\#()(\mathtt{this!}())$ can be written as $\#()\mathtt{this!}()$. The, possibly indexed and/or primed, metavariables $\mathtt{T}$, $\mathtt{V}$, $\mathtt{U}$, $\mathtt{S}$, $\mathtt{W}$ range over type expressions; $\mathtt{T}$ ranges over type expressions which are not closures; $\mathtt{X}$, $\mathtt{Y}$, $\mathtt{Z}$ range over type variables; $\mathtt{N}$, $\mathtt{P}$, $\mathtt{Q}$ range over class types; $\mathtt{C}$, $\mathtt{D}$, $\mathtt{E}$ range over class names; $\mathtt{f}$, $\mathtt{g}$ range over field names; $\mathtt{e}$, $\mathtt{v}$, $\mathtt{d}$ range over expressions; $\mathtt{x}$, $\mathtt{y}$ range over variable names and $\mathtt{M}$, $\mathtt{K}$, $\mathtt{L}$ and $\mathtt{m}$ range respectively, over methods, constructors, classes, and method names. $[x/y]e$ denotes the result of replacing y by x in e. Eventually $FV(\overline{\mathtt{T}})$ denotes the set of type variables in $\overline{\mathtt{T}}$.

2.2 Syntax

The abstract syntax of FGJ is at the beginning of **Table1**, followed by the syntactic rules that extend FGJ with (generic) interfaces and anonymous class object creation, defining language FGAJ. A type interface $\mathtt{I}\langle\overline{\mathtt{T}}\rangle$ is an interface name $\mathtt{I}$ and a list $\overline{\mathtt{T}}$ of the type expressions that bind the type variables $\overline{\mathtt{X}}$ of the interface declaration (see, rules $\mathtt{T}_{\mathrm{FGAJ}}$ and $\mathtt{L}_{\mathrm{FGJ}}$). In Java, a type interface may have subtypes, moreover classes may implement interfaces: We omit such features in FGAJ , since we consider interfaces only in combination with the mechanism of anonymous class object creation. Analogously, we omit the use of classes in anonymous class object creation and restrict it to only interfaces (see, rule $\mathtt{e}_{\mathrm{FGAJ}}$). The use of classes, instead of interfaces, in anonymous class object creation, is more heavy since it involves method overriding: whose formalization requires additional rules. On the other hand, such complication is unnecessary for this paper aim, since translation $\mathcal{F}$ does not use such feature, The syntax of S-closures is the one adopted in [Rei09], it includes *lambda expressions* and *function types*, it is reported in the third box of **Table1** and extends FGJ in FGCJ defining the calculus studied in [BO10]. Lambda expressions consist of closures whose body is an expression and of closures whose body is a block: Since sequencing and assignment are omitted in FGJ as well as in FGCJ, the body of a closure can only be an expression (see rule $\mathtt{F}_{\text{FGCJ}}$). Closure types extend types as rule $\mathtt{T}_{\text{FGAJ}}$ shows. A closure type $\#\mathtt{T}(\overline{\mathtt{T}})$ specifies the type sequence $(\overline{\mathtt{T}})$, possibly empty (standing for the type unit), of the arguments and the type $\mathtt{T}$ of the result. An example of closure is $\#(\mathtt{Integer\ x}, \mathtt{Integer\ y})\ (\mathtt{x}+\mathtt{y})$ which has two arguments $\mathtt{x}$ and $\mathtt{y}$, has body $\mathtt{x}+\mathtt{y}$, and type $\#\mathtt{Integer}(\mathtt{Integer}, \mathtt{Integer})$. No new generic variables can be introduced when defining a closure (reasons can be found in [Rei10]) but of course generic variables (introduced in class or method declarations) can occur in the type expressions of the arguments or be used inside closure body.

Table 1 : **Syntax**	
FGJ	
$\mathtt{T} ::= \mathtt{X} \mid \mathtt{N}$	$(\mathtt{T}_{\text{FGJ}})$
$\mathtt{N} ::= \mathtt{C}\langle\overline{\mathtt{T}}\rangle$	$(\mathtt{N}_{\text{FGJ}})$
$\mathtt{L} ::= \mathtt{class}\ \mathtt{C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}}; \mathtt{K}\ \overline{\mathtt{M}}\}$	$(\mathtt{L}_{\text{FGJ}})$
$\mathtt{K} ::= \mathtt{C}(\overline{\mathtt{T}}\ \overline{\mathtt{f}})\{\mathtt{super}(\overline{\mathtt{f}}); \mathtt{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};\}$	$(\mathtt{K}_{\text{FGJ}})$
$\mathtt{M} ::= \langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\mathtt{T}\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\}$	$(\mathtt{M}_{\text{FGJ}})$
$\mathtt{e} ::= \mathtt{x} \mid \mathtt{e.f} \mid \mathtt{e.m}\langle\overline{\mathtt{T}}\rangle(\overline{\mathtt{e}}) \mid \mathtt{new}\ \mathtt{N}(\overline{\mathtt{e}}) \mid (\mathtt{N})\mathtt{e}$	$(\mathtt{e}_{\text{FGJ}})$
IA: *Extensions for Interfaces and Anonymous Class Objects*	
$\mathtt{T} ::= \mathtt{I}\langle\overline{\mathtt{T}}\rangle$	$(\mathtt{T}_{\text{FGAJ}})$
$\mathtt{L} ::= \mathtt{interface}\ \mathtt{I}\ \langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\{\overline{\mathtt{H}}\}$	$(\mathtt{L}_{\text{FGAJ}})$
$\mathtt{H} ::= \langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\mathtt{T}\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})$	$(\mathtt{H}_{\text{FGAJ}})$
$\mathtt{e} ::= \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\ \{\overline{\mathtt{M}}\}$	$(\mathtt{e}_{\text{FGAJ}})$
Cl: *Extensions for Closures*	
$\mathtt{T} ::= \#\mathtt{T}(\overline{\mathtt{T}})$	$(\mathtt{T}_{\text{FGCJ}})$
$\mathtt{e} :: \mathtt{F} \mid \mathtt{e}\ !\ (\overline{\mathtt{e}})$	$(\mathtt{e}_{\text{FGCJ}})$
$\mathtt{F} ::= \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\mathtt{e}$	$(\mathtt{F}_{\text{FGCJ}})$
FGAJ = FGJ + IA FGACJ = FGJ + IA + Cl FGCJ = FGJ + Cl	

Eventually, at the bottom of **Table1**, the syntactic structure of the various calculi, considered in the paper, is resumed. For space convenience, the reduction rules of the semantics as well as the typing rules are not given in separate tables for each calculus. In fact, since compositionality of the semantics (we use), the rules of the various constructs are the same in all calculi containing such a construct. However, for the reader convenience, in all tables, but **Table 3**, the rules for each calculus, FGJ, FGAJ, FGCJ, FGACJ, have a label which is indexed by the name of the minimal calculus including the construct, involved in the rule. Note that $\mathtt{C}\langle\overline{\mathtt{T}}\rangle$ include $\mathtt{Object}$(since $\overline{\mathtt{T}}$ may be the empty sequence and $\mathtt{C}$ may be $\mathtt{Object}$) hence generic variables in classes and methods can be instantiated with types $\mathtt{T}$ that include interfaces or closures.

2.3 Programs

A program defines a mutually recursive scope for a collection of classes and interfaces that are: (1) well formed according to the syntax in **Table 1**, (2) well typed according to the typing rules GT-CLASS$_{\text{FGJ}}$ and GT-INTERF$_{\text{FGAJ}}$ of **Tables 4a**. The reduction semantics, `Red`, of a program is the set of all pairs (`e`,`e`') such that `e` is any closed and well typed expression (i.e. $\emptyset, \emptyset \vdash \mathtt{e} : \mathtt{T}$, for a type `T`, using the typing rules) that can be expressed using classes and interfaces of the program, and `e`' is such that $\mathtt{e} \longrightarrow \mathtt{e}'$, using

Table 2: Computation

Computation

$$\frac{\mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}}{(\mathtt{new\ N}(\overline{\mathtt{e}})).\mathtt{f}_i \longrightarrow \mathtt{e}_i} \quad (\text{GR-FIELD}_{\text{FGJ}})$$

$$\frac{\mathit{mbody}(\mathtt{m}\langle\overline{\mathtt{V}}\rangle, \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}}{(\mathtt{new\ N}(\overline{\mathtt{e}})).\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{d}}) \longrightarrow [\overline{\mathtt{d}}/\overline{\mathtt{x}}, \mathtt{new\ N}(\overline{\mathtt{e}})/\mathtt{this}]\mathtt{e}} \quad (\text{GR-INVK}_{\text{FGJ}})$$

$$\frac{\emptyset \vdash \mathtt{N} <: \mathtt{P}}{(\mathtt{P})(\mathtt{new\ N}(\overline{\mathtt{e}})) \longrightarrow \mathtt{new\ N}(\overline{\mathtt{e}})} \quad (\text{GR-CAST}_{\text{FGJ}})$$

$$\#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\mathtt{e}!(\overline{\mathtt{d}}) \longrightarrow [\overline{\mathtt{d}}/\overline{\mathtt{x}}, \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\mathtt{e}/\mathtt{this}]\mathtt{e} \quad (\text{GR-INV-CLOS}_{\text{FGCJ}})$$

$$\frac{\mathit{mbody}(\mathtt{m}\langle\overline{\mathtt{V}}\rangle, \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}) = \overline{\mathtt{x}}.\mathtt{e}}{(\mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}).\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{d}}) \longrightarrow [\overline{\mathtt{d}}/\overline{\mathtt{x}}, \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}/\mathtt{this}]\mathtt{e}} \quad (\text{GR-INVK-ANONYM}_{\text{FGAJ}})$$

Congruence

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{e}_0'}{\mathtt{e}_0.\mathtt{f} \longrightarrow \mathtt{e}_0'.\mathtt{f}} \quad (\text{GRC-FIELD}_{\text{FGJ}})$$

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{e}_0'}{\mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{T}}\rangle(\overline{\mathtt{e}}) \longrightarrow \mathtt{e}_0'.\mathtt{m}\langle\overline{\mathtt{T}}\rangle(\overline{\mathtt{e}})} \quad (\text{GRC-T-INV}_{\text{FGJ}})$$

$$\frac{\mathtt{e}_i \longrightarrow \mathtt{e}_i'}{\mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{T}}\rangle(\ldots, \mathtt{e}_i, \ldots) \longrightarrow \mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{T}}\rangle(\ldots, \mathtt{e}_i' \ldots)} \quad (\text{GRC-INV-ARG}_{\text{FGJ}})$$

$$\frac{\mathtt{e}_i \longrightarrow \mathtt{e}_i'}{\mathtt{new\ N}(\ldots, \mathtt{e}_i, \ldots) \longrightarrow \mathtt{new\ N}(\ldots, \mathtt{e}_i', \ldots)} \quad (\text{GRC-NEW}_{\text{FGJ}})$$

$$\frac{\mathtt{e} \longrightarrow \mathtt{e}'}{(\mathtt{N})\mathtt{e} \longrightarrow (\mathtt{N})\mathtt{e}'} \quad (\text{GRC-CAST}_{\text{FGJ}})$$

$$\frac{\mathtt{e} \longrightarrow \mathtt{e}'}{\#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\mathtt{e} \longrightarrow \#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\mathtt{e}'} \quad (\text{GRC-CLOS-VAL}_{\text{FGCJ}})$$

$$\frac{\mathtt{e} \longrightarrow \mathtt{e}'}{\mathtt{e}!(\overline{\mathtt{e}}) \longrightarrow \mathtt{e}'!(\overline{\mathtt{e}})} \quad (\text{GRC-INV-CLOS}_{\text{FGCJ}})$$

$$\frac{\mathtt{e}_i \longrightarrow \mathtt{e}_i'}{\mathtt{e}!(\ldots, \mathtt{e}_i, \ldots) \longrightarrow \mathtt{e}!(\ldots, \mathtt{e}_i', \ldots)} \quad (\text{GRC-CLOS-ARG}_{\text{FGCJ}})$$

the reduction semantic below: Formally, Let $\mathtt{P}$ be a program (in FGJ, FGAJ, FGACJ), then $\mathtt{Red(P)} = \{(\mathtt{e} \longrightarrow \mathtt{e'}) \mid \emptyset, \emptyset \vdash \mathtt{e} : \mathtt{T} \;\; for \;\; \mathtt{T} \in \mathtt{P}\}$.

2.4 Semantics: Reduction

The reduction semantics is given through the inference rules in **Table 2**, which define the reduction relation $\mathtt{e} \longrightarrow \mathtt{e'}$ that says that "expression $\mathtt{e}$ reduces to expression $\mathtt{e'}$ in one step". The set of expressions which cannot be further reduced is the set of *normal*

Table 3: Classes and Interfaces

Subclassing

$$\mathtt{C} \trianglelefteq \mathtt{C} \qquad \frac{\mathtt{C} \trianglelefteq \mathtt{D} \qquad \mathtt{D} \trianglelefteq \mathtt{E}}{\mathtt{C} \trianglelefteq \mathtt{E}} \qquad \frac{\mathtt{class\ C}\langle \overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}} \rangle \triangleleft \mathtt{D}\ \{\overline{\mathtt{S}}\ \overline{\mathtt{f}}; \mathtt{K}\ \overline{\mathtt{M}}\}}{\mathtt{C} \trianglelefteq \mathtt{D}}$$

Auxiliary functions

$$fields(\mathtt{Object}) = \circ \qquad \text{(F-Object)}$$

$$\frac{\mathtt{class\ C}\langle \overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}} \rangle \triangleleft \mathtt{N}\ \{\overline{\mathtt{S}}\ \overline{\mathtt{f}}; \mathtt{K}\ \overline{\mathtt{M}}\} \qquad fields([\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}) = \overline{\mathtt{U}}\ \overline{\mathtt{g}}}{fields(\mathtt{C}\langle \overline{\mathtt{T}} \rangle) = \overline{\mathtt{U}}\ \overline{\mathtt{g}}, [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}\ \overline{\mathtt{f}}} \qquad \text{(F-Class)}$$

$$\frac{\mathtt{class\ C}\langle \overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}} \rangle \triangleleft \mathtt{N}\ \{\overline{\mathtt{S}}\ \overline{\mathtt{f}}; \mathtt{K}\ \overline{\mathtt{M}}\} \qquad \langle \overline{\mathtt{Y}} \triangleleft \overline{\mathtt{P}} \rangle \mathtt{U\ m}\ (\overline{\mathtt{U}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\} \in \overline{\mathtt{M}}}{mtype(\mathtt{m}, \mathtt{C}\langle \overline{\mathtt{T}} \rangle) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}](\langle \overline{\mathtt{Y}} \triangleleft \overline{\mathtt{P}} \rangle \overline{\mathtt{U}} \to \mathtt{U})} \qquad \text{(MT-Class)}$$

$$\frac{\mathtt{class\ C}\langle \overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}} \rangle \triangleleft \mathtt{N}\ \{\overline{\mathtt{S}}\ \overline{\mathtt{f}}; \mathtt{K}\ \overline{\mathtt{M}}\} \qquad \mathtt{m} \notin \overline{\mathtt{M}}}{mtype(\mathtt{m}, \mathtt{C}\langle \overline{\mathtt{T}} \rangle) = mtype(\mathtt{m}, [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N})} \qquad \text{(MT-Super)}$$

$$\frac{\mathtt{interface\ I}\langle \overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}} \rangle\ \{\overline{\mathtt{H}}\} \qquad \langle \overline{\mathtt{Y}} \triangleleft \overline{\mathtt{P}} \rangle \mathtt{U\ m}(\overline{\mathtt{U}}\ \overline{\mathtt{x}}) \in \overline{\mathtt{H}}}{mtype(\mathtt{m}, \mathtt{I}\langle \overline{\mathtt{T}} \rangle) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}](\langle \overline{\mathtt{Y}} \triangleleft \overline{\mathtt{P}} \rangle \overline{\mathtt{U}} \to \mathtt{U})} \qquad \text{(MT-Interface)}$$

$$\frac{\mathtt{class\ C}\langle \overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}} \rangle \triangleleft \mathtt{N}\ \{\overline{\mathtt{S}}\ \overline{\mathtt{f}}; \mathtt{K}\ \overline{\mathtt{M}}\} \qquad \langle \overline{\mathtt{Y}} \triangleleft \overline{\mathtt{P}} \rangle \mathtt{U\ m}\ (\overline{\mathtt{U}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\} \in \overline{\mathtt{M}}}{mbody(\mathtt{m}\langle \overline{\mathtt{V}} \rangle, \mathtt{C}\langle \overline{\mathtt{T}} \rangle) = \overline{\mathtt{x}}.[\overline{\mathtt{T}}/\overline{\mathtt{X}}, \overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{e}} \qquad \text{(MB-Class)}$$

$$\frac{\mathtt{class\ C}\langle \overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}} \rangle \triangleleft \mathtt{N}\ \{\overline{\mathtt{S}}\ \overline{\mathtt{f}}; \mathtt{K}\ \overline{\mathtt{M}}\} \qquad \mathtt{m} \notin \overline{\mathtt{M}}}{mbody(\mathtt{m}\langle \overline{\mathtt{V}} \rangle, \mathtt{C}\langle \overline{\mathtt{T}} \rangle) = mbody(\mathtt{m}\langle \overline{\mathtt{V}} \rangle, [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N})} \qquad \text{(MB-Super)}$$

$$\frac{\mathtt{interface\ I}\langle \overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}} \rangle\ \{...\} \qquad \langle \overline{\mathtt{Y}} \triangleleft \overline{\mathtt{P}} \rangle \mathtt{U\ m}\ (\overline{\mathtt{U}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\} \in \overline{\mathtt{M}}}{mbody(\mathtt{m}\langle \overline{\mathtt{V}} \rangle, \mathtt{new\ I}\langle \overline{\mathtt{T}} \rangle()\{\overline{\mathtt{M}}\}) = \overline{\mathtt{x}}.[\overline{\mathtt{T}}/\overline{\mathtt{X}}, \overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{e}} \qquad \text{(MB-Interface)}$$

Auxiliary predicates

$$\mathtt{override}(\mathtt{m}, \mathtt{Object}, \langle \overline{\mathtt{Y}} \triangleleft \overline{\mathtt{P}} \rangle \overline{\mathtt{T}} \to \mathtt{T}_0) \qquad \text{(Over-Object)}$$

$$\frac{\begin{array}{l} mtype(\mathtt{m}, \mathtt{N}) = \langle \overline{\mathtt{Z}} \triangleleft \overline{\mathtt{Q}} \rangle \overline{\mathtt{U}} \to \mathtt{U}_0\ \mathtt{implies} \\ \qquad ((\overline{\mathtt{P}}, \overline{\mathtt{T}}) = [\overline{\mathtt{Y}}/\overline{\mathtt{Z}}](\overline{\mathtt{Q}}, \overline{\mathtt{U}})\ \mathtt{and}\ \overline{\mathtt{Y}} <: \overline{\mathtt{P}} \vdash \mathtt{T}_0 <: [\overline{\mathtt{Y}}/\overline{\mathtt{Z}}]\mathtt{U}_0) \end{array}}{\mathtt{override}(\mathtt{m}, \mathtt{N}, \langle \overline{\mathtt{Y}} \triangleleft \overline{\mathtt{P}} \rangle \overline{\mathtt{T}} \to \mathtt{T}_0)} \qquad \text{(Over)}$$

DCast

$$\frac{dcast(\mathtt{C}, \mathtt{D}) \qquad dcast(\mathtt{D}, \mathtt{E})}{dcast(\mathtt{C}, \mathtt{E})} \qquad \frac{\mathtt{class\ C}\langle \overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}} \rangle \triangleleft \mathtt{D}\langle \overline{\mathtt{T}} \rangle\ \{\ldots\} \qquad \overline{\mathtt{X}} = FV(\overline{\mathtt{T}})}{dcast(\mathtt{C}, \mathtt{D})} \qquad \text{(DCast)}$$

forms and constitute values of the calculus. In FGACJ values are objects, constructed out of an anonymous or named class, and of closures. Hence the grammatical category v defines the syntactic form of the values (domain) of the calculus FGACJ:

$$\begin{aligned}\mathtt{v} ::= {} & \mathtt{new}\ \mathtt{N}(\overline{\mathtt{v}}) \\ & \mid \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\} \\ & \mid \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\mathtt{e}\end{aligned}$$

This structure of values results from the reduction rules of the calculus. The rules indexed by FGJ are the same as those of calculus FGJ introduced in [IPW01], and those

