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Joint assignment, scheduling and routing models to Home Care optimization: a pattern based approach

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Abstract

The design of efficient Home Care Services is a quite recent and challenging problem motivated by the ever increasing age of population and the consequent need to reduce hospitalization costs. We are given a weekly planning horizon, a set of operators each characterized by a skill and a set of patients each requiring a set of visits also characterized by a skill. We propose an integrated model that jointly addresses the following problems: (i) the assignment of operators to patients guaranteeing the appropriate levels of skill; (ii) the scheduling of visits in the planning horizon; and (iii) the determination of a set of tours indicating the sequence of patients each operator must visit in every day of the week. Several variants of this model are investigated. All of them use the pattern as a key tool to formulate the problem. A pattern is an a priori given profile specifying a possible schedule for a given set of visits possibly characterized by differnt skills. Computational results on a set of real instances are analysed. They show that the selection of the pattern generation policy is crucial to solve large instances efficiently.

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Introduction

Nowadays, the ever increasing average age of population, at least in industrialized countries, and the increased costs for the consequently required care, compel the medical care units (hospitals and so on) to offer home care services in an attempt to limit costs. Elderly people have in fact varying degrees of need for assistance and medical treatment, and it may be advantageous to allow them to live in their own homes as long as possible, since a long-term stay in a nursing home can be much more costly for the social insurance system than a treatment at home providing assistance to the required level. Even more important, medical treatments carried out at patients home impact favorably on their quality of life. Therefore, home care services are a cost-effective and flexible instrument in the social system.

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Interestingly, in home care setting the minimization of the operating costs, that is a common objective of the stakeholders (either private or public) providing the service, and the maximization of some measures of patient satisfaction, that is an objective demanded by the patients, are not conflicting objectives as it happens in other settings.

In this paper we address a relevant optimization problem arising in Home Care; specifically, given a planning horizon W, usually a week, a set of patients with an associated *care plan*, i.e. weekly requests each of them demanding a specific skill to be operated, and a set of operators also characterized by a specific skill, the problem asks to schedule the patient requests during the planning horizon, assign the operators to the patients by taking into account the compatibility between request and operator skills, and determine the tour each operator has to perform in every day of the planning horizon.

An objective function typically used to guide the home care decisions is the balancing of the workload among the operators. Hence we maximize the minimum operator utilization factor, where the *operator utilization factor* is the total workload of the operator in W over his/her maximum possible workload. An alternative balancing function is investigated, which penalizes the tours with excessive traveling time; i.e., it prevents undesiderable solutions where operators spend a lot of time moving among patient's homes, to increase their own utilization factor.

Relevant Quality of Service requisites are taken into account during the optimization process. One of them is the so-called *continuity of care*, expressed by imposing that at most T operators can be assigned to each patient in the considered planning horizon, where T is given. Another relevant requisite, the *maximum daily workload*, states that the workload of each operator in each day, i.e. the sum of the daily service times and traveling times of the operator, can not exceed the duration of the operator workday.

The Home Care context under investigation thus involves joint assignment, scheduling and routing decisions over a given planning horizon. In the state-of-the-art literature Home Care problems are usually solved in cascade: first the operators are assigned to the patients on a geographical basis so as to match the skills required by a service at patient's home with the skill owned by the operator; second, the schedule of each operator is determined, usually operator-wise. Some Vehicle Routing Problem (VRP)-like formulations exist in the literature, which however generally deal with a daily planning horizon. To the best of our knowledge there are only three exceptions. In [3] the weekly planning is viewed as the union of independent daily plannings. The other two studies ([14] , [17]) address a problem which is similar to the one analysed in this paper, seeking for an optimal weekly plan. However, no exact approach is proposed to solve the overall problem, but two-stage solution approaches are presented. In fact, in the literature tailored metaheuristic approaches are usually proposed to solve Home Care problems rather than exact approaches.

On the other hand, here the Home Care problem is formulated by jointly addressing assignment, scheduling and routing decisions over a weekly planning horizon, by maintaining its strong relationship with the VRP, and specifically with Skill VRP ([6], [7]), from which it inherits the skill based structure. We propose a new Integer Linear Programming (ILP) formulation, with some variants, that extends one of the Skill VRP models proposed in [6] and [7], and specifically the one with the lower level of disaggregation among the three proposed (due to the complexity of Home Care problems). In fact, the model is based on a suitable use of skill information for both patients and operators in order to state the compatibility between the skills required for operating the patient requests and the operator skills.

An innovative modelling device to combine the three levels of decisions is that services are offered according to a set of a priori given *patterns*: three policies to generate patterns are designed and discussed in the following. The relevant QoS constraints mentioned before, i.e. continuity of care and maximum daily workload, are incorporated into the models.

The ILP models have been enhanced also by means of valid inequalities aimed at breaking the symmetric structure that usually characterizes the Home Care instances and the underlying logistics network. These models have been validated by means of 40 runs on 10 large instances using real data from a northern Italian medical care unit. Specifically, each run is obtained combining a particular pattern generation policy with a particular objective function. In addition, the data set comprises also three randomly generated instances used to assess the impact of increasing number of high skill requests. These instances have been generated starting from real instances. The obtained results, although preliminary, are very interesting. First of all, the experiments show that determining a good but limited set of patterns appears to be crucial for efficiently solving the Linear Programming (LP) relaxation, thus helping in computing very good quality solutions. In particular, one of the proposed pattern generation approaches, which is based on the solution of an auxiliary network flow problem, proved to be very effective in selecting a small number of patterns of good quality. Also the proposed symmetry management valid inequalities proved to be crucial either to halve the optimum percentage gap, or to find feasible solutions in the most critical cases. Therefore the main achievement is that, by properly selecting the pattern generation mechanism and suitably using the symmetry management tool, in almost all the cases the proposed models were able to compute very good quality solutions (with an optimum percentage gap within 0.32% for the solved instances), although the problem solution required large computational time and memory resources. However a large but affordable consumption of such resources does not seem to be an issue when the focus is a difficult planning problem that has to be solved once a week. Anyway, due to the large size of the tested instances, which reflects the large size of real Home Care instances, and due to the complexity of the involved decisions (i.e. scheduling, assignment and routing on a weekly planning horizon), the determination of very good quality solutions in almost all the performed experiments reveals, in our opinion, the potentiality of the proposed ILP models in successfully addressing still larger Home Care instances. The design of decomposition methods based on the proposed ILP models, and tailored to solve even larger Home Care instances, is