Table 4: Typing Rules

$$\Delta; \Gamma \vdash \mathtt{x} : \Gamma(\mathtt{x}) \qquad (\text{GT-Var}_{\text{FGJ}})$$

$$\frac{\Delta; \Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}}{\Delta; \Gamma \vdash \mathtt{e}_0.\mathtt{f}_i : \mathtt{T}_i} \qquad (\text{GT-Field}_{\text{FGJ}})$$

$$\frac{\begin{array}{c}\mathit{mtype}(\mathtt{m}, \mathit{bound}_\Delta(\mathtt{T}_0)) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \to \mathtt{U} \\ \Delta; \Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta \vdash \overline{\mathtt{V}}\ \mathtt{ok} \qquad \Delta \vdash \overline{\mathtt{V}} <: [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}} \\ \Delta; \Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}} \qquad \Delta \vdash \overline{\mathtt{S}} <: [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}}\end{array}}{\Delta; \Gamma \vdash \mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{e}}) : [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}} \qquad (\text{GT-Inv}_{\text{FGJ}})$$

$$\frac{\begin{array}{c}\mathit{mtype}(\mathtt{m}, \mathtt{I}\langle\overline{\mathtt{T}}\rangle) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \to \mathtt{U} \\ \Delta; \Gamma \vdash \mathtt{e}_0 : \mathtt{I}\langle\overline{\mathtt{T}}\rangle \qquad \Delta \vdash \overline{\mathtt{V}}\ \mathtt{ok} \qquad \Delta \vdash \overline{\mathtt{V}} <: [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}} \\ \Delta; \Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}} \qquad \Delta \vdash \overline{\mathtt{S}} <: [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}}\end{array}}{\Delta; \Gamma \vdash \mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{e}}) : [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}} \qquad (\text{GT-AnonymInv}_{\text{FGAJ}})$$

$$\frac{\begin{array}{c}\Delta \vdash \mathtt{N}\ \mathtt{ok} \qquad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \\ \Delta; \Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}} \qquad \Delta \vdash \overline{\mathtt{S}} <: \overline{\mathtt{T}}\end{array}}{\Delta; \Gamma \vdash \mathtt{new}\ \mathtt{N}(\overline{\mathtt{e}}) : \mathtt{N}} \qquad (\text{GT-New}_{\text{FGJ}})$$

$$\frac{\Delta \vdash \mathtt{I}\langle\overline{\mathtt{T}}\rangle\ \mathtt{ok} \qquad \Delta; \Gamma \vdash \overline{\mathtt{M}}\ \ \mathtt{OK\,IN}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle}{\Delta; \Gamma \vdash \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\} : \mathtt{I}\langle\overline{\mathtt{T}}\rangle} \qquad (\text{GT-AnonymNew}_{\text{FGAJ}})$$

$$\frac{\Delta; \Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta \vdash \mathit{bound}_\Delta(\mathtt{T}_0) <: \mathtt{N}}{\Delta; \Gamma \vdash (\mathtt{N})\mathtt{e}_0 : \mathtt{N}} \qquad (\text{GT-UCast}_{\text{FGJ}})$$

$$\frac{\begin{array}{c}\Delta; \Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta \vdash \mathtt{N}\ \mathtt{ok} \qquad \Delta \vdash \mathtt{N} <: \mathit{bound}_\Delta(\mathtt{T}_0) \\ \mathtt{N} = \mathtt{C}\langle\overline{\mathtt{T}}\rangle \qquad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{D}\langle\overline{\mathtt{T}}\rangle \qquad \mathit{dcast}(\mathtt{C}, \mathtt{D})\end{array}}{\Delta; \Gamma \vdash (\mathtt{N})\mathtt{e}_0 : \mathtt{N}} \qquad (\text{GT-DCast}_{\text{FGJ}})$$

$$\frac{\begin{array}{c}\Delta; \Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta \vdash \mathtt{N}\ \mathtt{ok} \\ \mathtt{N} = \mathtt{C}\langle\overline{\mathtt{T}}\rangle \qquad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{D}\langle\overline{\mathtt{U}}\rangle \qquad \mathtt{C} \not\trianglelefteq \mathtt{D} \qquad \mathtt{D} \not\trianglelefteq \mathtt{C}\end{array}}{\Delta; \Gamma \vdash (\mathtt{N})\mathtt{e}_0 : \mathtt{N}} \qquad (\text{GT-SCast}_{\text{FGJ}})$$

$$\frac{\Delta \vdash \overline{\mathtt{T}}\ \mathtt{ok} \qquad \Delta; \Gamma, \overline{\mathtt{x}} : \overline{\mathtt{T}}, \mathtt{this} : \#\mathtt{T}(\overline{\mathtt{T}}) \vdash \mathtt{e}\ : \mathtt{T}}{\Delta; \Gamma \vdash \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\ \mathtt{e} : \#\mathtt{T}(\overline{\mathtt{T}})} \qquad (\text{GT-Closure}_{\text{FGCJ}})$$

$$\frac{\Delta; \Gamma \vdash \mathtt{e} : \#\mathtt{T}(\overline{\mathtt{T}}) \qquad \Delta; \Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}} \qquad \Delta \vdash \overline{\mathtt{S}} <: \overline{\mathtt{T}}}{\Delta; \Gamma \vdash \mathtt{e!}(\overline{\mathtt{e}}) : \mathtt{T}} \qquad (\text{GT-Closure-Inv}_{\text{FGCJ}})$$

indexed by FGCJ are the same as those of the calculus FGCJ introduced in [BO10]. In particular, they include rule GR-Invk-Clos that reduces a closure invocation replacing it by the closure body in which the formal parameters are replaced by the corresponding actual ones, and `this` is replaced by the closure itself, thus allowing *recursive closures*. We have only one new rule, GR-Invk-Anonym$_{\text{FGAJ}}$, which is indexed by FGACJ and gives the semantics of invocation with anonymous class objects. The rule is similar to the one of method invocation with object of named classes. In fact, the two kinds of invocation may be formulated similarly provided that the auxiliary functions `mtype` and `mbody`, introduced in **Table3**, are suitably extended to select the type and the body of anonymous class objects (see rules MT-Interface and MB-Interface). Moreover, since anonymous class object creation is formulated as a new expression that extends the calculus FGJ (resp. FGCJ), the rules of congruence of [IPW01] (resp. [BO10]) are unchanged.

2.5 Semantics: Typing

The typing rules are given through inference rules that use two different kinds of environment, Δ (for type variables) and Γ (for value variables), and five different typing judgements: one for each different term structure of the language. A (well formed) type

Table 4a: Typing Rules

Classes, Interfaces, Methods

$$\frac{\begin{array}{c}\Delta = \overline{\mathtt{X}}<:\overline{\mathtt{N}},\ \overline{\mathtt{Y}}<:\overline{\mathtt{P}} \qquad \Delta \vdash \overline{\mathtt{T}},\mathtt{T},\overline{\mathtt{P}}\ \mathtt{ok} \\ \Delta;\overline{\mathtt{x}}:\overline{\mathtt{T}},\mathtt{this}:\mathtt{C}\langle\overline{\mathtt{X}}\rangle \vdash \mathtt{e}_0:\mathtt{S} \qquad \Delta\vdash \mathtt{S}<:\mathtt{T} \\ \mathtt{class}\ \mathtt{C}\langle\overline{\mathtt{X}}\lhd\overline{\mathtt{N}}\rangle\lhd\mathtt{N}\{...\} \qquad \mathtt{override}(\mathtt{m},\mathtt{N},\langle\overline{\mathtt{Y}}\lhd\overline{\mathtt{P}}\rangle\overline{\mathtt{T}}\to\mathtt{T})\end{array}}{\langle\overline{\mathtt{Y}}\lhd\overline{\mathtt{P}}\rangle\mathtt{T}\ \ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e}_0;\}\ \mathtt{OK\,IN}\ \mathtt{C}\langle\overline{\mathtt{X}}\lhd\overline{\mathtt{N}}\rangle} \qquad (\text{GT-Method}_{\text{FGJ}})$$

$$\frac{\overline{\mathtt{Y}}<:\overline{\mathtt{P}},\overline{\mathtt{X}}<:\overline{\mathtt{N}}\vdash\overline{\mathtt{T}},\mathtt{T},\overline{\mathtt{P}}\ \mathtt{ok}}{\langle\overline{\mathtt{Y}}\lhd\overline{\mathtt{P}}\rangle\mathtt{T}\ \ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\ \mathtt{OK\,IN}\ \mathtt{I}\langle\overline{\mathtt{X}}\lhd\overline{\mathtt{N}}\rangle} \qquad (\text{GT-Header}_{\text{FGAJ}})$$

$$\frac{\begin{array}{c}\Delta' = \Delta,\overline{\mathtt{X}}<:\overline{\mathtt{N}},\ \overline{\mathtt{Y}}<:\overline{\mathtt{P}} \qquad \Delta';\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}},\mathtt{this}:\mathtt{I}\langle\overline{\mathtt{V}}\rangle\vdash\mathtt{e}_0:\mathtt{S} \\ \Delta'\vdash\overline{\mathtt{T}},\mathtt{T},\overline{\mathtt{P}}\ \mathtt{ok} \qquad \Delta'\vdash\overline{\mathtt{V}}<:[\overline{\mathtt{V}}/\overline{\mathtt{X}}]\overline{\mathtt{N}} \qquad \Delta'\vdash\mathtt{S}<:\mathtt{T} \\ \mathtt{interface}\ \mathtt{I}\langle\overline{\mathtt{X}}\lhd\overline{\mathtt{N}}\rangle\{\overline{\mathtt{H}}\} \qquad \langle\overline{\mathtt{Y}}\lhd\overline{\mathtt{P}}\rangle\mathtt{T}\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\ \in\ \overline{\mathtt{H}}\end{array}}{\Delta;\Gamma\vdash\langle\overline{\mathtt{Y}}\lhd\overline{\mathtt{P}}\rangle\mathtt{T}\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\uparrow\mathtt{e}_0;\}\ \mathtt{OK\,IN}\ \mathtt{I}\langle\overline{\mathtt{V}}\rangle} \qquad (\text{GT-Anonym}_{\text{FGAJ}})$$

$$\frac{\begin{array}{c}\overline{\mathtt{X}}<:\overline{\mathtt{N}}\vdash\overline{\mathtt{N}},\mathtt{N},\overline{\mathtt{T}}\ \mathtt{ok} \qquad \overline{\mathtt{M}}\ \ \mathtt{OK\,IN}\ \mathtt{C}\langle\overline{\mathtt{X}}\lhd\overline{\mathtt{N}}\rangle \\ \mathit{fields}(\mathtt{N})=\overline{\mathtt{U}}\ \overline{\mathtt{g}} \qquad \mathtt{K}=\mathtt{C}(\overline{\mathtt{U}}\ \overline{\mathtt{g}},\overline{\mathtt{T}}\ \overline{\mathtt{f}})\{\text{super}(\overline{\mathtt{g}});\ \text{this}.\overline{\mathtt{f}}=\overline{\mathtt{f}};\}\end{array}}{\mathtt{class}\ \mathtt{C}\langle\overline{\mathtt{X}}\lhd\overline{\mathtt{N}}\rangle\lhd\mathtt{N}\{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\mathtt{K}\ \overline{\mathtt{M}}\}\ \mathtt{OK}} \qquad (\text{GT-Class}_{\text{FGJ}})$$

$$\frac{\overline{\mathtt{X}}<:\overline{\mathtt{N}}\vdash\overline{\mathtt{N}}\ \mathtt{ok} \qquad \overline{\mathtt{H}}\ \ \mathtt{OK\,IN}\ \mathtt{I}\langle\overline{\mathtt{X}}\lhd\overline{\mathtt{N}}\rangle}{\mathtt{interface}\ \mathtt{I}\langle\overline{\mathtt{X}}\lhd\overline{\mathtt{N}}\rangle\{\overline{\mathtt{H}}\}\ \mathtt{OK}} \qquad (\text{GT-Interf}_{\text{FGAJ}})$$

environment Δ is a mapping from type variables to (well formed, in Δ) types written as a list of $\mathtt{X}<:\mathtt{T}$, meaning that type variable `X` must be bound to a subtype of type `T`: $\Delta(\mathtt{X}) = \mathtt{T}$ if Δ contains $\mathtt{X}<:\mathtt{T}$, undefined otherwise (i.e. $\mathtt{X} \notin dom(\Delta)$). An environment Γ is a mapping from variables to types written as a list of $\mathtt{x} : \mathtt{T}$ meaning that "`x` has type `T`": $\Gamma(\mathtt{x}) = \mathtt{T}$ if Γ contains $\mathtt{x} : \mathtt{T}$, undefined otherwise (i.e. $\mathtt{x} \notin dom(\Gamma)$). When needed and without loss of generality, variable renaming is used to avoid name collision among environment bindings. The judgement for a (generic) type `T` (see **Table 5**) has the form

Table 5: Subtypes	
Subtypes	
$bound_{\Delta}(\mathtt{X}) = \Delta(\mathtt{X})$	(B-VAR$_{\text{FGJ}}$)
$bound_{\Delta}(\mathtt{N}) = \mathtt{N}$	(B-CLASS$_{\text{FGJ}}$)
$\Delta \vdash \mathtt{T} <: \mathtt{T}$	(S-REFL$_{\text{FGJ}}$)
$\dfrac{\Delta \vdash \mathtt{S} <: \mathtt{T} \qquad \Delta \vdash \mathtt{T} <: \mathtt{U}}{\Delta \vdash \mathtt{S} <: \mathtt{U}}$	(S-TRANS$_{\text{FGJ}}$)
$\Delta \vdash \mathtt{X} <: \Delta(\mathtt{X})$	(S-VAR$_{\text{FGJ}}$)
$\dfrac{\mathtt{class}\ \mathtt{C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\{\ldots\}}{\Delta \vdash \mathtt{C}\langle\overline{\mathtt{T}}\rangle <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}}$	(S-CLASS$_{\text{FGJ}}$)
Well-formed types	
$\Delta \vdash$ Object $\mathtt{ok}$	(WF-OBJECT$_{\text{FGJ}}$)
$\dfrac{\mathtt{X} \in dom(\Delta)}{\Delta \vdash \mathtt{X}\ \mathtt{ok}}$	(WF-VAR$_{\text{FGJ}}$)
$\dfrac{\mathtt{class}\ \mathtt{C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\{\ldots\} \qquad \Delta \vdash \overline{\mathtt{T}}\ \mathtt{ok} \qquad \Delta \vdash \overline{\mathtt{T}} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}}{\Delta \vdash \mathtt{C}\langle\overline{\mathtt{T}}\rangle\ \mathtt{ok}}$	(WF-CLASS$_{\text{FGJ}}$)
$\dfrac{\mathtt{interface}\ \mathtt{I}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\{\ldots\} \qquad \Delta \vdash \overline{\mathtt{T}}\ \mathtt{ok} \qquad \Delta \vdash \overline{\mathtt{T}} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}}{\Delta \vdash \mathtt{I}\langle\overline{\mathtt{T}}\rangle\ \mathtt{ok}}$	(WF-INTERF$_{\text{FGAJ}}$)
$\dfrac{\Delta \vdash \overline{\mathtt{T}}\ \mathtt{ok} \qquad \Delta \vdash \mathtt{T}\ \mathtt{ok}}{\Delta \vdash \#\mathtt{T}(\overline{\mathtt{T}})\ \mathtt{ok}}$	(WF-CLOSURE$_{\text{FGCJ}}$)

$\Delta \vdash \mathtt{T}\ \mathtt{ok}$ meaning that "T is a well-formed type in the (well formed) type environment Δ". The typing judgements for subtyping (see **Table 5**) has the form $\Delta \vdash \mathtt{S} <: \mathtt{T}$ meaning that "S is a subtype of T in Δ". The judgement for classes (see rule GT-CLASS$_{\text{FGJ}}$ in **Table 4a**) has the form C OK meaning that "C is well typed". The typing judgements for methods (see GT-METHOD$_{\text{FGJ}}$ in **Table 4a**) has the form M OK IN C meaning that "M is well typed when its declaration occurs in class C"[1]. The judgement for expressions (see the rules of **Table 4**) has the form $\Delta; \Gamma \vdash \mathtt{e} : \mathtt{T}$ meaning that expression e has type T in the typing environment Δ and in the (variable) en-

[1] For methods in instances, the judgement is the same than for classes but the inference has to consider that these methods are defined for interface instantiations instead of interfaces. These instances may be defined within other methods [hence the presence of Γ for non local variables] that are in classes [hence the presence of Δ for the type variables of the class or of the method in which the instance is defined]). In particular, GT-ANONYM$_{\text{FGAJ}}$ is introduced for rule GT-ANONYMNEW$_{\text{FGAJ}}$ that defines the type of $\mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\})$ in a way that checks that all the methods in $\overline{\mathtt{M}}$ are well defined and correctly typed: Not the only methods on which the object is invoked. In the different choice in which we limit correctness to the only methods on which the object is invoked, then rules GT-ANONYMINV$_{\text{FGAJ}}$ and GT-ANONYMNEW$_{\text{FGAJ}}$ have to be modified to check correctness only for the method involved in the invocation

vironment Γ. The typing rules are contained in **Table 4** and extends those of FGJ. Two rules have been added for closure construction and closure invocation. Such rules simply assert the correctness of the involved types. Four rules have been added for typing (GT-AnonymInv$_{\text{FGAJ}}$), judgement OK (GT-Interface$_{\text{FGAJ}}$), judgement OK IN (GT-Interf$_{\text{FGAJ}}$, GT-Anonym$_{\text{FGAJ}}$). The rules for subtypes and wellformed types are reported in **Table 5**.

3 Properties

Semantics is useful to prove language properties: We extend to FGACJ type soundness and backward compatibility already proved for FGJ [IPW01] and for FGCJ [BO10]. Then, we extend to FGACJ the closure abstraction property already proved for FGCJ [BO11b]. All Lemma and Theorem proofs are deferred to the Appendix A.

Theorem 1 (Progress). *Suppose* $\mathtt{e}$ *is a well-typed expression. If* $\mathtt{e}$ *includes as a subexpression:*

1. $\mathtt{new\ N(\overline{e}).f}$ *then* $fields(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$, *for some* $\overline{\mathtt{T}}$ *and* $\overline{\mathtt{f}}$, *and* $\mathtt{f} \in \overline{\mathtt{f}}$.
2. $\mathtt{new\ N(\overline{e}).m\langle\overline{V}\rangle(\overline{d})}$ *then* $mbody(\mathtt{m\langle\overline{V}\rangle, N}) = \overline{\mathtt{x}}.\mathtt{e}_0$, *for some* $\overline{\mathtt{x}}$ *and* $\mathtt{e}_0$, *and* $|\overline{\mathtt{x}}| = |\overline{\mathtt{d}}|$.
3. $\mathtt{new\ I\langle\overline{T}\rangle()\{\overline{M}\}.m\langle\overline{V}\rangle(\overline{d})}$ *then* $mbody(\mathtt{m\langle\overline{V}\rangle, new\ I\langle\overline{T}\rangle()\{\overline{M}\}}) = \overline{\mathtt{x}}.\mathtt{e}_0$, *for some* $\overline{\mathtt{x}}$ *and* $\mathtt{e}_0$, *and* $|\overline{\mathtt{x}}| = |\overline{\mathtt{d}}|$.
4. $\mathtt{F!(\overline{d})}$ *then* $\mathtt{F} = \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\ \mathtt{e}_0$, *for some* $\overline{\mathtt{T}}$, $\overline{\mathtt{x}}$ *and* $\mathtt{e}_0$, *and* $|\overline{\mathtt{x}}| = |\overline{\mathtt{d}}|$. □

Theorem 2 (Type Soundness). *If* $\emptyset;\emptyset \vdash_{\text{FGACJ}} \mathtt{e : T}$ *and* $\mathtt{e} \rightarrow^*_{\text{FGACJ}} \mathtt{e}'$ *with* $\mathtt{e}'$ *a normal form, then* $\mathtt{e}'$ *is a value* $\mathtt{w}$ *with* $\emptyset;\emptyset \vdash_{\text{FGACJ}} \mathtt{w : S}$ *and* $\emptyset \vdash_{\text{FGACJ}} \mathtt{S} <: \mathtt{T}$. □

Theorem 3 (Abstraction Property). *Let* $\Delta \vdash_{\text{FGACJ}} \mathtt{T\ ok}$, $\mathtt{H}[\bullet]$ *be any context,* $\mathtt{G}[\bullet]$ *be any context of type* $(\Gamma, \mathtt{T})$[2] *and with no free occurrences of* `this`. *Let* $\mathtt{e}_2$ *be any expression such that its free variables are not bound in* $\mathtt{e}_1 \equiv \mathtt{G}[\mathtt{e}_2]$ *(but possibly, in* $\mathtt{H}[\bullet]$*). Then* $\mathtt{H}[(\#(\mathtt{T\ x})\mathtt{G}[\mathtt{x}])!(\mathtt{e}_2)] \approx \mathtt{H}[\mathtt{e}_1]$, *for any fresh variable* $\mathtt{x}$. □

Theorem 4 (Backward compatibility). *If an* FGACJ *program is well typed under the* FGCJ *rules it is also well typed under the* FGACJ *rules. Moreover, for all* FGCJ *programs* e *and* e' *(whether well typed or not)* $\mathtt{e} \rightarrow_{\text{FGACJ}} \mathtt{e}' \iff \mathtt{e} \rightarrow_{\text{FGCJ}} \mathtt{e}'$. □

4 The Translation Semantics of Java Simple Closures

The translation semantics $\mathcal{F}[\![]\!]_\tau$ of S-closures has been defined in [BO11b], it is based on the structures of *interfaces*, *anonymous classes*, and *classes* (of variable objects) and translates S-closures into a composition of such structures. In this paper, we simplifies $\mathcal{F}[\![]\!]_\tau$ in order to apply it to FGACJ, instead of Java 1.5 extended with S-closures and to translate onto FGAJ instead of ordinary Java 1.5. FGACJ (resp FGAJ) is used as a minimal core calculus for Java 1.5 extended with closures (resp. ordinary Java 1.5), to formalize the translation semantics of closures with anonymous class objects. It is shown in Fig. 2, where J is a syntactic projection of Java 1.5 onto FGAJ.

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[2] A *context of type* $(\Gamma, \mathtt{T})$ is any context $\mathtt{H}[\bullet]$, in FGCJ, such that $\Delta; \Gamma, \mathtt{x} : \mathtt{T} \vdash \mathtt{H}[\mathtt{x}] : \mathtt{S}$ for some $\Delta \vdash \mathtt{T\ ok}$ and type $\mathtt{S}$, and fresh variable $\mathtt{x}$[BO11b]. The self reference `this` occurs bound, in a context (or an expression), only when it occurs inside a closure or a method defined in such a context (or expression). In all the other cases, `this` occurs free.

$$\begin{tikzcd} e\in \text{Java1.5}^{E} \arrow[r, "\mathcal{F}"] \arrow[d, "J"'] & e'\in \text{Java1.5} \arrow[d, "J"] \\ J(e)\in \text{FGACJ} \arrow[r, "\mathcal{F}"'] & J(e')\in \text{FGAJ} \end{tikzcd}$$

Fig. 2. FGACJ is a Minimal Core Calculus for Translation $\mathcal{F}$ on Java 1.5 with S-Closures

A first and most evident simplification, is the elimination of parameter τ. In the original translation τ contained bindings for Java variables, which are not present in FGACJ [3]. In **Table 6** we report the definition of $\mathcal{F}\llbracket\rrbracket$ restricted to FGACJ.

Table 6: $\mathcal{F}\llbracket\rrbracket$ Translation Semantics	
Translation Rules	
1t. $\mathcal{F}\llbracket \mathtt{T}\rrbracket = \mathtt{T}$	with $\mathtt{T} \equiv \mathtt{X}$
2t. $\mathcal{F}\llbracket \mathtt{T}\rrbracket = \mathtt{I\$n}\langle \mathcal{F}\llbracket \overline{\mathtt{S}}\rrbracket \mathcal{F}\llbracket \mathtt{S}\rrbracket\rangle$	with $\mathtt{T} \equiv \#\mathtt{S}(\overline{\mathtt{S}}),\ \mathtt{n} = \lvert\overline{\mathtt{T}}\rvert$
3t. $\mathcal{F}\llbracket \mathtt{T}\rrbracket = \mathtt{A}\langle \mathcal{F}\llbracket \overline{\mathtt{T}}\rrbracket\rangle$	with $\mathtt{T} \equiv \mathtt{A}\langle\overline{\mathtt{T}}\rangle,\ (\mathtt{A} \equiv \mathtt{C}$ or $\mathtt{A} \equiv \mathtt{I})$
1l. $\mathcal{F}\llbracket \mathtt{L}\rrbracket = \mathtt{class\ C}\langle \overline{\mathtt{X}} \lhd \mathcal{F}\llbracket\overline{\mathtt{N}}\rrbracket\rangle \lhd \mathcal{F}\llbracket \mathtt{N}\rrbracket\ \{$ $\mathcal{F}\llbracket\overline{\mathtt{T}}\rrbracket\ \overline{\mathtt{f}}; \mathcal{F}\llbracket \mathtt{K}\rrbracket \mathcal{F}\llbracket\overline{\mathtt{M}}\rrbracket\}$	with $\mathtt{L} \equiv \mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}}; \mathtt{K}\ \overline{\mathtt{M}}\}$
2l. $\mathcal{F}\llbracket \mathtt{L}\rrbracket = \mathtt{interface\ I}\langle \overline{\mathtt{X}} \lhd \mathcal{F}\llbracket\overline{\mathtt{N}}\rrbracket\rangle\{\mathcal{F}\llbracket\overline{\mathtt{H}}\rrbracket\}$	with $\mathtt{L} \equiv \mathtt{interface\ I}\ \langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\{\overline{\mathtt{H}}\}$
k. $\mathcal{F}\llbracket \mathtt{K}\rrbracket = \mathtt{C}(\mathcal{F}\llbracket\overline{\mathtt{T}}\rrbracket\ \overline{\mathtt{f}})\{\mathtt{super}(\overline{\mathtt{f}}); \mathtt{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};\}$	with $\mathtt{K} \equiv \mathtt{C}(\overline{\mathtt{T}}\ \overline{\mathtt{f}})\{\mathtt{super}(\overline{\mathtt{f}}); \mathtt{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};\}$
m. $\mathcal{F}\llbracket \mathtt{M}\rrbracket = \langle\overline{\mathtt{X}} \lhd \mathcal{F}\llbracket\overline{\mathtt{N}}\rrbracket\rangle\ \mathcal{F}\llbracket \mathtt{T}\rrbracket\ \mathtt{m}(\mathcal{F}\llbracket\overline{\mathtt{T}}\rrbracket\ \overline{\mathtt{x}})\{\uparrow \mathcal{F}\llbracket \mathtt{e}\rrbracket\}$	with $\mathtt{M} \equiv \langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\mathtt{T\ m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e}\}$
h. $\mathcal{F}\llbracket \mathtt{H}\rrbracket = \langle\overline{\mathtt{X}} \lhd \mathcal{F}\llbracket\overline{\mathtt{N}}\rrbracket\rangle\ \mathcal{F}\llbracket \mathtt{T}\rrbracket\ \mathtt{m}(\mathcal{F}\llbracket\overline{\mathtt{T}}\rrbracket\ \overline{\mathtt{x}})$	with $\mathtt{H} \equiv \langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\mathtt{T\ m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})$
1e. $\mathcal{F}\llbracket \mathtt{e}\rrbracket = \mathtt{x}$	with $\mathtt{e} \equiv \mathtt{x}$
2e. $\mathcal{F}\llbracket \mathtt{e}\rrbracket = \mathcal{F}\llbracket \mathtt{e}_0\rrbracket.\mathtt{f}$	with $\mathtt{e} \equiv \mathtt{e}_0.\mathtt{f}$
3e. $\mathcal{F}\llbracket \mathtt{e}\rrbracket = \mathcal{F}\llbracket \mathtt{e}_0\rrbracket.\mathtt{m}\langle\mathcal{F}\llbracket\overline{\mathtt{T}}\rrbracket\rangle(\mathcal{F}\llbracket\overline{\mathtt{e}}\rrbracket)$	with $\mathtt{e} \equiv \mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{T}}\rangle(\overline{\mathtt{e}})$
4e. $\mathcal{F}\llbracket \mathtt{e}\rrbracket = \mathtt{new}\ \mathcal{F}\llbracket \mathtt{N}\rrbracket(\mathcal{F}\llbracket\overline{\mathtt{e}}\rrbracket)$	with $\mathtt{e} \equiv \mathtt{new\ N}(\overline{\mathtt{e}})$
5e. $\mathcal{F}\llbracket \mathtt{e}\rrbracket = (\mathcal{F}\llbracket \mathtt{N}\rrbracket)(\mathcal{F}\llbracket \mathtt{e}_0\rrbracket)$	with $\mathtt{e} \equiv (\mathtt{N})\mathtt{e}_0$
6e. $\mathcal{F}\llbracket \mathtt{e}\rrbracket = \mathcal{F}\llbracket \mathtt{e}_0\rrbracket.\mathtt{invoke}(\mathcal{F}\llbracket\overline{\mathtt{e}}\rrbracket)$	with $\mathtt{e} \equiv \mathtt{e}_0!(\overline{\mathtt{e}})$
7e. $\mathcal{F}\llbracket \mathtt{e}\rrbracket = \mathtt{new\ I\$n}\langle\mathcal{F}\llbracket\overline{\mathtt{T}}\rrbracket\mathcal{F}\llbracket \mathtt{T}\rrbracket\rangle()\{$ $\mathcal{F}\llbracket \mathtt{T}\rrbracket\ \mathtt{invoke}(\mathcal{F}\llbracket\overline{\mathtt{T}}\rrbracket\ \overline{\mathtt{x}})\{\uparrow \mathcal{F}\llbracket \mathtt{e}_0\rrbracket\}\}$	with $\mathtt{e} \equiv \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\mathtt{e}_0,\ \mathtt{n} = \lvert\overline{\mathtt{T}}\rvert$ and $\Delta;\Gamma \vdash \mathtt{e} : \#\mathtt{T}(\overline{\mathtt{T}})$
8e. $\mathcal{F}\llbracket \mathtt{e}\rrbracket = \mathtt{new\ I}\langle\mathcal{F}\llbracket\overline{\mathtt{T}}\rrbracket\rangle()\{\mathcal{F}\llbracket\overline{\mathtt{M}}\rrbracket\}$	with $\mathtt{e} \equiv \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}$
Translation Structures	
$\mathtt{interface\ I\$n}\langle\overline{\mathtt{X}}, \mathtt{X} \lhd \overline{\mathtt{Object}}, \mathtt{Object}\ \rangle\{\mathtt{X\ invoke}(\overline{\mathtt{X}}\ \overline{\mathtt{x}})\}$	*with* $\mathtt{n} = \lvert\overline{\mathtt{X}}\rvert$

The inference rules of the definition are written following the compact notation used in [BO11a], however:

$$\mathcal{F}\llbracket U\rrbracket = L' \ \ \texttt{with}\ U \equiv L \ \ \texttt{and}\ C_1, ..., C_n \quad \texttt{stands for:} \quad \frac{C_1, ..., C_n, Z_1, ..., Z_k}{L \ \rightarrow_{\mathcal{F}}\ L'_Z}$$

[3] FGACJ contains parameters and class fields whose life cycle is different from the one of non-local variables [BO11a]

where: U ranges over the syntactic domains of the language, $L \in U_{\text{FGACJ}}$ (i.e. syntactic domain U of FGACJ), $L' \in U_{\text{FGAJ}}$ (i.e. domain U of language FGAJ), premises C_i are judgments (possibly including typing judgments), $\rightarrow_{\mathcal{F}}$ is the translation judgement. Eventually, let $\mathcal{F}[\![l_1]\!]$,..,$\mathcal{F}[\![l_k]\!]$ be all the translated forms occurring in L' (with k=0 when none occurs). Then, L'_Z is L' where $\mathcal{F}[\![l_i]\!]$ is replaced by l'_i, for each $i \in [1..k]$, and $Z_i \equiv l_i \;\rightarrow_{\mathcal{F}}\; l'_i$.

Translation $\mathcal{F}[\![]\!]$ assigns meaning to closures by mapping closures of FGACJ into method objects of FGAJ ($\mathcal{F}$-rule 7e), type closures of FGACJ into FGAJ interfaces for method objects ($\mathcal{F}$-rule 2t), and closure invocation of FGACJ into FGAJ invocations of the method wrapped in the method object associated to the closure ($\mathcal{F}$-rule 6e). The remaining $\mathcal{F}$-rules express a sort of congruence of the $\mathcal{F}$-rules above and allow to apply such $\mathcal{F}$-rules in each subterm of the FGACJ program.

Our final goal here is to prove that the two semantics (reduction and translation semantics) commute. This is expressed primarily by Theorem 6 and also by Theorem 5. For technical convenience, we extend $\mathcal{F}$ to the environments Γ and Δ: for each Γ (resp. Δ), $\mathcal{F}[\![\Gamma]\!]$ (resp. $\mathcal{F}[\![\Delta]\!]$) is the environment such that for each variable `x` (resp. `X`), $\mathcal{F}[\![\Gamma]\!](\mathtt{x}) = \mathcal{F}[\![\Gamma(\mathtt{x})]\!]$ (resp. $\mathcal{F}[\![\Delta]\!](\mathtt{x}) = \mathcal{F}[\![\Delta(\mathtt{X})]\!]$. Moreover, we write $\vdash_{\text{FGAJ}}$ if derivation $\vdash$ uses only rules of the calculus FGAJ. We write $\vdash_{\text{FGACJ}}$ if derivation uses, in addition, rules of the calculus FGCJ.

Theorem 5 (Expression Typing Preservation). *Let* $\Delta; \Gamma \vdash_{FGACJ} \mathtt{e} : \mathtt{T}$ *in a program. Then,* $\mathcal{F}[\![\Delta]\!]; \mathcal{F}[\![\Gamma]\!] \vdash_{FGAJ} \mathcal{F}[\![\mathtt{e}]\!] : \mathcal{F}[\![\mathtt{T}]\!]$ *in the* $\mathcal{F}$*-program.* □

Proposition 1 (Program Typing Preservation). *Let* P *be any well typed* FGACJ *program, i.e.* `A OK` *for each class and interface* `A` *of* P*. Then,* $\mathcal{F}[\![P]\!]$ *is a well typed program in* FGAJ*.* □

Theorem 6 (Execution Preservation). *Let* $\Delta; \Gamma \vdash_{FGACJ} \mathtt{e} : \mathtt{T}$ *and* $\mathtt{e} \rightarrow_{FGACJ} \mathtt{e}'$ *in a program. Then,* $\mathcal{F}[\![\mathtt{e}]\!] \rightarrow_{FGAJ} \mathcal{F}[\![\mathtt{e}']\!]$ *in the* $\mathcal{F}$*-program.* □

Proposition 2 (Semantics Equivalence). *Let* `P` *be any well typed* FGACJ *program. Then, if* $(\mathtt{e}, \mathtt{e}') \in \mathtt{Red(P)}$ *then* $(\mathcal{F}[\![\mathtt{e}]\!], \mathcal{F}[\![\mathtt{e}']\!]) \in \mathtt{Red}(\mathcal{F}[\![\mathtt{P}]\!])$*.* □

Example 1. Consider a program with two classes (of FGAJ, for simplicity):

```
class A {Object x; A (Object x){super(x); this.x=x;}}
class B {Object x; B (Object x){super(x); this.x=x;}}
```

Let $e \equiv$ `(#(B x)(new A(x)))!(new B(new Object()))` be an expression (of FGACJ) for the program.: Reducing e and then translating, elsewhere translating e and then reducing, yield the same term.

reduction $e \rightarrow_{\text{FGACJ}}$

$e \rightarrow_{GR-Inv-Clos_{\text{FGCJ}}}$ [`new B(new Object())/x,#(B x)(new A(x))/this`] `new A(x)`

$\equiv$ `new A(new B(new Object()))`

translation $\mathcal{F}[\![e]\!]$

$\mathcal{F}[\![$`#A(B x)(new A x)!(new B(new Object))`$]\!]$

=$\mathcal{F}[\![$`#A(B x)(new A x)`$]\!]$`.invoke(`$\mathcal{F}[\![$`new B(new Object())`$]\!]$

=`new` $I\$1$ <B, A> (){$\mathcal{F}[\![$`A invoke(B x){new A(x)}`$]\!]$`.invoke(`$\mathcal{F}[\![$`new B(new Object())`$]\!]$)

=`new` $I\$1$ <B, A> (){$\mathcal{F}[\![$`A invoke(B x){new A(x)}`$]\!]$}`.invoke(new B(new Object()))`

$=$`new` $I\$1$ `<B,A>`$()\{$`A invoke(B x)`$\{\uparrow$ `new A(x)`$\}\}$`.invoke(new B(new Object ()))`
reduction $\mathcal{F}[\![e]\!] \rightarrow_{\text{FGAJ}}$
`new` $I\$1$ `<B,A>`$()\{$`A invoke(B x)`$\{\uparrow$ `new A(x)`$\}\}$
`.invoke(new B(new Object ()))` $\rightarrow_{GR-Invk-Anonym_{\text{FGAJ}}}$
`[new B(new Object())/x]new A(x)`
$\equiv$ `new A(new B(new Object()))`

where `interface I$1`$\langle$`X` $\lhd$ `Object`, `Y` $\lhd$ `Object`$\rangle\{$`X invoke(Y x);`$\}$ is defined.

5 Conclusions

We proved that the reduction semantics defined for S-closures, in the Straw-man proposal version [Rei09], and the translation semantics $\mathcal{F}[\![P]\!]$ implementing S-closure, as interface based callbacks, commute preserving typing and computation. The translation semantics required interfaces and anonymous objects hence we have extended the calculus FGCJ with such features and have proved that the semantic properties, type safety and abstraction, are also preserved. Open problems and interesting questions that deserve further investigation are (i) closure conversion, (ii) contracovariant closures, (iii) closures with `this` transparency.
(i) Closure conversion: It is an expression $\texttt{cnv}(\texttt{e},\texttt{T})$ which specifies an expression `e`, which must compute a closure, and a type `T`, which must be a *Single Abstract Method* type [Rei10]. It converts the value of `e` into an object of type `T`. The interest for this kind of mechanism resides in the re-use [Rei09,BGR10,Rei10], in callback programming, of Java APIs by passing closures in method invocations, instead of class or interface object creators. The treatment requires to extend FGACJ, hence both the reduction semantics and the translation semantics, to cope with this kind of expressions and with a satisfactory characterization of SAM types.
(ii) Contracovariant closures: The closures in the calculus of this paper have reflection subtyping [Rei09,BGR10] which means that: Let $\Delta \vdash \#\texttt{T}(\overline{\texttt{T}}), \#\texttt{S}(\overline{\texttt{S}})$ `ok`. Then, by S-Refl$_{\text{FGJ}}$, $\Delta \vdash \#\texttt{T}(\overline{\texttt{T}}) <: \#\texttt{S}(\overline{\texttt{S}})$ if and only $\#\texttt{T}(\overline{\texttt{T}})$ and $\#\texttt{S}(\overline{\texttt{S}})$ are the same type, i.e. $\texttt{T} = \texttt{S}$ and $\overline{\texttt{T}} = \overline{\texttt{S}}$. Contracovariant closures have the following more general subtyping rule:

$$\frac{\Delta \vdash \texttt{T} <: \texttt{S} \qquad \Delta \vdash \overline{\texttt{S}} <: \overline{\texttt{T}}}{\Delta \vdash \#\texttt{T}(\overline{\texttt{T}}) <: \#\texttt{S}(\overline{\texttt{S}})}$$

In [BO11b], we proved type soundness for Java with contracovariant S-closures [Rei10]. Contracovariant subtyping greatly extends the programs applicability by recognizing the type soundness of programs running on closures having types which are contracovariant of the program expected types. However, contracovariance is an implicit subtyping relation which (is a subtyping introduction that) contrasts with the one adopted in Java for classes and interfaces, in which subtyping is introduced by means of explicit declarations (through the use of `extends` and `implements`). Hence, additional work is required deal with with the translation of contracovariant closures [Rei10] into Java anonymous class objects.
(iii) This transparency: In Java, the self-reference `this` may occur in an object initializer, constructor or instance method. Its meaning is a reference to the object being constructed or respectively, to the object for which the instance method is invoked. Accordingly, such a meaning of `this` is preserved in FGJ [IPW01] and, in FGAJ in this paper, where the occurrence of `this` is restricted to (instance) method bodies (since

initializers are not present in the calculus FGJ, while constructors have a stylized use of `this`). What is the meaning of `this` when it occurs within a S-closure? Since S-closures can occur only in method bodies we have two possibilities: A transparent definition of `this` which is not affected from the presence of closures [BGR10] or a non transparent definition [Rei10]. In the first case, the meaning of `this` is always the object for which the method, where `this` occurs, is invoked, regardless `this` is within a closure. In [BO10] we proved type safety of FGJ extended with S-closures having `this` transparency. In the second case, the meaning relies on the closure in which `this` occurs: In terms of the reduction semantics, given in this paper, `this` means a self-reference to the closure itself. In terms of the translation semantics, `this` means a self-reference to the anonymous class object in which the closure is mapped. This interpretation copes with the meaning given in [Rei10]. In this paper we have considered the latter one: Hence all the properties proved in the paper are for S-closure with non transparent `this`.

A Property, Lemma and Theorem Proofs

Theorem 1: proof The proof is based on the analysis of all well typed expressions that either are in normal form or fall in one of the above 4 cases and need be further reduced to obtain a normal form. As already stated in section 2.4, in FGACJ there are 3 possible normal forms i.e. values. They are: $\mathtt{new}\ \mathtt{N}(\overline{\mathtt{w}})$ (Object in FGJ), $\mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}$ (Object of anonymous class in FGAJ) and $\#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\mathtt{e}$ (Closure in FGCJ). □

Theorem 2: proof Immediate from Theorem 1, and Theorem 7 (on subject reduction, see Appendix B). □

Theorem 3: proof The set $\mathtt{e}[\bullet]$ of expression contexts of FGACJ is the same of FGCJ (see Definition 4.1 [BO11b]) extended with contexts for the new expression $\mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}$. But this expression cannot contain *bullet* since it is a value (see definition of $\mathtt{v}$ in Section 2.4), hence the set $\mathtt{e}[\bullet]$ is:

$$\mathtt{e}[\bullet] ::= \bullet \mid \mathtt{x} \mid \mathtt{e}[\bullet].\mathtt{f} \mid \mathtt{e}[\bullet].\mathtt{m}\langle\overline{\mathtt{T}}\rangle(\overline{\mathtt{e}[\bullet]}) \mid \mathtt{new}\ \mathtt{N}(\overline{\mathtt{e}[\bullet]}) \mid (\mathtt{N})\mathtt{e}[\bullet] \mid \mathtt{F}[\bullet] \mid \mathtt{e}[\bullet]!(\overline{\mathtt{e}[\bullet]}) \mid \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}$$
$$\mathtt{F}[\bullet] ::= \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\mathtt{e}[\bullet]$$

and the proof follows immediately from the Abstraction Property theorem on FGCJ [BO11b]. □

Theorem 4: proof All FGACJ sets of rules (from Table 2 to Table 5) include FGCJ rules. □

Theorem 5: proof By induction on typing derivation and case analysis on the last rule used. Two are the most intriguing cases: GT-CLOSURE$_{\text{FGCJ}}$ and GT-CLOSURE-INV$_{\text{FGCJ}}$

Case GT-VAR$_{\text{FGJ}}$: if $\Delta;\Gamma \vdash_{\text{FGACJ}} \mathtt{x} : \Gamma(\mathtt{x})$ then $\mathcal{F}[\![\Delta]\!];\mathcal{F}[\![\Gamma]\!] \vdash_{\text{FGJ}} \mathtt{x} : \mathcal{F}[\![\Gamma(\mathtt{x})]\!]$ since by definition $\mathcal{F}[\![\Gamma]\!](\mathtt{x}) = \mathcal{F}[\![\Gamma(\mathtt{x})]\!]$, for each variable $\mathtt{x}$ and derivation uses only rules of FGJ.

Case GT-FIELD$_{\text{FGJ}}$: $\quad \mathtt{e} \equiv \mathtt{e}_0.\mathtt{f}_i \qquad \mathtt{T} \equiv \mathtt{T}_i$

$$\Delta;\Gamma \vdash_{\text{FGACJ}} \mathtt{e}_0 : \mathtt{T}_0 \quad \text{and} \quad \mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$$

By induction hypothesis, $\mathcal{F}[\![\Delta]\!];\mathcal{F}[\![\Gamma]\!] \vdash_{\text{FGAJ}} \mathcal{F}[\![\mathtt{e}_0]\!] : \mathcal{F}[\![\mathtt{T}_0]\!]$, and by $\mathcal{F}$-rule k and by F-CLASS, $\mathit{fields}(\mathit{bound}_{\mathcal{F}[\![\Delta]\!]}(\mathcal{F}[\![\mathtt{T}_0]\!])) = \mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{f}}$. Then, by GT-FIELD$_{\text{FGJ}}$

$$\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\texttt{e}_0.\texttt{f}_i\rrbracket : \mathcal{F}\llbracket\texttt{T}_i\rrbracket.$$

Case GT-Inv$_{\text{FGJ}}$: $\texttt{e} \equiv \texttt{e}_0.\texttt{m}\langle\overline{\texttt{V}}\rangle(\overline{\texttt{e}}) \quad \Delta;\Gamma \vdash_{\text{FGACJ}} \texttt{e} : \texttt{T}$

$$\Delta;\Gamma \vdash_{\text{FGACJ}} \texttt{e}_0 : \texttt{T}_0 \quad \text{and} \quad \Delta \vdash_{\text{FGACJ}} \overline{\texttt{V}}\ \texttt{ok} \quad \text{and} \quad \Delta;\Gamma \vdash_{\text{FGACJ}} \overline{\texttt{e}} : \overline{\texttt{S}}$$
$$mtype(\texttt{m}, bound_\Delta(\texttt{T}_0)) = \langle\overline{\texttt{Y}} \lhd \overline{\texttt{P}}\rangle\overline{\texttt{U}} \to \texttt{U}$$
$$\Delta \vdash_{\text{FGACJ}} \overline{\texttt{S}} <: [\overline{\texttt{V}}/\overline{\texttt{Y}}]\overline{\texttt{U}} \quad \text{and} \quad \Delta \vdash_{\text{FGACJ}} \overline{\texttt{V}} <: [\overline{\texttt{V}}/\overline{\texttt{Y}}]\overline{\texttt{P}} \quad \text{and} \quad \texttt{T} \equiv [\overline{\texttt{V}}/\overline{\texttt{Y}}]\texttt{U}$$

By induction hypothesis, $\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\texttt{e}_0\rrbracket : \mathcal{F}\llbracket\texttt{T}_0\rrbracket$. By Lemma **6** $mtype(\texttt{m}, \mathcal{F}\llbracket bound_\Delta(\texttt{T}_0)\rrbracket) = \langle\overline{\texttt{Y}} \lhd \mathcal{F}\llbracket\overline{\texttt{P}}\rrbracket\rangle\mathcal{F}\llbracket\overline{\texttt{U}}\rrbracket \to \mathcal{F}\llbracket\texttt{U}\rrbracket$, and since Lemma 3 and Lemma 4 on $\overline{\texttt{V}}, \overline{\texttt{S}}, \overline{\texttt{U}}$, and by GT-Inv$_{\text{FGJ}}$

$$\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\texttt{e}_0\rrbracket.\texttt{m}\langle\mathcal{F}\llbracket\overline{\texttt{V}}\rrbracket\rangle(\mathcal{F}\llbracket\overline{\texttt{e}}\rrbracket) : \mathcal{F}\llbracket\texttt{T}\rrbracket.$$

Case GT-AnonymInv$_{\text{FGAJ}}$: $\texttt{e} \equiv \texttt{e}_0.\texttt{m}\langle\overline{\texttt{V}}\rangle(\overline{\texttt{e}}) \quad \Delta;\Gamma \vdash_{\text{FGACJ}} \texttt{e} : \texttt{T}$

$$\texttt{interface I}\langle\overline{\texttt{X}} \lhd \overline{\texttt{N}}\rangle\ \{\overline{\texttt{H}}\}\ \texttt{OK} \quad \text{and} \quad \langle\overline{\texttt{Y}} \lhd \overline{\texttt{P}}\rangle\texttt{U m}(\overline{\texttt{U}}\ \overline{\texttt{x}}) \in \overline{\texttt{H}}$$
$$\Delta;\Gamma \vdash_{\text{FGACJ}} \texttt{e}_0 : \texttt{I}\langle\overline{\texttt{T}}\rangle \quad \text{and} \quad \Delta, \overline{\texttt{X}}<:\overline{\texttt{N}} \vdash_{\text{FGACJ}} \overline{\texttt{V}}<:[\overline{\texttt{V}}/\overline{\texttt{Y}}]\overline{\texttt{P}} \quad \text{and} \quad \Delta;\Gamma \vdash_{\text{FGACJ}} \overline{\texttt{e}} : \overline{\texttt{S}}$$
$$\Delta \vdash_{\text{FGACJ}} \overline{\texttt{T}}<:[\overline{\texttt{T}}/\overline{\texttt{X}}]\overline{\texttt{N}},\ \overline{\texttt{S}} <: [\overline{\texttt{V}}/\overline{\texttt{Y}}]\overline{\texttt{U}} \quad \text{and} \quad \Delta \vdash_{\text{FGACJ}} \overline{\texttt{V}}\ \texttt{ok}$$

Hence, $\texttt{T}{=}[\overline{\texttt{V}}/\overline{\texttt{Y}}]\texttt{U}$. By induction, $\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\texttt{e}_0\rrbracket : \texttt{I}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle$ and, $\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\overline{\texttt{e}}\rrbracket : \mathcal{F}\llbracket\overline{\texttt{S}}\rrbracket$. By Lemma **3** and Lemma **4** on $\overline{\texttt{V}}, \overline{\texttt{S}}, \overline{\texttt{U}}$, and using GT-AnonymInv$_{\text{FGAJ}}$

$$\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\texttt{e}_0\rrbracket.\texttt{m}\langle\mathcal{F}\llbracket\overline{\texttt{V}}\rrbracket\rangle(\mathcal{F}\llbracket\overline{\texttt{e}}\rrbracket) : \mathcal{F}\llbracket\texttt{T}\rrbracket.$$

Case GT-New$_{\text{FGJ}}$: $\texttt{e} \equiv \texttt{new N}(\overline{\texttt{e}}) \quad \texttt{T} \equiv \texttt{N}$

$$\Delta \vdash \texttt{N ok} \quad \text{and} \quad fields(\texttt{N}) = \overline{\texttt{T}}\ \overline{\texttt{f}} \quad \text{and} \quad \Delta;\Gamma \vdash_{\text{FGACJ}} \overline{\texttt{e}} : \overline{\texttt{S}} \quad \text{and} \quad \Delta \vdash_{\text{FGACJ}} \overline{\texttt{S}}<:\overline{\texttt{T}}$$

By induction hypothesis, $\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\overline{\texttt{e}}\rrbracket : \mathcal{F}\llbracket\overline{\texttt{S}}\rrbracket$, and by $\mathcal{F}$-rule k and by F-Class, $fields(\mathcal{F}\llbracket\texttt{N}\rrbracket) = \mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\ \overline{\texttt{f}}$. Then, by Lemma **3** and GT-New$_{\text{FGJ}}$

$$\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \texttt{new}\ \mathcal{F}\llbracket\texttt{N}\rrbracket(\mathcal{F}\llbracket\overline{\texttt{e}}\rrbracket) : \mathcal{F}\llbracket\texttt{N}\rrbracket.$$

Case GT-AnonymNew$_{\text{FGAJ}}$: $\texttt{e} \equiv \texttt{new I}\langle\overline{\texttt{T}}\rangle()\{\overline{\texttt{M}}\} \quad \texttt{T} \equiv \texttt{I}\langle\overline{\texttt{T}}\rangle$

$$\Delta \vdash \texttt{I}\langle\overline{\texttt{T}}\rangle\ \texttt{ok} \quad \text{and} \quad \Delta;\Gamma \vdash_{\text{FGACJ}} \overline{\texttt{M}}\ \ \texttt{OK IN I}\langle\overline{\texttt{T}}\rangle$$

By Lemma **3**, $\mathcal{F}\llbracket\Delta\rrbracket \vdash_{\text{FGAJ}} \texttt{I}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle\ \texttt{ok}$. By Theorem **9**, $\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\overline{\texttt{M}}\rrbracket\ \texttt{OK IN I}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle$. Then, by GT-AnonymNew$_{\text{FGAJ}}$ on the transformed terms:

$$\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \texttt{new I}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle()\{\mathcal{F}\llbracket\overline{\texttt{M}}\rrbracket\} : \texttt{I}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle.$$

Case GT-UCast$_{\text{FGJ}}$: $\texttt{e} \equiv (\texttt{N})\texttt{e}_0 \quad \texttt{T} \equiv \texttt{N}$

$$\Delta;\Gamma \vdash \texttt{e}_0 : \texttt{T}_0 \quad \text{and} \quad \Delta \vdash bound_\Delta(\texttt{T}_0)<:\texttt{N}$$

By induction hypothesis, $\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\texttt{e}_0\rrbracket : \mathcal{F}\llbracket\texttt{T}_0\rrbracket$. By Lemma **3**, letting $\texttt{C}\langle\overline{\texttt{S}}\rangle = bound_\Delta(\texttt{T}_0)$ for some $\texttt{C}\langle\overline{\texttt{S}}\rangle$, $\mathcal{F}\llbracket\Delta\rrbracket \vdash \mathcal{F}\llbracket bound_\Delta(\texttt{T}_0)\rrbracket<:\mathcal{F}\llbracket\texttt{N}\rrbracket$. Then, using GT-UCast$_{\text{FGJ}}$ on the transformed terms concludes the case.

Case GT-DCast$_{\text{FGJ}}$: $\texttt{e} \equiv (\texttt{C}\langle\overline{\texttt{T}}\rangle)\texttt{e}_0 \quad \texttt{T} \equiv \texttt{C}\langle\overline{\texttt{T}}\rangle$

$$\Delta;\Gamma \vdash \texttt{e}_0 : \texttt{T}_0 \quad \text{and} \quad bound_\Delta(\texttt{T}_0) = \texttt{D}\langle\overline{\texttt{T}}\rangle \quad \text{and} \quad \Delta \vdash \texttt{C}\langle\overline{\texttt{T}}\rangle\ \texttt{ok} \quad \text{and}$$
$$\Delta \vdash \texttt{C}\langle\overline{\texttt{T}}\rangle<:\texttt{D}\langle\overline{\texttt{T}}\rangle \quad \text{and} \quad dcast(\texttt{C}, \texttt{D})$$

By induction hypothesis, $\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\texttt{e}_0\rrbracket : \mathcal{F}\llbracket\texttt{T}_0\rrbracket$. By Lemma **3**, $\mathcal{F}\llbracket\Delta\rrbracket \vdash \texttt{C}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle\ \texttt{ok}$ and $\mathcal{F}\llbracket\Delta\rrbracket \vdash \texttt{C}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle<:\texttt{D}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle$. By DCast, $dcast(\texttt{C}, \texttt{D})$ holds in the transformed program. Then, using GT-DCast$_{\text{FGJ}}$ on the transformed terms concludes the case.

Case GT-SCast$_{\text{FGJ}}$: $\texttt{e} \equiv (\texttt{C}\langle\overline{\texttt{T}}\rangle)\texttt{e}_0 \quad \texttt{T} \equiv \texttt{C}\langle\overline{\texttt{T}}\rangle$

$$\Delta;\Gamma \vdash \texttt{e}_0 : \texttt{T}_0 \quad \text{and} \quad bound_\Delta(\texttt{T}_0) = \texttt{D}\langle\overline{\texttt{T}}\rangle \quad \text{and} \quad \Delta \vdash \texttt{C}\langle\overline{\texttt{T}}\rangle\ \texttt{ok} \quad \text{and}$$
$$\Delta \vdash \texttt{C}\langle\overline{\texttt{T}}\rangle<:\texttt{D}\langle\overline{\texttt{U}}\rangle \quad \text{and} \quad \texttt{C} \not\trianglelefteq \texttt{D} \quad \text{and} \quad \texttt{D} \not\trianglelefteq \texttt{C}$$

By induction hypothesis, $\mathcal{F}\llbracket\Delta\rrbracket;\mathcal{F}\llbracket\Gamma\rrbracket \vdash_{\text{FGAJ}} \mathcal{F}\llbracket\texttt{e}_0\rrbracket : \mathcal{F}\llbracket\texttt{T}_0\rrbracket$. By Lemma **3**, $\mathcal{F}\llbracket\Delta\rrbracket \vdash \texttt{C}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle\ \texttt{ok}$ and $\mathcal{F}\llbracket\Delta\rrbracket \vdash \texttt{C}\langle\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\rangle<:\texttt{D}\langle\mathcal{F}\llbracket\overline{\texttt{U}}\rrbracket\rangle$. By subclassing, see **Table 3**, $\texttt{C} \not\trianglelefteq \texttt{D}$, $\texttt{D} \not\trianglelefteq \texttt{C}$ hold also in the transformed program. Then, using GT-SCast$_{\text{FGJ}}$ on the transformed terms concludes the case.

Case GT-closure$_{\text{FGCJ}}$: $\texttt{e} \equiv \#(\overline{\texttt{T}}\ \overline{\texttt{x}})\ \texttt{e}_0 \quad \texttt{T} \equiv \#\texttt{T}_0(\overline{\texttt{T}})$

$$\Delta_0 \vdash \overline{\texttt{T}}\ \texttt{ok} \quad \text{and} \quad \Delta_0;\Gamma_0,\overline{\texttt{x}}:\overline{\texttt{T}},\texttt{this}:\#\texttt{T}_0(\overline{\texttt{T}}) \vdash \texttt{e}_0 : \texttt{T}_0$$

By Lemma **3**, $\mathcal{F}\llbracket\Delta_0\rrbracket \vdash \mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket\ \texttt{ok}$. By induction, $\mathcal{F}\llbracket\Delta_0\rrbracket;\mathcal{F}\llbracket\Gamma_0\rrbracket,\overline{\texttt{x}}:\mathcal{F}\llbracket\overline{\texttt{T}}\rrbracket,\texttt{this}:\mathcal{F}\llbracket\#\texttt{T}_0(\overline{\texttt{T}})\rrbracket \vdash$

$\mathcal{F}[\![\mathtt{e}_0]\!] : \mathcal{F}[\![\mathtt{T}_0]\!]$, where, by $\mathcal{F}$-rule 2t, $\mathcal{F}[\![\#\mathtt{T}_0(\overline{\mathtt{T}})]\!]$=$\mathtt{I\$n}\langle\mathcal{F}[\![\overline{\mathtt{T}}]\!]\mathcal{F}[\![\mathtt{T}_0]\!]\rangle$. Using GT-ANONYM$_{\text{FGAJ}}$ and letting, $\Delta \equiv \Delta_0$, $\Gamma \equiv \Gamma_0$, $\overline{\mathtt{N}} \equiv \overline{\mathtt{Object}}$ ($\overline{\mathtt{Y}} \equiv \circ \equiv \overline{\mathtt{P}}$ be the empty sequence), $\overline{\mathtt{V}} \equiv \mathcal{F}[\![\overline{\mathtt{T}}]\!]\mathcal{F}[\![\mathtt{T}_0]\!]$, $\mathtt{m} \equiv \mathtt{invoke}$, $\mathtt{I} \equiv \mathtt{I\$n}$ (as defined in **Table 6** - translation Structures), we obtain: $\mathcal{F}[\![\Delta_0]\!]; \mathcal{F}[\![\Gamma_0]\!] \vdash \mathtt{M}\ \mathtt{OK\,IN}\ \mathtt{I}\langle\mathcal{F}[\![\overline{\mathtt{T}}]\!]\mathcal{F}[\![\mathtt{T}_0]\!]\rangle$,
for $\mathtt{M} \equiv \mathcal{F}[\![\mathtt{T}_0]\!]\ \mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{x}})\{\uparrow \mathcal{F}[\![\mathtt{e}_0]\!];\}$. Using GT-ANONYMNEW$_{\text{FGAJ}}$ and letting, $\overline{\mathtt{M}} \equiv \mathtt{M}$, we obtain: $\mathcal{F}[\![\Delta_0]\!]; \mathcal{F}[\![\Gamma_0]\!] \vdash \mathtt{new\ I}\langle\mathcal{F}[\![\overline{\mathtt{T}}]\!]\mathcal{F}[\![\mathtt{T}_0]\!]\rangle()\{\overline{\mathtt{M}}\}\text{:}\ \mathtt{I}\langle\mathcal{F}[\![\overline{\mathtt{T}}]\!]\mathcal{F}[\![\mathtt{T}_0]\!]\rangle$
By $\mathcal{F}$-rule 7e, $\mathtt{new\ I}\langle\mathcal{F}[\![\overline{\mathtt{T}}]\!]\mathcal{F}[\![\mathtt{T}_0]\!]\rangle()\{\overline{\mathtt{M}}\} = \mathcal{F}[\![\#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\ \mathtt{e}_0]\!]$, and by $\mathcal{F}$-rule 2t, $\mathtt{I}\langle\mathcal{F}[\![\overline{\mathtt{T}}]\!]\mathcal{F}[\![\mathtt{T}_0]\!]\rangle = \mathcal{F}[\![\#\mathtt{T}_0(\overline{\mathtt{T}})]\!]$: This concludes the case.

Case GT-CLOSURE-INV$_{\text{FGCJ}}$: $\mathtt{e} \equiv \mathtt{e}_0\,!\,(\overline{\mathtt{e}}) \qquad \mathtt{T} \equiv \mathtt{T}_0$

$$\Delta_0; \Gamma_0 \vdash \mathtt{e}_0 : \#\mathtt{T}_0(\overline{\mathtt{T}}_0) \ \text{ and } \ \Delta_0; \Gamma_0 \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}}_0 \ \text{ and } \ \Delta_0 \vdash \overline{\mathtt{S}}_0 <: \overline{\mathtt{T}}_0$$

By induction, $\mathcal{F}[\![\Delta_0]\!]; \mathcal{F}[\![\Gamma_0]\!] \vdash \mathcal{F}[\![\mathtt{e}_0]\!] : \mathcal{F}[\![\#\mathtt{T}_0(\overline{\mathtt{T}}_0)]\!]$, and, $\mathcal{F}[\![\Delta_0]\!]; \mathcal{F}[\![\Gamma_0]\!] \vdash \mathcal{F}[\![\overline{\mathtt{e}}]\!] : \mathcal{F}[\![\overline{\mathtt{S}}_0]\!]$, where, by $\mathcal{F}$-rule 2t, $\mathcal{F}[\![\#\mathtt{T}_0(\overline{\mathtt{T}}_0)]\!]$=$\mathtt{I\$n}\langle\mathcal{F}[\![\overline{\mathtt{T}}_0]\!]\mathcal{F}[\![\mathtt{T}_0]\!]\rangle$. By Lemma **3**, $\mathcal{F}[\![\Delta_0]\!] \vdash \mathcal{F}[\![\overline{\mathtt{S}}_0]\!] <: \mathcal{F}[\![\overline{\mathtt{T}}_0]\!]$. Using GT-INTERF$_{\text{FGAJ}}$ (and GT-HEADER$_{\text{FGAJ}}$ of **Table 4a**) and letting $\mathtt{I} \equiv \mathtt{I\$n}$, $\overline{\mathtt{X}} \equiv \overline{\mathtt{X}}_0\mathtt{X}_0$, with n=$|\overline{\mathtt{X}}|$, $\overline{\mathtt{N}} \equiv \overline{\mathtt{Object}}$, we have:

$$\mathtt{interface\ I}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\{\overline{\mathtt{H}}\}\ \mathtt{OK} \ \text{ and } \ \overline{\mathtt{H}} \equiv \mathtt{X}_0\ \mathtt{invoke}(\overline{\mathtt{X}}_0\ \overline{\mathtt{x}})$$

and by MT-INTERFACE, letting $\mathtt{m} \equiv \mathtt{invoke}$, $\overline{\mathtt{T}} \equiv \mathcal{F}[\![\overline{\mathtt{T}}_0]\!]\mathcal{F}[\![\mathtt{T}_0]\!]$, $\overline{\mathtt{X}}_0 \equiv \overline{\mathtt{U}} \equiv \mathcal{F}[\![\overline{\mathtt{T}}_0]\!]$ and $\mathtt{X}_0 \equiv \mathtt{U} \equiv \mathcal{F}[\![\mathtt{T}_0]\!]$,

$$mtype(\mathtt{invoke}, \mathtt{I\$n}\langle\mathcal{F}[\![\overline{\mathtt{T}}_0]\!]\mathcal{F}[\![\mathtt{T}_0]\!]\rangle) = \mathcal{F}[\![\overline{\mathtt{T}}_0]\!] \to \mathcal{F}[\![\mathtt{T}_0]\!].$$

eventually, by GT-ANONYMINV$_{\text{FGAJ}}$ and letting, $\Delta \equiv \Delta_0$, $\overline{\mathtt{V}}$ be the empty sequence, $\overline{\mathtt{S}} \equiv \mathcal{F}[\![\overline{\mathtt{S}}_0]\!]$ we have $\mathcal{F}[\![\Delta_0]\!] \vdash \mathcal{F}[\![\overline{\mathtt{S}}_0]\!] <: \overline{\mathtt{T}}_0$ holds and also: $\mathcal{F}[\![\Delta_0]\!]; \mathcal{F}[\![\Gamma]\!] \vdash \mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{m}(\overline{\mathtt{e}}_0) : \mathcal{F}[\![\mathtt{T}_0]\!]$. It concludes the case and the proof. □

Proposition1: proof. By case analysis on the typing rules on classes and instances of Table 4a.

Case GT-CLASS$_{\text{FGJ}}$: $\mathtt{A} \equiv \mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\{\overline{\mathtt{T}}\ \overline{\mathtt{f}}; \mathtt{K}\ \overline{\mathtt{M}}\}$

$$\overline{\mathtt{X}} <: \overline{\mathtt{N}} \vdash \overline{\mathtt{N}}, \mathtt{N}, \overline{\mathtt{T}}\ \mathtt{ok} \qquad \overline{\mathtt{M}}\ \ \mathtt{OK\,IN}\ \mathtt{C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle$$
$$fields(\mathtt{N}) = \overline{\mathtt{U}}\ \overline{\mathtt{g}} \qquad \mathtt{K} = \mathtt{C}(\overline{\mathtt{U}}\ \overline{\mathtt{g}}, \overline{\mathtt{T}}\ \overline{\mathtt{f}})\{\text{super}(\overline{\mathtt{g}});\ \text{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};\}$$

By Lemma **3**, let $\Delta \equiv \overline{\mathtt{X}} <: \overline{\mathtt{N}}$. Then, $\mathcal{F}[\![\Delta]\!] \vdash \mathcal{F}[\![\overline{\mathtt{N}}]\!], \mathcal{F}[\![\mathtt{N}]\!], \mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \mathtt{ok}$;
By Theorem **8**, $\mathcal{F}[\![\overline{\mathtt{M}}]\!]\ \ \mathtt{OK\,IN}\ \mathtt{C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle$ (since, $\mathcal{F}[\![\mathtt{C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle]\!]$ reduces to $\mathtt{C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle$);
By F-CLASS of Table 3, on $\mathcal{F}[\![\mathtt{A}]\!]$, and:

- by $\mathcal{F}$-rule 1l, $fields(\mathcal{F}[\![\mathtt{N}]\!]) = \mathcal{F}[\![\overline{\mathtt{U}}]\!]\ \overline{\mathtt{g}}$
- by $\mathcal{F}$-rule k, $\mathcal{F}[\![\mathtt{K}]\!] = \mathtt{C}(\mathcal{F}[\![\overline{\mathtt{U}}]\!]\ \overline{\mathtt{g}}, \mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{f}})\{\text{super}(\overline{\mathtt{g}});\ \text{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};\}$.

Hence, by GT-CLASS$_{\text{FGJ}}$, $\mathcal{F}[\![\mathtt{A}]\!]\ \mathtt{OK}$.

Case GT-INTERF$_{\text{FGAJ}}$: $\mathtt{A} \equiv \mathtt{interface\ I}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\{\overline{\mathtt{H}}\}$

$$\overline{\mathtt{X}} <: \overline{\mathtt{N}} \vdash \overline{\mathtt{N}}\ \mathtt{ok} \qquad \overline{\mathtt{H}}\ \ \mathtt{OK\,IN}\ \mathtt{I}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle$$

By Lemma **3**, $\overline{\mathtt{X}} <: \mathcal{F}[\![\overline{\mathtt{N}}]\!] \vdash \mathcal{F}[\![\overline{\mathtt{N}}]\!]\ \mathtt{ok}$, and by Theorem **8**, $\mathcal{F}[\![\overline{\mathtt{H}}]\!]\ \ \mathtt{OK\,IN}\ \mathtt{I}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle$ in the $\mathcal{F}$-program, hence by GT-INTERF$_{\text{FGAJ}}$, $\mathcal{F}[\![\mathtt{A}]\!]\ \mathtt{OK}$ holds too in the $\mathcal{F}$-program. □

Theorem 6: proof. By case analysis on the last rule used in the computation. The most intriguing case is GR-INV-CLOS$_{\text{FGCJ}}$.

Case GR-FIELD$_{\text{FGJ}}$: $\mathtt{e} \equiv (\mathtt{new\ C}\langle\overline{\mathtt{S}}_0\rangle(\overline{\mathtt{e}})).\mathtt{f}_i \qquad \mathtt{e}' \equiv \mathtt{e}_i$

$$fields(\mathtt{C}\langle\overline{\mathtt{S}}_0\rangle) = \overline{\mathtt{S}}\ \overline{\mathtt{f}}$$

By F-CLASS and $\mathcal{F}$-rule 1l, let $\mathtt{L} \equiv \mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}}; ...\}$ then $\mathcal{F}[\![\mathtt{L}]\!] \equiv \mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle \lhd \mathcal{F}[\![\mathtt{N}]\!]\ \{\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{f}}; ...\}$, and $fields(\mathtt{C}\langle\mathcal{F}[\![\overline{\mathtt{S}}_0]\!]\rangle) = \mathcal{F}[\![\overline{\mathtt{S}}]\!]\ \overline{\mathtt{f}}$, hence by GR-FIELD$_{\text{FGJ}}$:

$$\mathtt{new\ C}\langle\mathcal{F}[\![\overline{\mathtt{S}}_0]\!]\rangle(\mathcal{F}[\![\overline{\mathtt{e}}]\!]).\mathtt{f}_i \longrightarrow \mathcal{F}[\![\mathtt{e}_i]\!].$$

Case GR-INVK$_{\text{FGJ}}$: $\mathtt{e} \equiv (\mathtt{new\ C}\langle\overline{\mathtt{S}}_0\rangle(\overline{\mathtt{e}})).\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{d}}) \qquad \mathtt{e}' \equiv [\overline{\mathtt{d}}/\overline{\mathtt{x}}, \mathtt{new\ C}\langle\overline{\mathtt{S}}_0\rangle(\overline{\mathtt{e}})/\mathtt{this}]\mathtt{e}_0$

$$mbody(\mathtt{m}\langle\overline{\mathtt{V}}\rangle, \mathtt{C}\langle\overline{\mathtt{S}}_0\rangle) = \overline{\mathtt{x}}.\mathtt{e}_0$$

By MB-CLASS (or MB-SUPER, if $\mathtt{m} \notin \overline{\mathtt{M}}$) and $\mathcal{F}$-rule 1l, let $\mathtt{L} \equiv \mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\ \{...;..\overline{\mathtt{M}}\}$

and $\mathtt{m} \in \overline{\mathtt{M}}$, then $\mathcal{F}[\![\mathtt{L}]\!] \equiv \mathtt{class}\ \mathtt{C}\langle \overline{\mathtt{X}} \triangleleft \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle \triangleleft \mathcal{F}[\![\mathtt{N}]\!]\ \{...; ..\mathcal{F}[\![\overline{\mathtt{M}}]\!]\}$, and $\mathit{mbody}(\mathtt{m}\langle \mathcal{F}[\![\overline{\mathtt{V}}]\!]\rangle, \mathtt{C}\langle \mathcal{F}[\![\overline{\mathtt{S}}_0]\!]\rangle)$ $= \overline{\mathtt{x}}.\mathcal{F}[\![\mathtt{e}_0]\!]$, hence by GR-INV$_{\text{FGJ}}$:

$\mathtt{new}\ \mathtt{C}\langle \mathcal{F}[\![\overline{\mathtt{S}}_0]\!]\rangle(\mathcal{F}[\![\overline{\mathtt{e}}]\!])).\mathtt{m}\langle \mathcal{F}[\![\overline{\mathtt{V}}]\!]\rangle(\mathcal{F}[\![\overline{\mathtt{d}}]\!]) \longrightarrow [\mathcal{F}[\![\overline{\mathtt{d}}]\!]/\overline{\mathtt{x}}, \mathtt{new}\ \mathtt{C}\langle \mathcal{F}[\![\overline{\mathtt{S}}_0]\!]\rangle(\mathcal{F}[\![\overline{\mathtt{e}}]\!])/\mathtt{this}]\mathcal{F}[\![\mathtt{e}_0]\!]$.

Case GR-CAST$_{\text{FGCJ}}$ $\quad \mathtt{e} \equiv (\mathtt{P})(\mathtt{new}\ \mathtt{N}(\overline{\mathtt{e}})) \quad \mathtt{e}' \equiv \mathtt{new}\ \mathtt{N}(\overline{\mathtt{e}})$

$$\emptyset \vdash \mathtt{N} <: \mathtt{P}$$

By Lemma **3**, $\emptyset \vdash \mathcal{F}[\![\mathtt{N}]\!] <: \mathcal{F}[\![\mathtt{P}]\!]$, hence by GR-CAST$_{\text{FGCJ}}$:

$$(\mathcal{F}[\![\mathtt{P}]\!])(\mathtt{new}\ \mathcal{F}[\![\mathtt{N}]\!](\mathcal{F}[\![\overline{\mathtt{e}}]\!])) \longrightarrow \mathtt{new}\ \mathcal{F}[\![\mathtt{N}]\!](\mathcal{F}[\![\overline{\mathtt{e}}]\!]).$$

Case GR-INV-CLOS$_{\text{FGCJ}}$: $\quad \mathtt{e} \equiv \#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\mathtt{e}_0!(\overline{\mathtt{d}}) \quad \mathtt{e}' \equiv [\overline{\mathtt{d}}/\overline{\mathtt{x}}, \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\mathtt{e}_0/\mathtt{this}]\mathtt{e}_0$

Assuming: $\Delta; \Gamma \vdash_{\text{FGACJ}} \#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\mathtt{e}_0!(\overline{\mathtt{d}}) : \mathtt{T}$, for some Δ, Γ and $\mathtt{T}$. Then:

By $\mathcal{F}$-rule 6e, $\mathcal{F}[\![\#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\mathtt{e}_0!(\overline{\mathtt{d}})]\!] = \mathcal{F}[\![\#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\mathtt{e}_0]\!].\mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{d}}]\!])$, and by $\mathcal{F}$-rule 7e,

$$\mathcal{F}[\![\#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\mathtt{e}_0]\!] = \mathtt{new}\ \mathtt{I\$n}\langle \mathcal{F}[\![\,\overline{\mathtt{T}}\,]\!]\mathcal{F}[\![\,\mathtt{T}'\,]\!]\rangle()\{\overline{\mathtt{M}}\}$$

for $\mathtt{n} = |\overline{\mathtt{T}}|$ and $\Delta; \Gamma \vdash \#(\overline{\mathtt{T}}\,\overline{\mathtt{x}})\mathtt{e}_0 : \#\mathtt{T}'(\overline{\mathtt{T}})$ and $\overline{\mathtt{M}} \equiv \mathcal{F}[\![\mathtt{T}']\!]\ \mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{x}})\{\uparrow \mathcal{F}[\![\mathtt{e}_0]\!]\}$ (with $\mathtt{T}' \equiv \mathtt{T}$, since the assumption on the type of $\mathtt{e}$);

By MB-INTERFACE, letting $\mathtt{A} \equiv \mathtt{new}\ \mathtt{I\$n}\langle \mathcal{F}[\![\,\overline{\mathtt{T}}\,]\!]\mathcal{F}[\![\,\mathtt{T}\,]\!]\rangle()\{\overline{\mathtt{M}}\}$, we have

$$\mathit{mbody}(\mathtt{invoke}, \mathtt{A}) = \overline{\mathtt{x}}.\mathcal{F}[\![\mathtt{e}_0]\!]$$

By GR-INVK-ANONYM$_{\text{FGAJ}}$, on the transformed terms:

$$\mathtt{A}.\mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{d}}]\!]) \rightarrow [\mathcal{F}[\![\overline{\mathtt{d}}]\!]/\overline{\mathtt{x}}, \mathtt{A}/\mathtt{this}]\mathcal{F}[\![\mathtt{e}_0]\!]$$

that we can apply since, by Theorem **9**, $\mathtt{A}$, $\mathcal{F}[\![\overline{\mathtt{T}}]\!]$, $\mathcal{F}[\![\mathtt{T}]\!]$, $\mathcal{F}[\![\overline{\mathtt{d}}]\!]$, $\mathcal{F}[\![\mathtt{e}_0]\!]$ are terms of FGAJ, and by Theorem **5**, are all well typed terms. This concludes the case.

Case GR-INVK-ANONYM$_{\text{FGAJ}}$.

$$\mathtt{e} \equiv (\mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}).\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{d}}) \quad \mathtt{e}' \equiv [\overline{\mathtt{d}}/\overline{\mathtt{x}}, \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}/\mathtt{this}]\mathtt{e}_0$$
$$\mathit{mbody}(\mathtt{m}\langle\overline{\mathtt{V}}\rangle, \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}) = \overline{\mathtt{x}}.\mathtt{e}_0$$

By MB-INTERFACE and $\mathcal{F}$-rule 1l, let $\mathtt{L} \equiv \mathtt{interface}\ \mathtt{I}\langle\overline{\mathtt{X}} \triangleleft \overline{\mathtt{N}}\rangle\ \{\overline{\mathtt{H}}\}$ and $\langle\overline{\mathtt{Y}} \triangleleft \overline{\mathtt{P}}\rangle\mathtt{U}\ \mathtt{m}\ (\overline{\mathtt{U}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e}_0'; \} \in \overline{\mathtt{M}}$ with $\mathtt{e}_0 \equiv [\overline{\mathtt{T}}/\overline{\mathtt{X}}, \overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{e}_0'$. Then $\mathcal{F}[\![\mathtt{L}]\!] \equiv \mathtt{interface}\ \mathtt{I}\langle\overline{\mathtt{X}} \triangleleft \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle\ \{\mathcal{F}[\![\overline{\mathtt{H}}]\!]\}$ is defined since $\mathcal{F}$-rule 2l, and $\langle\overline{\mathtt{Y}} \triangleleft \mathcal{F}[\![\overline{\mathtt{P}}]\!]\rangle\mathcal{F}[\![\mathtt{U}]\!]\ \mathtt{m}\ (\mathcal{F}[\![\overline{\mathtt{U}}]\!]\ \overline{\mathtt{x}})\{\uparrow \mathcal{F}[\![\mathtt{e}_0']\!]; \} \in \mathcal{F}[\![\overline{\mathtt{M}}]\!]$ is defined since $\mathcal{F}$-rule m, hence by MB-INTERFACE on the transformed terms:

$$\mathit{mbody}(\mathtt{m}\langle\mathcal{F}[\![\overline{\mathtt{V}}]\!]\rangle, \mathtt{new}\ \mathtt{I}\langle\mathcal{F}[\![\overline{\mathtt{T}}]\!]\rangle()\{\mathcal{F}[\![\overline{\mathtt{M}}]\!]\}) = \overline{\mathtt{x}}.[\mathcal{F}[\![\overline{\mathtt{T}}]\!]/\overline{\mathtt{X}}, \mathcal{F}[\![\overline{\mathtt{V}}]\!]/\overline{\mathtt{Y}}]\mathcal{F}[\![\mathtt{e}_0']\!]$$
$$(\mathtt{new}\ \mathtt{I}\langle[\mathcal{F}[\![\overline{\mathtt{T}}]\!]\rangle()\{\mathcal{F}[\![\overline{\mathtt{M}}]\!]\}).\mathtt{m}\langle\mathcal{F}[\![\overline{\mathtt{V}}]\!]\rangle(\mathcal{F}[\![\overline{\mathtt{d}}]\!]) \longrightarrow \mathcal{F}[\![[\overline{\mathtt{d}}/\overline{\mathtt{x}}, \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}/\mathtt{this}]\mathtt{e}_0]\!].$$

since $[\mathcal{F}[\![\overline{\mathtt{T}}]\!]/\overline{\mathtt{X}}, \mathcal{F}[\![\overline{\mathtt{V}}]\!]/\overline{\mathtt{Y}}]\mathcal{F}[\![\mathtt{e}_0']\!] = \mathcal{F}[\![\mathtt{e}_0]\!]$ and Lemma **4** □

Proposition 2: proof. Immediately from the Theorem 6. □

B Auxiliary Lemmas and Theorems

Lemmas A.2.1 through A.2.5 and A.2.7 through A.2.9 in [IPW01] remain valid for FGACJ without proof extensions and are not reported here. Proof of Lemma A.2.6 need to be extended to consider rules (WF-INTERF$_{\text{FGAJ}}$) and (WF-CLOSURE$_{\text{FGCJ}}$), analogously proofs of Lemma A.2.10 and A.2.11 need to be extended to consider rules (GT-ANONYMINV$_{\text{FGAJ}}$) and (GT-ANONYMNEW$_{\text{FGAJ}}$), and cases for (GT-CLOSURE$_{\text{FGCJ}}$) and (GT-CLOSURE-INV$_{\text{FGCJ}}$), whose proof needs to be modified since S-closures are not contro-covariant. Furthermore, the proof of Theorem 7 needs to be extended to case GR-INVK-ANONYM$_{\text{FGAJ}}$, while cases GR-INV-CLOS$_{\text{FGCJ}}$, GRC-CLOS-VAL$_{\text{FGCJ}}$and GRC-CLOS-ARG$_{\text{FGCJ}}$proved in [BO10] are still valid. Eventually, two new lemmas are necessary: Lemma 1 to assert that type substitution preserves method signatures and Lemma 2, analogous to Lemma A.2.12, to assert correctness of *mtype* and *mbody* defined on interfaces.

Lemma A.2.6. *If* $\Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{T}\ \mathtt{ok}$ *and* $\Delta_1 \vdash \overline{\mathtt{U}}<:[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$ *with* $\Delta_1 \vdash \overline{\mathtt{U}}\ \mathtt{ok}$ *and none of* $\overline{\mathtt{X}}$ *appearing in* Δ_1*, then* $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}\ \mathtt{ok}$.

Proof: The proof is given by induction on the derivation of $\Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{T}\ \mathtt{ok}$ with a case analysis on the last rule. We extend proof in [IPW01] with the analysis of the new cases.
Case WF-Interf$_{\text{FGAJ}}$. The same as case WF-Class:

$$\mathtt{T} = \mathtt{I}\langle\overline{\mathtt{T}}\rangle, \qquad \Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{T}}\ \mathtt{ok}$$
$$\Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{T}}<:[\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\ \overline{\mathtt{P}}$$
$$\mathtt{interface\ I}\langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\{\ldots\}$$

By induction hypothesis, $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}\ \mathtt{ok}$. By Lemma A.2.5, $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}<:[\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}}$ and, **since** $\overline{\mathtt{Y}}<:\overline{\mathtt{P}} \vdash \overline{\mathtt{P}}$ by rule GT-Interf$_{\text{FGAJ}}$, $\overline{\mathtt{P}}$ does not include any $\overline{\mathtt{X}}$ as free variable. Thus $[\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}} = [[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}}$, and by WF-Interf$_{\text{FGAJ}}$we have $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}\rangle\ \mathtt{ok}$.
Case WF-Closure$_{\text{FGCJ}}$. $\mathtt{T} = \#\mathtt{T}_0(\overline{\mathtt{T}})$. By induction hypothesis: $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0\ \mathtt{ok}$ and $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}\ \mathtt{ok}$ hence premises of WF-Closure are satisfied. □

Lemma 1. *Let* $\Delta, \overline{\mathtt{Z}}<:\overline{\mathtt{V}} \vdash \overline{\mathtt{V}}\ \mathtt{ok}$ *and* $\Delta, \overline{\mathtt{Z}}<:\overline{\mathtt{V}} \vdash \overline{\mathtt{T}}\ \mathtt{ok}$ *and none of* $\overline{\mathtt{Z}}$ *appears in* Δ*. If* $mtype(\mathtt{m}, \mathtt{I}\langle\overline{\mathtt{T}}\rangle) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{W}} \to \mathtt{W}$*, then* $mtype(\mathtt{m}, \mathtt{I}\langle[\overline{\mathtt{V}}/\overline{\mathtt{Z}}]\overline{\mathtt{T}}\rangle) = \langle\overline{\mathtt{Y}} \lhd [\overline{\mathtt{V}}/\overline{\mathtt{Z}}]\overline{\mathtt{P}}\rangle[\overline{\mathtt{V}}/\overline{\mathtt{Z}}]\overline{\mathtt{W}} \to [\overline{\mathtt{V}}/\overline{\mathtt{Z}}]\mathtt{W}$

Proof: By rule MT-Interface there exists $\mathtt{interface\ I}\langle\overline{\mathtt{X}}\lhd\overline{\mathtt{N}}\rangle\ \{\overline{\mathtt{H}}\}$ and $\langle\overline{\mathtt{Y}}\lhd\overline{\mathtt{P}}\rangle\mathtt{U}\ \mathtt{m}(\overline{\mathtt{U}}\ \overline{\mathtt{x}}) \in \overline{\mathtt{H}}$ and $\langle\overline{\mathtt{Y}}\lhd\overline{\mathtt{P}}\rangle\overline{\mathtt{W}} \to \mathtt{W} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}](\langle\overline{\mathtt{Y}}\lhd\overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \to \mathtt{U})$. Without loss of generality $\overline{\mathtt{X}}$ and $\overline{\mathtt{Y}}$ and $\overline{\mathtt{Z}}$ are distinct, hence and by rule MT-Interface $mtype(\mathtt{m}, \mathtt{I}\langle[\overline{\mathtt{V}}/\overline{\mathtt{Z}}]\overline{\mathtt{T}}\rangle) = [\overline{\mathtt{V}}/\overline{\mathtt{Z}}][\overline{\mathtt{T}}/\overline{\mathtt{X}}](\langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \to \mathtt{U}$ $= (\langle\overline{\mathtt{Y}}\lhd[\overline{\mathtt{V}}/\overline{\mathtt{Z}}][\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{P}}\rangle[\overline{\mathtt{V}}/\overline{\mathtt{Z}}][\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \to [\overline{\mathtt{V}}/\overline{\mathtt{Z}}][\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$. Letting $[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U} = \mathtt{W}$ and $[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} = \overline{\mathtt{W}}$ finishes the proof. □

Lemma A.2.10. *If* $\Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2; \Gamma \vdash \mathtt{e} : \mathtt{T}$ *and* $\Delta_1 \vdash \overline{\mathtt{U}}<:[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$ *where* $\Delta_1 \vdash \overline{\mathtt{U}}\ \mathtt{ok}$ *and none of* $\overline{\mathtt{X}}$ *appears in* Δ_1*, then* $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e} : \mathtt{S}$ *for some* $\mathtt{S}$ *such that* $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S}<:[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$

Proof: As in [IPW01], the proof is given by induction and case analysis on the last rule used to infer the type of $\mathtt{e}$. We consider here, only the new cases.
Case GT-AnonymInv$_{\text{FGAJ}}$: Similarly to GT-Inv$_{\text{FGJ}}$:

$$\mathtt{e} = \mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{e}}) \qquad \Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2; \Gamma \vdash \mathtt{e}_0 : \mathtt{I}\langle\overline{\mathtt{T}}\rangle$$
$$mtype(\mathtt{m}, \mathtt{I}\langle\overline{\mathtt{T}}\rangle) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{W}} \to \mathtt{W}_0$$
$$\Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{V}}\ \mathtt{ok} \qquad \Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{V}}<:[\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}}$$
$$\Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2; \Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}} \qquad \Delta_1, \overline{\mathtt{X}}<:\overline{\mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{S}}<:[\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{W}} \qquad \mathtt{T} = [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{W}_0$$

By induction we have:

$$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 : \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}\rangle$$
$$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{e}} : \overline{\mathtt{S}}'$$
$$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; \vdash \overline{\mathtt{S}}'<:[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}$$

By Lemma A.2.6, we have: $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}\ \mathtt{ok}$. Furthermore, without loss of generality we can assume $\overline{\mathtt{X}}$ and $\overline{\mathtt{Y}}$ are distinct and none of $\overline{\mathtt{Y}}$ appears in $\overline{\mathtt{U}}$, hence by Lemma A.2.5 we have: $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}<:[\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}} \quad (= [[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}})$, and
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}<:[\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{W}} \quad (= [[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{W}})$, and
by transitivity, $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{S}}'<:[\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{W}}$. By Lemma 1 we have: $mtype(\mathtt{m}, \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}\rangle) = \langle\overline{\mathtt{Y}} \lhd [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{P}}\rangle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}} \to [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W}_0$ hence by GT-AnonymNew$_{\text{FGAJ}}$ $\mathtt{S} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{W}_0$ which

finishes the case.

Case GT-ANONYMNEW$_{\text{FGAJ}}$: $\quad \mathtt{e} = \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{W}}\rangle()\{\overline{\mathtt{M}}\} \qquad \Delta = \Delta_1, \overline{\mathtt{X}}{<:}\overline{\mathtt{N}}, \Delta_2$

$$\Delta; \Gamma \vdash \overline{\mathtt{M}}\ \ \mathtt{OK\,IN}\ \ \mathtt{I}\langle\overline{\mathtt{W}}\rangle \qquad \Delta \vdash \mathtt{I}\langle\overline{\mathtt{W}}\rangle\mathtt{ok}$$

By Lemma **A.2.6**

$$(1) \qquad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}\rangle\mathtt{ok}$$

Moreover, we prove below, that:

$$(2) \qquad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{M}}\ \ \mathtt{OK\,IN}\ \ \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}\rangle$$

Then, from (1) and (2), by using rule GT-ANONYMNEW$_{\text{FGAJ}}$ we have:

$$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash \mathtt{new}\ \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}\rangle\{[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{M}}\} : \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}\rangle$$

which concludes the case since $\mathtt{new}\ \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}\rangle\{[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{M}}\} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}$ and letting $\mathtt{S} \equiv \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}\rangle$, $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S}{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{I}\langle\overline{\mathtt{W}}\rangle$ holds.

We prove here, that (2) holds.

Let $\mathtt{M} = \langle\overline{\mathtt{Y}}{<:}\overline{\mathtt{P}}\rangle\mathtt{T}_0\ \mathtt{m}(\overline{\mathtt{S}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\}$ be any method in $\overline{\mathtt{M}}$: We prove that $[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{M}$ is well-formed (in $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma$). By GT-ANONYM$_{\text{FGAJ}}$: $\mathtt{interface}\ \mathtt{I}\langle\overline{\mathtt{Z}}{<:}\overline{\mathtt{Q}}\rangle\{\overline{\mathtt{H}}\}$ and $\langle\overline{\mathtt{Y}}{<:}\overline{\mathtt{P}}\rangle\mathtt{T}_0\ \mathtt{m}(\overline{\mathtt{S}}\ \overline{\mathtt{x}}) \in \overline{\mathtt{H}}$ with $\Delta' = \Delta_1, \overline{\mathtt{X}}{<:}\overline{\mathtt{N}}, \Delta_2, \overline{\mathtt{Y}}{<:}\overline{\mathtt{P}}, \overline{\mathtt{Z}}{<:}\overline{\mathtt{Q}}$ and $\Delta_1, \overline{\mathtt{X}}{<:}\overline{\mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{S}}, \mathtt{T}_0, \overline{\mathtt{P}}\ \mathtt{ok}$ and $\Delta'; \Gamma, \overline{\mathtt{x}} : \overline{\mathtt{S}}, \mathtt{this} : \mathtt{I}\langle\overline{\mathtt{T}}\rangle \vdash \mathtt{e} : \mathtt{S}_0$ and $\Delta' \vdash \mathtt{S}_0{<:}\mathtt{T}_0$. Letting $\Delta'_2 = \Delta_2, \overline{\mathtt{Y}}{<:}\overline{\mathtt{P}}, \overline{\mathtt{Z}}{<:}\overline{\mathtt{Q}}$, by Lemma **A.2.6** we have $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta'_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{P}}\ \mathtt{ok}$. By induction we have:

$$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta'_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma, \overline{\mathtt{x}} : [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}, \mathtt{this} : \mathtt{I}\langle\ [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}\rangle \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e} : \mathtt{S}'$$

for $\mathtt{S}'$ such that $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta'_2 \vdash \mathtt{S}'{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{S}_0$, and, since Lemma **A.2.5** (see [IPW01]), $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta'_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{S}_0{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0$ and by rule S-TRANS $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta'_2 \vdash \mathtt{S}'{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0$.

Hence, by GT-ANONYM$_{\text{FGAJ}}$:

$$\langle\overline{\mathtt{Y}}{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{P}}\rangle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0\ \mathtt{m}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\}\ \mathtt{OK\,IN}\ \mathtt{I}\langle[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}\rangle$$

which concludes the proof.

Case GT-CLOSURE$_{\text{FGCJ}}$: $\quad \mathtt{e} = \#(\overline{\mathtt{W}}\ \overline{\mathtt{w}})\ \mathtt{e}_0 \qquad \mathtt{T} = \#\mathtt{W}(\overline{\mathtt{W}})$

$$\Delta = \Delta_1, \overline{\mathtt{X}}{<:}\overline{\mathtt{N}}, \Delta_2$$

$$\Delta; \Gamma, \overline{\mathtt{w}} : \overline{\mathtt{W}}, \mathtt{this} : \#\mathtt{W}(\overline{\mathtt{W}}) \vdash \mathtt{e}_0 : \mathtt{W} \qquad \Delta \vdash \#\mathtt{W}(\overline{\mathtt{W}})\ \mathtt{ok}$$

By induction: $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma, \overline{\mathtt{w}} : [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}, \mathtt{this} : \#[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}) \vdash\ [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 : \mathtt{S}'$ for a type $\mathtt{S}'$ such that $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S}'{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W}$.

By GT-CLOSURE$_{\text{FGCJ}}$: $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash\ \#([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}\ \overline{\mathtt{w}})[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 : \#\mathtt{S}'([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}})$ with $[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W} = \mathtt{S}'$ (because of $\mathtt{this} : \#[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}})$ in the rule premises). Hence,

$$([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}\ \overline{\mathtt{w}})[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e} \text{ and } \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \#\mathtt{S}'([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}){<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}](\#\mathtt{W}(\overline{\mathtt{W}}))$$

conclude the case.

Case GT-CLOSURE-INV$_{\text{FGCJ}}$: $\quad \mathtt{e} = \mathtt{e}_0\,!\,(\overline{\mathtt{e}}) \qquad \mathtt{T} = \mathtt{W}$

$$\Delta = \Delta_1, \overline{\mathtt{X}}{<:}\overline{\mathtt{N}}, \Delta_2$$

$$\Delta; \Gamma \vdash \mathtt{e}_0 : \#\mathtt{W}(\overline{\mathtt{W}}) \qquad \Delta; \Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}}{<:}\overline{\mathtt{W}}$$

By induction $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{e}} : \overline{\mathtt{S}}'$ such that $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{S}}'{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}$. By Lemma **A.2.5**, since $\Delta \vdash \overline{\mathtt{S}}{<:}\overline{\mathtt{W}}$ then $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}$ and by S-TRANS$_{\text{FGJ}}$, $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{S}}'{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}}$. By induction $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 : \mathtt{S}'$ such that $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S}'{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\#(\mathtt{W}(\overline{\mathtt{W}})) = \#[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{W}})$. Since closure subtyping, $\mathtt{S}' = \#[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W}([\overline{\mathtt{U}}/\overline{\mathtt{X}}](\overline{\mathtt{W}}))$ by S-REFL$_{\text{FGJ}}$. Hence, by rule GT-CLOSURE-INV$_{\text{FGCJ}}$ $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0\,!\,([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{e}}) : [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W}$. Letting $\mathtt{S}'' \equiv [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W}$, then $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S}''{<:}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{W}$, by S-REFL$_{\text{FGJ}}$, concludes the case. □

Lemma A.2.11. *If* $\Delta; \Gamma, \overline{\mathtt{x}} : \overline{\mathtt{T}} \vdash \mathtt{e} : \mathtt{T}$ *and* $\Delta; \Gamma \vdash \overline{\mathtt{d}} : \overline{\mathtt{S}}$ *where* $\Delta \vdash \overline{\mathtt{S}}{<:}\overline{\mathtt{T}}$*, then* $\Delta; \Gamma \vdash [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e} : \mathtt{S}$ *for some* $\mathtt{S}$ *such that* $\Delta \vdash \mathtt{S}{<:}\mathtt{T}$

Proof: As in [IPW01], the proof is given by induction and case analysis on the last rule used to infer the type of $\mathtt{e}$. We consider here, only the new cases.

Case GT-AnonymInv$_{\text{FGAJ}}$: Similarly to GT-Inv$_{\text{FGJ}}$.

$$\mathtt{e} = \mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{e}}) \qquad \mathtt{T} = [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}$$
$$\Delta;\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \mathtt{e}_0 : \mathtt{I}\langle\overline{\mathtt{T}}'\rangle \qquad mtype(\mathtt{m},\mathtt{I}\langle\overline{\mathtt{T}}'\rangle) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \to \mathtt{U}$$
$$\Delta \vdash \overline{\mathtt{V}}\ \mathtt{ok} \qquad \Delta \vdash \overline{\mathtt{V}} <: [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}}$$
$$\Delta;\Gamma,\overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}} \qquad \Delta \vdash \overline{\mathtt{S}} <: [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}}$$

By induction $\Delta;\Gamma \vdash [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \mathtt{S}'$ for $\Delta \vdash \mathtt{S}' <: \mathtt{I}\langle\overline{\mathtt{T}}'\rangle$ and, since interface subtyping, $\Delta \vdash \mathtt{S}' = \mathtt{I}\langle\overline{\mathtt{T}}'\rangle$ by S-Refl$_{\text{FGJ}}$. Moreover, by induction $\Delta;\Gamma \vdash [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}} : \overline{\mathtt{W}}$ for $\Delta \vdash \overline{\mathtt{W}} <: \overline{\mathtt{S}}$. By GT-AnonymInv$_{\text{FGAJ}}$ $\Delta;\Gamma \vdash [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e} : [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}$. Letting $\mathtt{S} \equiv [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}$, then $\Delta \vdash \mathtt{S} <: \mathtt{T}$, by S-Refl$_{\text{FGJ}}$, concludes the case.

Case GT-AnonymNew$_{\text{FGAJ}}$: Trivial.

Case GT-Closure$_{\text{FGCJ}}$: $\quad \mathtt{e} = \#(\overline{\mathtt{W}}\ \overline{\mathtt{y}})\mathtt{e}_0 \qquad \mathtt{T} = \#\mathtt{W}(\overline{\mathtt{W}})$

$$\Delta \vdash \overline{\mathtt{W}}\ \mathtt{ok} \qquad \Delta;\Gamma,\overline{\mathtt{y}}:\overline{\mathtt{W}},\mathtt{this}:\#\mathtt{W}(\overline{\mathtt{W}}) \vdash \mathtt{e}_0 : \mathtt{W}$$

Without loss of generality let $[\overline{\mathtt{d}}/\overline{\mathtt{x}}](\#(\overline{\mathtt{W}}\ \overline{\mathtt{y}})\mathtt{e}_0) = \#(\overline{\mathtt{W}}\ \overline{\mathtt{y}})[\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0$ (since variable renaming). By induction, let $\Delta;\Gamma,\overline{\mathtt{y}}:\overline{\mathtt{W}},\mathtt{this}:\#\mathtt{W}(\overline{\mathtt{W}}) \vdash [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \mathtt{W}'$ for some $\mathtt{W}'$ such that $\Delta \vdash \mathtt{W}' <: \mathtt{W}$. By GT-Closure$_{\text{FGCJ}}$ we have: $\Delta;\Gamma \vdash \#(\overline{\mathtt{W}}\ \overline{\mathtt{y}})[\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \#\mathtt{W}(\overline{\mathtt{W}})$ only with $\Delta \vdash \mathtt{W} = \mathtt{W}'$. Letting $\mathtt{S} \equiv \#\mathtt{W}(\overline{\mathtt{W}})$, $\Delta \vdash \mathtt{S} <: \mathtt{T}$, by S-Refl$_{\text{FGJ}}$, concludes the case.

Case GT-Closure-Inv$_{\text{FGCJ}}$: $\quad \mathtt{e} = \mathtt{e}_0!(\overline{\mathtt{e}}) \qquad \mathtt{T} = \mathtt{W}$

$$\Delta;\Gamma \vdash \mathtt{e}_0 : \#\mathtt{W}(\overline{\mathtt{W}}) \qquad \Delta;\Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}} \qquad \Delta \vdash \overline{\mathtt{S}} <: \overline{\mathtt{W}}$$

By induction, $\Delta;\Gamma \vdash [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \mathtt{S}'$ for some $\mathtt{S}'$ such that $\Delta \vdash \mathtt{S}' <: \#\mathtt{W}(\overline{\mathtt{W}})$ and, since closure subtyping, $\Delta \vdash \mathtt{S}' = \#\mathtt{W}(\overline{\mathtt{W}})$ by S-Refl$_{\text{FGJ}}$. Moreover, by induction, $\Delta;\Gamma \vdash [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}} : \overline{\mathtt{S}}'$ for some $\overline{\mathtt{S}}'$ such that $\Delta \vdash \overline{\mathtt{S}}' <: \overline{\mathtt{W}}$. Hence, by GT-Closure-Inv$_{\text{FGCJ}}$ we have: $\Delta;\Gamma \vdash ([\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0)!([\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}):\mathtt{W}$. By factoring: $\Delta;\Gamma \vdash [\overline{\mathtt{d}}/\overline{\mathtt{x}}](\mathtt{e}_0!(\overline{\mathtt{e}})):\mathtt{W}$. Letting $\mathtt{S} \equiv \mathtt{W}$, $\Delta \vdash \mathtt{S} <: \mathtt{T}$, by S-Refl$_{\text{FGJ}}$, concludes the case. □

Lemma 2. *If* $mtype(\mathtt{m},\mathtt{I}\langle\overline{\mathtt{T}}\rangle) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \to \mathtt{U}$ *and* $mbody(\mathtt{m}\langle\overline{\mathtt{V}}\rangle, \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}) = \overline{\mathtt{x}}.\mathtt{e}_0$ *where* $\Delta;\Gamma \vdash \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\} : \mathtt{I}\langle\overline{\mathtt{T}}\rangle$ *and* $\Delta \vdash \overline{\mathtt{V}}\ \mathtt{ok}$ *and* $\Delta \vdash \overline{\mathtt{V}} <: [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}}$. *Then* $\mathtt{S}$ *exists such that* $\Delta \vdash \mathtt{S} <: [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}$ *and* $\Delta \vdash \mathtt{S}\ \mathtt{ok}$ *and* $\Delta;\Gamma,\overline{\mathtt{x}}:[\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}},\mathtt{this}:\mathtt{I}\langle\overline{\mathtt{T}}\rangle \vdash \mathtt{e}_0 : \mathtt{S}$.

Proof By MB-Interface

$$mbody(\mathtt{m}\langle\overline{\mathtt{V}}\rangle, \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}) = \overline{\mathtt{x}}.\mathtt{e}_0$$
$$\mathtt{interface}\ \mathtt{I}\langle\overline{\mathtt{X}}' \lhd \overline{\mathtt{N}}'\rangle\ \{\ldots\} \qquad \langle\overline{\mathtt{Y}}' \lhd \overline{\mathtt{P}}'\rangle\mathtt{U}'\ \mathtt{m}(\overline{\mathtt{U}}'\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\} \in \overline{\mathtt{M}}$$
$$\mathtt{e}_0 = [\overline{\mathtt{T}}/\overline{\mathtt{X}}', \overline{\mathtt{V}}/\overline{\mathtt{Y}}']\mathtt{e}$$

By GT-AnonymNew$_{\text{FGAJ}}$

$$\Delta;\Gamma \vdash \mathtt{new}\ \mathtt{I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\} : \mathtt{I}\langle\overline{\mathtt{T}}\rangle$$
$$\Delta \vdash \mathtt{I}\langle\overline{\mathtt{T}}\rangle\ \mathtt{ok} \qquad \Delta;\Gamma \vdash \overline{\mathtt{M}}\ \ \mathtt{OK\,IN}\ \ \mathtt{I}\langle\overline{\mathtt{T}}\rangle$$

Hence, $\Delta;\Gamma \vdash \langle\overline{\mathtt{Y}}' \lhd \overline{\mathtt{P}}'\rangle\mathtt{U}'\ \mathtt{m}(\overline{\mathtt{U}}'\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\}\ \ \mathtt{OK\,IN}\ \ \mathtt{I}\langle\overline{\mathtt{T}}\rangle$.

By GT-Anonym$_{\text{FGAJ}}$, letting $\Gamma' = \Gamma,\overline{\mathtt{x}}:\overline{\mathtt{U}}',\mathtt{this}:\mathtt{I}\langle\overline{\mathtt{T}}\rangle$ and $\Delta' = \Delta,\overline{\mathtt{X}}' <: \overline{\mathtt{N}}',\overline{\mathtt{Y}}' <: \overline{\mathtt{P}}'$, we have:

$$\Delta;\Gamma \vdash \langle\overline{\mathtt{Y}}' \lhd \overline{\mathtt{P}}'\rangle\mathtt{U}'\ \mathtt{m}(\overline{\mathtt{U}}'\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\}\ \mathtt{OK\,IN}\ \ \mathtt{I}\langle\overline{\mathtt{T}}\rangle$$
$$\mathtt{interface}\ \mathtt{I}\langle\overline{\mathtt{X}}' \lhd \overline{\mathtt{N}}'\rangle\{\overline{\mathtt{H}}\} \qquad \langle\overline{\mathtt{Y}}' \lhd \overline{\mathtt{P}}'\rangle\mathtt{U}'\ \mathtt{m}(\overline{\mathtt{U}}'\ \overline{\mathtt{x}}) \in \overline{\mathtt{H}}$$
$$\Delta';\Gamma' \vdash \mathtt{e}:\mathtt{S}_0 \qquad \Delta' \vdash \overline{\mathtt{T}} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}']\overline{\mathtt{N}}' \qquad \Delta' \vdash \mathtt{S}_0 <: \mathtt{U}'$$

By MT-Interface

$$mtype(\mathtt{m},\mathtt{I}\langle\overline{\mathtt{T}}\rangle) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \to \mathtt{U}$$
$$\mathtt{interface}\ \mathtt{I}\langle\overline{\mathtt{X}}'' \lhd \overline{\mathtt{N}}''\rangle\ \{\overline{\mathtt{H}}''\} \qquad \langle\overline{\mathtt{Y}}'' \lhd \overline{\mathtt{P}}''\rangle\mathtt{U}''\ \mathtt{m}(\overline{\mathtt{U}}''\ \overline{\mathtt{x}}) \in \overline{\mathtt{H}}''$$
$$\overline{\mathtt{P}} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}'']\overline{\mathtt{P}}'' \qquad \overline{\mathtt{U}} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}'']\overline{\mathtt{U}}'' \qquad \mathtt{U} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}'']\mathtt{U}''.$$

Since unicity of the interface names in the program, and unicity of method names in each interface, we have:

$$\overline{\mathtt{Y}} = \overline{\mathtt{Y}}' = \overline{\mathtt{Y}}'' \qquad \overline{\mathtt{X}} = \overline{\mathtt{X}}' = \overline{\mathtt{X}}'' \qquad \overline{\mathtt{P}} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{P}}' \qquad \overline{\mathtt{U}} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}' \qquad \mathtt{U} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}'$$

(Since $(\overline{\mathtt{X}} \cup \overline{\mathtt{Y}}) \cap (\overline{\mathtt{V}} \cup \overline{\mathtt{T}}) = \{\}$) From $\Delta'; \Gamma' \vdash \mathtt{e} : \mathtt{S}_0$, i.e. $\Delta, \overline{\mathtt{X}}' \texttt{<:} \overline{\mathtt{N}}', \overline{\mathtt{Y}}' \texttt{<:} \overline{\mathtt{P}}'; \Gamma' \vdash \mathtt{e} : \mathtt{S}_0$, by Lemma **A.2.10** we have $\Delta, \overline{\mathtt{Y}} \texttt{<:} [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{P}}'; \Gamma, \overline{\mathtt{x}} : [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}', \mathtt{this} : \mathtt{I}\langle\overline{\mathtt{T}}\rangle \vdash [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{e} : \mathtt{S}'_0$ and, since $\overline{\mathtt{P}} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{P}}'$, $\overline{\mathtt{U}} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}'$:

$$(1) \qquad \Delta, \overline{\mathtt{Y}} \texttt{<:} \overline{\mathtt{P}}; \Gamma, \overline{\mathtt{x}} : \overline{\mathtt{U}}, \mathtt{this} : \mathtt{I}\langle\overline{\mathtt{T}}\rangle \vdash [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{e} : \mathtt{S}'_0$$

for $\mathtt{S}'_0$ such that $\Delta, \overline{\mathtt{Y}} \texttt{<:} \overline{\mathtt{P}} \vdash \mathtt{S}'_0 \texttt{<:} [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{S}_0$.
From $\Delta' \vdash \mathtt{S}_0 \texttt{<:} \mathtt{U}'$, by Lemma **A.2.5** ([IPW01], pag. 429): $\Delta, \overline{\mathtt{Y}} \texttt{<:} \overline{\mathtt{P}} \vdash [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{S}_0 \texttt{<:} [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}'$. Since $\mathtt{U} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}'$: $\Delta, \overline{\mathtt{Y}} \texttt{<:} \overline{\mathtt{P}} \vdash [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{S}_0 \texttt{<:} \mathtt{U}$. By S-Trans$_{\text{FGJ}}$: $\Delta, \overline{\mathtt{Y}} \texttt{<:} \overline{\mathtt{P}} \vdash \mathtt{S}'_0 \texttt{<:} \mathtt{U}$. Finally, from (1), by Lemma **A.2.10**:

$$(2) \qquad \Delta; \Gamma, \overline{\mathtt{x}} : [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}}, \mathtt{this} : \mathtt{I}\langle\overline{\mathtt{T}}\rangle \vdash [\overline{\mathtt{V}}/\overline{\mathtt{Y}}][\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{e} : \mathtt{S}''_0$$

for $\mathtt{S}''_0$ such that $\Delta \vdash \mathtt{S}''_0 \texttt{<:} [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{S}'_0$. From $\Delta, \overline{\mathtt{Y}} \texttt{<:} \overline{\mathtt{P}} \vdash \mathtt{S}'_0 \texttt{<:} \mathtt{U}$, by Lemma **A.2.5**, $\Delta \vdash [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{S}'_0 \texttt{<:} [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}$. Then, using S-Trans$_{\text{FGJ}}$ and letting $\mathtt{S} = \mathtt{S}''_0$ finishes the proof. □

Theorem 7 (Subject reduction). *If $\Delta; \Gamma \vdash \mathtt{e} : \mathtt{T}$ and $\mathtt{e} \rightarrow \mathtt{e}'$ then $\Delta; \Gamma \vdash \mathtt{e}' : \mathtt{T}'$, for some $\mathtt{T}'$ such that $\Delta \vdash \mathtt{T}' \texttt{<:} \mathtt{T}$*

Proof. By induction on the reduction $\mathtt{e} \rightarrow \mathtt{e}'$, with a case analysis on the reduction rule used. It extends the proof, [IPW01] (pp. 435-436), of the corresponding theorem for FGJ with the following additional case.
Case GR-Invk-Anonym$_{\text{FGAJ}}$: Similarly to GR-Invk$_{\text{FGJ}}$

$$\mathtt{e} = \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}.\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{d}}) \qquad \mathit{mbody}(\mathtt{m}\langle\overline{\mathtt{V}}\rangle, \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}) = \overline{\mathtt{x}}.\mathtt{e}_0$$
$$\mathtt{e}' = [\overline{\mathtt{d}}/\overline{\mathtt{x}}, \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}/\mathtt{this}]\mathtt{e}_0$$

By GT-AnonymInv$_{\text{FGAJ}}$, let $\mathtt{e}'_0 \equiv \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}$, we have:

$$\Delta; \Gamma \vdash \mathtt{e}'_0.\mathtt{m}\langle\overline{\mathtt{V}}\rangle(\overline{\mathtt{d}}) : \mathtt{T} \qquad \mathtt{T} \equiv [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}$$
$$\mathit{mtype}(\mathtt{m}, \mathtt{I}\langle\overline{\mathtt{T}}\rangle) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \rightarrow \mathtt{U}$$
$$\Delta; \Gamma \vdash \mathtt{e}'_0 : \mathtt{I}\langle\overline{\mathtt{T}}\rangle \qquad \Delta \vdash \overline{\mathtt{V}}\ \mathtt{ok} \qquad \Delta \vdash \overline{\mathtt{V}} \texttt{<:} [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{P}}$$
$$\Delta; \Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{S}} \qquad \Delta \vdash \overline{\mathtt{S}} \texttt{<:} [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}}$$

By GT-AnonymNew$_{\text{FGAJ}}$ we have:

$$\Delta; \Gamma \vdash \mathtt{e}'_0 : \mathtt{I}\langle\overline{\mathtt{T}}\rangle$$
$$\Delta \vdash \mathtt{I}\langle\overline{\mathtt{T}}\rangle\ \mathtt{ok} \qquad \Delta; \Gamma \vdash \overline{\mathtt{M}}\ \ \mathtt{OK\,IN\ \ I}\langle\overline{\mathtt{T}}\rangle$$

By Lemma **2**, $\Delta; \Gamma, \overline{\mathtt{x}} : [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}}, \mathtt{this} : \mathtt{I}\langle\overline{\mathtt{T}}\rangle \vdash \mathtt{e}_0 : \mathtt{S}$ for some $\mathtt{S}$ such that $\Delta \vdash \mathtt{S} \texttt{<:} [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}$. Then, by Lemma **A.2.1** and Lemma **A.2.11**, $\Delta; \Gamma \vdash \mathtt{e}'_0 : \mathtt{S}'$ for some $\mathtt{S}'$ such that $\Delta \vdash \mathtt{S}' \texttt{<:} [\overline{\mathtt{V}}/\overline{\mathtt{Y}}]\mathtt{U}$, by S-Trans$_{\text{FGJ}}$. □

Lemma 3 ($\mathcal{F}[\![]\!]$ Preserves Types Structure). *(a) If $\Delta \vdash \overline{\mathtt{T}} <: \overline{\mathtt{U}}$, then $\mathcal{F}[\![\Delta]\!] \vdash \mathcal{F}[\![\overline{\mathtt{T}}]\!] <: \mathcal{F}[\![\overline{\mathtt{U}}]\!]$. (b) If $\Delta \vdash \overline{\mathtt{T}}\ \mathtt{ok}$, then $\mathcal{F}[\![\Delta]\!] \vdash \mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \mathtt{ok}$.*

Proof (Part a) By case analysis on subtypes in Table 5.
Cases S-Refl$_{\text{FGJ}}$, S-Trans$_{\text{FGJ}}$, S-Var$_{\text{FGJ}}$ are immediate.
Case S-Class$_{\text{FGJ}}$ $\Delta \vdash\ \mathtt{C}\langle\overline{\mathtt{T}}\rangle <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}$ for class $\mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\{\ldots\}$
By $\mathcal{F}$-rule 1l, $\mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle \lhd \mathcal{F}[\![\mathtt{N}]\!]\{\ldots\}$ is in the $\mathcal{F}$-program and by Theorem **9**, $\mathcal{F}[\![\overline{\mathtt{T}}]\!]$ is a type, and by S-Class$_{\text{FGJ}}$, $\mathcal{F}[\![\Delta]\!] \vdash\ \mathtt{C}\langle\mathcal{F}[\![\overline{\mathtt{T}}]\!]\rangle <: [\mathcal{F}[\![\overline{\mathtt{T}}]\!]/\overline{\mathtt{X}}]\mathcal{F}[\![\mathtt{N}]\!]$
(Part b) By case analysis on well-formed types, and induction on the structure of the types.
Cases WF-Object$_{\text{FGJ}}$ and WF-Var$_{\text{FGJ}}$ are immediate.
Case WF-Class$_{\text{FGJ}}$, $\Delta \vdash\ \mathtt{C}\langle\overline{\mathtt{T}}\rangle\ \mathtt{ok}$ for class $\mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\{\ldots\}$, $\Delta \vdash\ \overline{\mathtt{T}}\ \mathtt{ok}$, $\Delta \vdash\ \overline{\mathtt{T}} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$. By induction, $\mathcal{F}[\![\Delta]\!] \vdash\ \mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \mathtt{ok}$, and by $\mathcal{F}$-rule 1l, $\mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle \lhd \mathcal{F}[\![\mathtt{N}]\!]\{\ldots\}$ is in the $\mathcal{F}$-program, and by part (a) of this Lemma, $\mathcal{F}[\![\Delta]\!] \vdash \mathcal{F}[\![\overline{\mathtt{T}}]\!] <: [\mathcal{F}[\![\overline{\mathtt{T}}]\!]/\overline{\mathtt{X}}]\mathcal{F}[\![\overline{\mathtt{N}}]\!]$. This completes the case.

Case WF-$\text{Interf}_{\text{FGAJ}}$ can be proved simply rephrasing the previous case (using interface and I, instead of class and C).
Case WF-$\text{Closure}_{\text{FGCJ}}$, $\Delta \vdash \#\mathtt{T}(\overline{\mathtt{T}})$ ok for types $\Delta \vdash \overline{\mathtt{T}}$ ok $\Delta \vdash \mathtt{T}$ ok. By $\mathcal{F}$-rule 2t, $\mathcal{F}[\![\#\mathtt{T}(\overline{\mathtt{T}})]\!] = \mathtt{I\$n}\langle \mathcal{F}[\![\overline{\mathtt{T}}]\!], \mathcal{F}[\![\,\mathtt{T}\,]\!]\rangle$ for $A \equiv$interface $\mathtt{I\$n}\langle \overline{\mathtt{X}}, \mathtt{X} \lhd\ \overline{\mathtt{O}}, \mathtt{O}\rangle\{\mathtt{X}\ \mathtt{invoke}(\overline{\mathtt{X}}\ \overline{\mathtt{x}})\}$ and $\mathtt{n} = |\overline{\mathtt{T}}|$ (and O standing for Object). By induction, $\mathcal{F}[\![\Delta]\!] \vdash \mathcal{F}[\![\overline{\mathtt{T}}]\!]$ ok $\mathcal{F}[\![\Delta]\!] \vdash \mathcal{F}[\![\mathtt{T}]\!]$ ok. By WF-Interf: $\mathcal{F}[\![\Delta]\!] \vdash \mathtt{I\$n}\langle \mathcal{F}[\![\overline{\mathtt{T}}]\!], \mathcal{F}[\![\,\mathtt{T}\,]\!]\rangle$ ok (since we can assume[4], $\Delta \vdash \mathtt{T} <: \mathtt{Object}$, for Δ and T). □

Lemma 4 ($\mathcal{F}[\![]\!]$ Preserves Term Substitution). $\mathcal{F}[\![\ [\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}\ ]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\ \mathcal{F}[\![\mathtt{e}]\!]$.

Proof By cases on the form of expression and by induction on the structure.
Case $\mathtt{e} \equiv \mathtt{x}$. Then either (a) $[\overline{\mathtt{e}}/\overline{\mathtt{y}}]\mathtt{e} = \mathtt{e}_i$ (case $\mathtt{x} \equiv \mathtt{y}_i$), or (b) $[\overline{\mathtt{e}}/\overline{\mathtt{y}}]\mathtt{e} = \mathtt{x}$ (case $\mathtt{x} \neq \mathtt{y}_i$, for all i). Hence, (a) $\mathcal{F}[\![\ [\overline{\mathtt{e}}/\overline{\mathtt{y}}]\mathtt{e}\ ]\!] = \mathcal{F}[\![\mathtt{e}_i]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{y}}]\ \mathcal{F}[\![\mathtt{y}_i]\!]$ (by $\mathcal{F}$-rule 1e); (b) $\mathcal{F}[\![\ [\overline{\mathtt{e}}/\overline{\mathtt{y}}]\mathtt{x}\ ]\!] = \mathcal{F}[\![\mathtt{x}]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{y}}]\ \mathcal{F}[\![\mathtt{x}]\!]$.
Case $\mathtt{e} \equiv \mathtt{e}_0.\mathtt{f}$. Then $[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0.\mathtt{f} = ([\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0).\mathtt{f}$, since $\mathtt{f} \neq \mathtt{x}_i$ for all i. By $\mathcal{F}$-rule 2e, $\mathcal{F}[\![([\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0).\mathtt{f}]\!] = \mathcal{F}[\![\ [\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0\ ]\!].\mathtt{f}$. By induction $\mathcal{F}[\![\ [\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0\ ]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\ \mathcal{F}[\![\mathtt{e}_0]\!]$, $\mathcal{F}[\![\ [\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0.\mathtt{f}\ ]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{f}$. Then, $[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{f} = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!].[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathtt{f}$, since $\mathtt{f} = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathtt{f}$. By factoring $[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]$:

$$[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!].[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathtt{f} = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}](\mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{f}) = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}]\!]$$

Case $\mathtt{e} \equiv \mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{T}}\rangle(\overline{\mathtt{e}}')$. Then $[\overline{\mathtt{e}}/\overline{\mathtt{x}}](\mathtt{e}_0.\mathtt{m}\langle\overline{\mathtt{T}}\rangle(\overline{\mathtt{e}}')) = ([\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0).\mathtt{m}\langle\overline{\mathtt{T}}\rangle([\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}')$, since m and $\overline{\mathtt{T}}$ do not contain term variables. Hence, by $\mathcal{F}$-rule 3e, $\mathcal{F}[\![([\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0).\mathtt{m}\langle\overline{\mathtt{T}}\rangle([\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}')]\!] = \mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0]\!].\mathtt{m}\langle\overline{\mathtt{T}}\rangle(\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}']\!])$, and by induction $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!]$ and $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}']\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!]$ and by factoring $[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]$:

$$\begin{aligned}\mathcal{F}[\![\ [\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}\ ]\!] &= [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{m}\langle\overline{\mathtt{T}}\rangle([\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!])\\ &= [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!].([\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathtt{m})\langle([\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\overline{\mathtt{T}})\rangle([\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!])\\ &= [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}](\mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{m}\langle\overline{\mathtt{T}})\rangle(\mathcal{F}[\![\overline{\mathtt{e}}']\!])) = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}](\mathcal{F}[\![\mathtt{e}]\!]\end{aligned}$$

Case $\mathtt{e} \equiv \mathtt{new}\ \mathtt{N}(\overline{\mathtt{e}}')$. Then $[\overline{\mathtt{e}}/\overline{\mathtt{x}}](\mathtt{new}\ \mathtt{N}(\overline{\mathtt{e}}')) = \mathtt{new}\ \mathtt{N}([\overline{\mathtt{e}}]/\overline{\mathtt{x}}]\overline{\mathtt{e}}')$, since N cannot contain term variables. Hence, by $\mathcal{F}$-rule 4e, $\mathcal{F}[\![\mathtt{new}\ \mathtt{N}([\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}')]\!] = \mathtt{new}\ \mathtt{N}(\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}']\!])$. By induction $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}']\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!]$, and by factoring $[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]$, $\mathtt{new}\ \mathtt{N}(\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}']\!]) = \mathtt{new}\ \mathtt{N}([\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!]) = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{new}\ \mathtt{N}(\overline{\mathtt{e}}')]\!]$.
Case $\mathtt{e} \equiv (\mathtt{N})\mathtt{e}_0$. A rephrasing of the case above where (N), $\mathtt{e}_0$ and $\mathcal{F}$-rule 5e are replacing new N, $\overline{\mathtt{e}}'$ and $\mathcal{F}$-rule 4e.
Case $\mathtt{e} \equiv \mathtt{e}_0!(\overline{\mathtt{e}}')$. Then $[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e} = [\overline{\mathtt{e}}/\overline{\mathtt{x}}](\mathtt{e}_0!(\overline{\mathtt{e}}')) = ([\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0)!([\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}')$. Hence by $\mathcal{F}$-rule 6e, $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}]\!] = \mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0]\!].\mathtt{invoke}(\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}']\!])$. By induction $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}_0]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!]$ and $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}']\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!]$, we have $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}]\!] = ([\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!])\ .\mathtt{invoke}([\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!])$. By factoring $[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]$, $([\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0]\!]).\mathtt{invoke}([\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\ \overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!]) = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}](\mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{e}}']\!]))$. Then by $\mathcal{F}$-rule 6e,

$$[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}](\mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{e}}']\!]) = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}_0!(\overline{\mathtt{e}}')]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}]\!]$$

Case $\mathtt{e} \equiv \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}}')\mathtt{e}_0$. Then, assumed without loss of generality, $\overline{\mathtt{x}} \cap \overline{\mathtt{x}}' = \emptyset$:

$$[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e} = [\overline{\mathtt{e}}/\overline{\mathtt{x}}](\#(\overline{\mathtt{T}}\ \overline{\mathtt{x}}')\mathtt{e}_0) = \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}}')[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}'.$$

Hence by $\mathcal{F}$-rule 7e, with $\Delta; \Gamma \vdash \mathtt{e} : \#\mathtt{T}(\overline{\mathtt{T}})$ and $\mathtt{n} = |\overline{\mathtt{T}}|$:

$$\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}]\!] = \mathtt{new}\ \ \mathtt{I\$n}\langle \mathcal{F}[\![\,\overline{\mathtt{T}}\,]\!]\mathcal{F}[\![\,\mathtt{T}\,]\!]\rangle()\{\mathcal{F}[\![\mathtt{T}]\!]\ \mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{x}}')\{\uparrow \mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}']\!]\}.$$

[4] The Java type system includes the axiom on the top class: $\Delta \vdash \mathtt{T} <: \mathtt{Object}$. In the type systems of the calculi, considered in the paper, including FGJ, FGCJ, FGAJ and FGACJ, the axiom is not included since it would be necessary only because of the simplification done in the class and interface header structure: forced to have a superclass, i.e. " $\lhd$ N". In Java, Object is the default superclass: the same can be in our calculi, or may not since such axiom is never used in all properties proofs.

By induction $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}']\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!]$:

$\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}]\!] = \mathtt{new\ \ I\$n}\langle\mathcal{F}[\![\,\overline{\mathtt{T}}\,]\!]\mathcal{F}[\![\,\mathtt{T}\,]\!]\rangle()\{\mathcal{F}[\![\mathtt{T}]\!]\ \mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{x}}')\{\uparrow [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{e}}']\!]\}$.

and by factoring $[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]$:

$[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}](\mathtt{new\ \ I\$n}\langle\mathcal{F}[\![\,\overline{\mathtt{T}}\,]\!]\mathcal{F}[\![\,\mathtt{T}\,]\!]\rangle()\{\mathcal{F}[\![\mathtt{T}]\!]\ \mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{x}}')\{\uparrow \mathcal{F}[\![\overline{\mathtt{e}}']\!]\} = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}](\mathcal{F}[\![\mathtt{e}]\!])$.

Case $\mathtt{e} \equiv \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\overline{\mathtt{M}}\}$. Then $[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e} = \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{M}}\}$. By $\mathcal{F}$-rule 8e, $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\mathtt{e}]\!] = \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{M}}]\!]\}$. By induction $\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{M}}]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{M}}]\!]$:

$$\mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\mathcal{F}[\![[\overline{\mathtt{e}}/\overline{\mathtt{x}}]\overline{\mathtt{M}}]\!]\} = \mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{M}}]\!]\}$$

and then by factoring $[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]$:

$$\mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{[\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\overline{\mathtt{M}}]\!] = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}](\mathtt{new\ I}\langle\overline{\mathtt{T}}\rangle()\{\mathcal{F}[\![\overline{\mathtt{M}}]\!]\}) = [\mathcal{F}[\![\overline{\mathtt{e}}]\!]/\overline{\mathtt{x}}]\mathcal{F}[\![\mathtt{e}]\!] \qquad \square$$

Lemma 5 ($\mathcal{F}[\![]\!]$ Preserves Overriding). *If* $\mathtt{override}(\mathtt{m},\mathtt{N},\mathtt{T})$ *is inferred for a program in* FGACJ *then,* $\mathtt{override}(\mathtt{m},\mathcal{F}[\![\mathtt{N}]\!],\mathcal{F}[\![\mathtt{T}]\!])$ *is inferred for the $\mathcal{F}$-program.*

Proof Immediate since rule Over involves (type) variable renaming. $\square$

Theorem 8 ($\mathcal{F}[\![]\!]$ Preserves Methods). *Let* $\mathtt{M}$ *be a method in a class* $\mathtt{C}$ *(resp. interface* $\mathtt{I}$*) of a program in* FGACJ*. If* $\mathtt{M\ OK\ IN\ C}$ *(resp.,* $\mathtt{M\ OK\ IN\ I}$*) then* $\mathcal{F}[\![\mathtt{M}]\!]\ \mathtt{OK\ IN}\ \mathcal{F}[\![\mathtt{C}]\!]$ *(resp.* $\mathcal{F}[\![\mathtt{M}]\!]\ \mathtt{OK\ IN}\ \mathcal{F}[\![\mathtt{I}]\!]$*) in the $\mathcal{F}$-program. Moreover, if* $\overline{\mathtt{H}}\ \mathtt{OK\ IN\ I}$ *in the program then* $\mathcal{F}[\![\overline{\mathtt{H}}]\!]\ \mathtt{OK\ IN}\ \mathcal{F}[\![\mathtt{I}]\!]$ *in the $\mathcal{F}$-program.*

Proof We prove separately, the cases of methods definitions in classes, anonymous classes and interfaces according to the first three rules in Table **4.a**.

Case GT-Method$_{\text{FGJ}}$. Let $\mathtt{M} \equiv \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\mathtt{T}\ \ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\uparrow\ \mathtt{e}_0;\}\ \mathtt{OK\ IN\ C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle$. By GT-Method$_{\text{FGJ}}$, we have for $\Delta = \overline{\mathtt{X}}{<:}\overline{\mathtt{N}},\ \overline{\mathtt{Y}}{<:}\overline{\mathtt{P}}$, that the followings hold (since it is the only rule that applies to $\mathtt{M}$): $\Delta;\overline{\mathtt{x}} : \overline{\mathtt{T}},\mathtt{this} : \mathtt{C}\langle\overline{\mathtt{X}}\rangle \vdash \mathtt{e}_0 : \mathtt{S}$, and $\Delta \vdash \mathtt{S}{<:}\mathtt{T}$, and $\mathtt{override}(\mathtt{m},\mathtt{N},\langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{T}} \to \mathtt{T}_0)$. Then, by Theorem **5** we have: $\mathcal{F}[\![\Delta]\!];\overline{\mathtt{x}} : \mathcal{F}[\![\overline{\mathtt{T}}]\!],\mathtt{this} : \mathtt{C}\langle\overline{\mathtt{X}}\rangle \vdash \mathcal{F}[\![\mathtt{e}_0]\!] : \mathcal{F}[\![\mathtt{S}]\!]$; By Lemma **3** we have: $\mathcal{F}[\![\Delta]\!] \vdash \mathcal{F}[\![\mathtt{S}]\!]{<:}\mathcal{F}[\![\mathtt{T}]\!]$; By Lemma **5** we have: $\mathtt{override}(\mathtt{m},\mathcal{F}[\![\mathtt{N}]\!],\langle\overline{\mathtt{Y}} \lhd \mathcal{F}[\![\overline{\mathtt{P}}]\!]\rangle\mathcal{F}[\![\overline{\mathtt{T}}]\!] \to \mathcal{F}[\![\mathtt{T}_0]\!])$. Then, by GT-Method$_{\text{FGJ}}$, $\mathcal{F}[\![\mathtt{M}]\!]\ \mathtt{OK\ IN}\ \mathcal{F}[\![\mathtt{C}]\!]$.

Case GT-Anonym$_{\text{FGAJ}}$. Let $\mathtt{M} \equiv \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\mathtt{T}\ \ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\uparrow\ \mathtt{e}_0;\}$ and for some Δ, Γ and $\overline{\mathtt{V}}$, $\Delta;\Gamma \vdash \mathtt{M\ OK\ IN\ I}\langle\overline{\mathtt{V}}\rangle$. By GT-Anonym$_{\text{FGAJ}}$, for $\Delta' = \Delta,\ \overline{\mathtt{X}}{<:}\overline{\mathtt{N}},\ \overline{\mathtt{Y}}{<:}\overline{\mathtt{P}}$, we have: $\Delta';\Gamma,\overline{\mathtt{x}} : \overline{\mathtt{T}},\mathtt{this} : \mathtt{I}\langle\overline{\mathtt{V}}\rangle \vdash \mathtt{e}_0 : \mathtt{S}$, and $\Delta' \vdash \overline{\mathtt{T}},\mathtt{T},\overline{\mathtt{P}}\ \mathtt{ok}$, and $\Delta' \vdash \overline{\mathtt{V}}{<:}[\overline{\mathtt{V}}/\overline{\mathtt{X}}]\overline{\mathtt{N}},\mathtt{S}{<:}\mathtt{T}$, and $\mathtt{interface\ I}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\{\overline{\mathtt{H}}\}$, and $\langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\mathtt{T\ m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}}) \in \overline{\mathtt{H}}$. Then, by Theorem **5** we have: $\mathcal{F}[\![\Delta']\!];\mathcal{F}[\![\Gamma]\!],\overline{\mathtt{x}} : \mathcal{F}[\![\overline{\mathtt{T}}]\!],\mathtt{this} : \mathtt{I}\langle\mathcal{F}[\![\overline{\mathtt{V}}]\!]\rangle \vdash \mathcal{F}[\![\mathtt{e}_0]\!] : \mathcal{F}[\![\mathtt{S}]\!]$; By Lemma **3** we have: $\mathcal{F}[\![\Delta']\!] \vdash \mathcal{F}[\![\overline{\mathtt{T}}]\!],\mathcal{F}[\![\mathtt{T}]\!],\mathcal{F}[\![\overline{\mathtt{P}}]\!]\ \mathtt{ok}$, and $\mathcal{F}[\![\Delta']\!] \vdash \mathcal{F}[\![\overline{\mathtt{V}}]\!]{<:}\mathcal{F}[\![[\overline{\mathtt{V}}/\ \overline{\mathtt{X}}]\overline{\mathtt{N}}]\!],\mathcal{F}[\![\mathtt{S}]\!]{<:}\mathcal{F}[\![\mathtt{T}]\!]$. Eventually, by $\mathcal{F}$-rule 2l, $\mathtt{interface\ I}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle\{\mathcal{F}[\![\overline{\mathtt{H}}]\!]\}$ is in the $\mathcal{F}$-program, and by $\mathcal{F}$-rule h, $\langle\overline{\mathtt{Y}} \lhd \mathcal{F}[\![\overline{\mathtt{P}}]\!]\rangle\mathcal{F}[\![\mathtt{T}]\!]\ \mathtt{m}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{x}}) \in \mathcal{F}[\![\overline{\mathtt{H}}]\!]$. Then, by GT-Anonym$_{\text{FGAJ}}$, $\mathcal{F}[\![\mathtt{M}]\!]\ \mathtt{OK\ IN}\ \mathcal{F}[\![\mathtt{I}]\!]$.

Case GT-Header$_{\text{FGAJ}}$. Let $\mathtt{H\ OK\ IN\ I}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle$ for a method signature $\mathtt{H} \equiv \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\mathtt{T\ m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})$ of an interface in the program. By GT-Header$_{\text{FGAJ}}$, $\overline{\mathtt{Y}}{<:}\overline{\mathtt{P}},\overline{\mathtt{X}}{<:}\overline{\mathtt{N}} \vdash \overline{\mathtt{T}},\mathtt{T},\overline{\mathtt{P}}\ \mathtt{ok}$ holds in the program, and by Lemma **3**, $\overline{\mathtt{Y}}{<:}\mathcal{F}[\![\overline{\mathtt{P}}]\!],\overline{\mathtt{X}}{<:}\mathcal{F}[\![\overline{\mathtt{N}}]\!] \vdash \mathcal{F}[\![\overline{\mathtt{T}}]\!],\mathcal{F}[\![\mathtt{T}]\!],\mathcal{F}[\![\overline{\mathtt{P}}]\!]\ \mathtt{ok}$ holds in the $\mathcal{F}$-program. Hence by GT-Header$_{\text{FGAJ}}$ in the $\mathcal{F}$-program, $\mathcal{F}[\![\mathtt{H}]\!]\ \mathtt{OK\ IN}\ \mathcal{F}[\![\mathtt{I}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle]\!]$ $\square$

Lemma 6 ($\mathcal{F}[\![]\!]$ Preserves Class Method Types). *If* $\mathrm{mtype}(\mathtt{m},\mathtt{N}) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \to \mathtt{U}$ *then* $\mathrm{mtype}(\mathtt{m},\mathcal{F}[\![\mathtt{N}]\!]) = \langle\overline{\mathtt{Y}} \lhd \mathcal{F}[\![\overline{\mathtt{P}}]\!]\rangle\mathcal{F}[\![\overline{\mathtt{U}}]\!] \to \mathcal{F}[\![\mathtt{U}]\!]$

Proof By $\mathcal{F}$-rule 1l, if $\mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\ \{\overline{\mathtt{S}}\ \overline{\mathtt{f}};\mathtt{K}\ \overline{\mathtt{M}}\}$ is in the program then $\mathtt{class\ C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle \lhd \mathcal{F}[\![\mathtt{N}]\!]\ \{\overline{\mathtt{S}}\ \overline{\mathtt{f}};\mathcal{F}[\![\mathtt{K}]\!]\ \mathcal{F}[\![\overline{\mathtt{M}}]\!]\}$ is in the $\mathcal{F}$-program, and by $\mathcal{F}$-rule m, if $\langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\mathtt{U\ m}\ (\overline{\mathtt{U}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e};\} \in \overline{\mathtt{M}}$ then $\langle\overline{\mathtt{Y}} \lhd \mathcal{F}[\![\overline{\mathtt{P}}]\!]\rangle\mathcal{F}[\![\mathtt{U}]\!]\ \mathtt{m}\ (\mathcal{F}[\![\overline{\mathtt{U}}]\!]\ \overline{\mathtt{x}})\{\uparrow \mathcal{F}[\![\mathtt{e}]\!];\} \in \mathcal{F}[\![\overline{\mathtt{M}}]\!]$. Hence, by MT-Class, if $mtype(\mathtt{m},\mathtt{N}) = \langle\overline{\mathtt{Y}} \lhd \overline{\mathtt{P}}\rangle\overline{\mathtt{U}} \rightarrow \mathtt{U}$ is inferred from the program then $mtype(\mathtt{m},\mathcal{F}[\![\mathtt{N}]\!])= \langle\overline{\mathtt{Y}} \lhd \mathcal{F}[\![\overline{\mathtt{P}}]\!]\rangle\mathcal{F}[\![\overline{\mathtt{U}}]\!] \rightarrow \mathcal{F}[\![\mathtt{U}]\!]$ is inferred from the $\mathcal{F}$-program. $\square$

Theorem 9 ($\mathcal{F}[\![]\!]$ is Complete and preserves classes and interfaces). *(a) Let $u \in$ FGACJ be any term, including types, classes, interfaces, expressions, then $\mathcal{F}[\![u]\!]$ exists and $\mathcal{F}[\![u]\!] \in$ FGAJ. (b) If $\Delta \vdash \#\mathtt{T}(\overline{\mathtt{T}})$ ok for some $\mathtt{T},\overline{\mathtt{T}}$, then $\Delta \vdash A$ OK for $A \equiv$*`interface` $\mathtt{I\$n}\langle\overline{\mathtt{X}},\mathtt{X}\lhd \overline{\mathtt{O}},\mathtt{O}\rangle\{\mathtt{X}\ \mathtt{invoke}(\overline{\mathtt{X}}\ \overline{\mathtt{x}})\}$. *(c) If $\Delta \vdash u$ OK, then $\mathcal{F}[\![\Delta]\!] \vdash \mathcal{F}[\![u]\!]$ OK*

Proof
(Part a). For each syntactic form u of FGACJ exactly one rule of $\mathcal{F}$-system (**Table 6**) applies: The rule may requires the application of only a finite number of other rules of $\mathcal{F}$-system to the sub-terms of u. Let k be the constituent (sub-)terms of u. Then, the application of the rule may involve no more than the application of k rules in total. This proves that $\mathcal{F}[\![u]\!]$ exists. Rule 2t removes closure types, $\mathcal{F}$-rule 6e removes closure invocations, $\mathcal{F}$-rule 7e removes closure expressions, and no rule introduces closure types, closure invocations, closure expressions. This proves that $\mathcal{F}[\![u]\!]$ is in FGAJ
(Part b). Let $\#\mathtt{T}(\overline{\mathtt{T}}) \equiv u$. Then, interface $A \equiv$`interface` $\mathtt{I\$n}\langle\overline{\mathtt{X}},\mathtt{X}\lhd \overline{\mathtt{O}},\mathtt{O}\rangle\{\mathtt{X}\ \mathtt{invoke}(\overline{\mathtt{X}}\ \overline{\mathtt{x}})\}$ is added to the $\mathcal{F}$-program, $\mathtt{n} = |\overline{\mathtt{T}}|$ and `O` stands for `Object`. By GT-Header$_{\text{FGAJ}}$(since, $\overline{\mathtt{X}}{<:}\overline{\mathtt{O}},\mathtt{X}{<:}\mathtt{O} \vdash \overline{\mathtt{X}},\mathtt{X}$ ok), and by GT-Interf$_{\text{FGAJ}}$: A OK.
(Part c) We have two cases.
Case `class` $\mathtt{C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\mathtt{K}\ \overline{\mathtt{M}}\} \equiv u$. By GT-Class$_{\text{FGJ}}$, and by Lemma **3**, $\overline{\mathtt{X}} <: \mathcal{F}[\![\overline{\mathtt{N}}]\!] \vdash \mathcal{F}[\![\overline{\mathtt{N}}]\!],\mathcal{F}[\![\mathtt{N}]\!],\mathcal{F}[\![\overline{\mathtt{T}}]\!]$ ok, and by Theorem **8**, $\mathcal{F}[\![\overline{\mathtt{M}}]\!]$ OK IN $\mathtt{C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle$. Moreover, $\mathcal{F}[\![fields(\mathtt{N})]\!] = \mathcal{F}[\![\overline{\mathtt{U}}]\!]\ \overline{\mathtt{g}}$, and $\mathcal{F}[\![\mathtt{K}]\!] = \mathtt{C}(\mathcal{F}[\![\overline{\mathtt{U}}]\!]\ \overline{\mathtt{g}},\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{f}})\{\text{super}(\overline{\mathtt{g}});\text{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};\}$ are easy to obtain. Hence, by GT-Class$_{\text{FGJ}}$, `class` $\mathtt{C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle \lhd \mathcal{F}[\![\mathtt{N}]\!]\ \{\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{f}};\ \mathcal{F}[\![\mathtt{K}]\!]\ \mathcal{F}[\![\overline{\mathtt{M}}]\!]\}$ OK.
Case `interface` $\mathtt{I}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\{\overline{\mathtt{H}}\} \equiv u$. By GT-Interf$_{\text{FGAJ}}$, and Lemma **3**, $\overline{\mathtt{X}} <: \mathcal{F}[\![\overline{\mathtt{N}}]\!] \vdash \mathcal{F}[\![\overline{\mathtt{N}}]\!]$ ok, and Theorem **8**, $\mathcal{F}[\![\overline{\mathtt{H}}]\!]$ OK IN $\mathtt{I}\langle\overline{\mathtt{X}}\lhd\mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle$. Hence, by GT-Interf$_{\text{FGAJ}}$, `interface` $\mathtt{I}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle\{\mathcal{F}[\![\overline{\mathtt{H}}]\!]\}$ OK $\square$

Theorem 10 ($\mathcal{F}[\![]\!]$ is Idempotent). *Let $u \in$ FGACJ. Then $\mathcal{F}[\![\mathcal{F}[\![u]\!]]\!]{=}\mathcal{F}[\![u]\!]$*

Proof By case analysis on the last $\mathcal{F}$-rule used and induction on the application of $\mathcal{F}$ to the subterms of u.
Case 1t. Trivial
Case 2t. Let $u \equiv \#\mathtt{S}(\overline{\mathtt{S}})$. Let $\mathtt{Z} \equiv \mathcal{F}[\![\mathcal{F}[\![u]\!]]\!]$. By $\mathcal{F}$-rule 2t, $\mathtt{Z} = \mathcal{F}[\![\mathtt{I\$n}\langle\mathcal{F}[\![\overline{\mathtt{S}}]\!]\mathcal{F}[\![\mathtt{S}]\!]\rangle]\!]$. By $\mathcal{F}$-rule 3t, and then, by induction, $\mathtt{Z} = \mathtt{I\$n}\langle\mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{S}}]\!]]\!]\mathcal{F}[\![\mathcal{F}[\![\mathtt{S}]\!]]\!]\rangle = \mathtt{I\$n}\langle\mathcal{F}[\![\overline{\mathtt{S}}]\!]\mathcal{F}[\![\mathtt{S}]\!]\rangle = \mathcal{F}[\![u]\!]$.
Case 3t. Immediate by induction.
Case 1l. Let $u \equiv$ `class` $\mathtt{C}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\mathtt{K}\ \overline{\mathtt{M}}\}$. Let $\mathtt{Z} \equiv \mathcal{F}[\![\mathcal{F}[\![u]\!]]\!]$. By $\mathcal{F}$-rule 1l, $\mathtt{Z} = \mathcal{F}[\![$`class` $\mathtt{C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle \lhd \mathcal{F}[\![\mathtt{N}]\!]\ \{\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{f}};\mathcal{F}[\![\mathtt{K}]\!]\ \mathcal{F}[\![\overline{\mathtt{M}}]\!]\}]\!]$. By $\mathcal{F}$-rule 1l, $\mathtt{Z} =$ `class` $\mathtt{C}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{N}}]\!]]\!]\rangle \lhd \mathcal{F}[\![\mathcal{F}[\![\mathtt{N}]\!]]\!]\ \{\mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{T}}]\!]]\!]\ \overline{\mathtt{f}};\mathcal{F}[\![\mathcal{F}\ [\![\mathtt{K}]\!]]\!]\ \mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{M}}]\!]]\!]\}$. Then, by induction, $\mathtt{Z} = \mathcal{F}[\![u]\!]$.
Case 2l. Let $u \equiv$ `interface` $\mathtt{I}\langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\{\overline{\mathtt{H}}\}$. Let $\mathtt{Z} \equiv \mathcal{F}[\![\mathcal{F}[\![u]\!]]\!]$. By $\mathcal{F}$-rule 2l, twice, $\mathtt{Z} =$ `interface` $\mathtt{I}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{N}}]\!]]\!]\rangle\{\mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{H}}]\!]]\!]\}$. Then, by induction, $\mathtt{Z} =$ `interface` $\mathtt{I}\langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle\{\mathcal{F}[\![\overline{\mathtt{H}}]\!]\}$.
Case k. Let $u \equiv \mathtt{C}(\overline{\mathtt{T}}\ \overline{\mathtt{f}})\{\mathtt{super}(\overline{\mathtt{f}});\mathtt{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};\}$. Let $\mathtt{Z} = \mathcal{F}[\![\mathcal{F}[\![u]\!]]\!]$. By $\mathcal{F}$-rule k, twice, $\mathtt{Z} = \mathtt{C}(\mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{T}}]\!]]\!]\ \overline{\mathtt{f}})\{\mathtt{super}(\overline{\mathtt{f}});\mathtt{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};\}$. Then, by the induction, $\mathtt{Z} = \mathtt{C}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{f}})\{\mathtt{super}(\overline{\mathtt{f}});\mathtt{this}.\overline{\mathtt{f}} = \overline{\mathtt{f}};\} = \mathcal{F}[\![u]\!]$.
Case m. Let $u \equiv \langle\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\rangle\mathtt{T}\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\uparrow \mathtt{e}\}$. Let $\mathtt{Z} \equiv \mathcal{F}[\![\mathcal{F}[\![u]\!]]\!]$. By $\mathcal{F}$-rule m, twice, $\mathtt{Z} = \langle\overline{\mathtt{X}} \lhd \mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{N}}]\!]]\!]\rangle\mathcal{F}[\![\mathcal{F}[\![\mathtt{T}]\!]]\!]\ \mathtt{m}(\mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{T}}]\!]]\!]\ \overline{\mathtt{x}})\{\uparrow \mathcal{F}[\![\mathcal{F}[\![\mathtt{e}]\!]]\!]\}$. Then, by induction $\mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{N}}]\!]]\!] = \mathcal{F}[\![\overline{\mathtt{N}}]\!]$, $\mathcal{F}[\![\mathcal{F}[\![\mathtt{T}]\!]]\!] = \mathcal{F}[\![\mathtt{T}]\!]$, $\mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{T}}]\!]]\!] = \mathcal{F}[\![\overline{\mathtt{T}}]\!]$, $\mathcal{F}[\![\mathcal{F}[\![\mathtt{e}]\!]]\!] = \mathcal{F}[\![\mathtt{e}]\!]$, we have $\mathtt{Z} = \langle\overline{\mathtt{X}}\lhd\mathcal{F}[\![\overline{\mathtt{N}}]\!]\rangle\mathcal{F}[\![\mathtt{T}]\!]\ \mathtt{m}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{x}})\{\uparrow$

$\mathcal{F}[\![\mathtt{e}]\!]\} = \mathcal{F}[\![u]\!]$.
Cases h, 1e-5e and 8e are proved by trivial re-phrasings of the proof of the above case.
Case 6e. Let $u \equiv \mathtt{e}_0!(\overline{\mathtt{e}})$. Let $\mathtt{Z} \equiv \mathcal{F}[\![\mathcal{F}[\![u]\!]]\!]$. By $\mathcal{F}$-rule 6e, $\mathtt{Z} \equiv \mathcal{F}[\![\mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{e}}]\!])]\!]$. By $\mathcal{F}$-rule 3e, $\mathtt{Z} \equiv \mathcal{F}[\![\mathcal{F}[\![\mathtt{e}_0]\!]]\!].\mathtt{invoke}(\mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{e}}]\!]]\!])$. By induction $\mathcal{F}[\![\mathcal{F}[\![\mathtt{e}_0]\!]]\!] = \mathcal{F}[\![\mathtt{e}_0]\!]$ and $\mathcal{F}[\![\mathcal{F}[\![\mathtt{e}]\!]]\!] = \mathcal{F}[\![\mathtt{e}]\!]$, we have $\mathtt{Z} \equiv \mathcal{F}[\![\mathtt{e}_0]\!].\mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{e}}]\!]) = \mathcal{F}[\![u]\!]$.
Case 7e. Let $u \equiv \#(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\mathtt{e}$, with $|\overline{\mathtt{T}}| = n$ and $\Delta; \Gamma \vdash \mathtt{e} : \#\mathtt{T}(\overline{\mathtt{T}})$, for some Δ and Γ. Let $\mathtt{Z} \equiv \mathcal{F}[\![\mathcal{F}[\![u]\!]]\!]$. By $\mathcal{F}$-rule 7e, $\mathtt{Z} \equiv \mathcal{F}[\![\mathtt{new\ I\$n}\langle \mathcal{F}[\![\,\overline{\mathtt{T}}\,]\!]\mathcal{F}[\![\,\mathtt{T}\,]\!]\rangle()\{\mathcal{F}[\![\mathtt{T}]\!]\ \mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{x}})\{\uparrow \mathcal{F}[\![\mathtt{e}_0]\!]\}\}]\!]$. By $\mathcal{F}$-rule 8e, $\mathtt{Z} \equiv \mathtt{new\ I\$n}\langle \mathcal{F}[\![\mathcal{F}[\![\,\overline{\mathtt{T}}\,]\!]\mathcal{F}[\![\,\mathtt{T}\,]\!]]\!]\rangle()\{\mathcal{F}[\![\mathcal{F}[\![\mathtt{T}]\!]\ \mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\overline{\mathtt{x}})\{\uparrow \mathcal{F}[\![\mathtt{e}]\!]]\!]\}\}$. By induction $\mathcal{F}[\![\mathcal{F}[\![\,\overline{\mathtt{T}}\,]\!]\mathcal{F}[\![\,\mathtt{T}\,]\!]]\!] = \mathcal{F}[\![\mathcal{F}[\![\,\overline{\mathtt{T}}\,\mathtt{T}\,]\!]]\!] = \mathcal{F}[\![\,\overline{\mathtt{T}}\,\mathtt{T}\,]\!]$, by $\mathcal{F}$-rule m on $\mathtt{invoke}$, by induction $\mathcal{F}[\![\mathcal{F}[\![\mathtt{T}]\!]]\!] = \mathcal{F}[\![\mathtt{T}]\!]$, $\mathcal{F}[\![\mathcal{F}[\![\overline{\mathtt{T}}]\!]]\!] = \mathcal{F}[\![\overline{\mathtt{T}}]\!]$ and $\mathcal{F}[\![\mathcal{F}[\![\mathtt{e}]\!]]\!] = \mathcal{F}[\![\mathtt{e}]\!]$, we have $\mathtt{Z} \equiv \mathtt{new\ I\$n}\langle \mathcal{F}[\![\,\overline{\mathtt{T}}\,\mathtt{T}\,]\!]\rangle\ ()\{\mathcal{F}[\![\mathtt{T}]\!]\ \mathtt{invoke}(\mathcal{F}[\![\overline{\mathtt{T}}]\!]\ \overline{\mathtt{x}})\{\uparrow \mathcal{F}[\![\mathtt{e}]\!]]\!]\}\} = \mathcal{F}[\![u]\!]$. $\square$

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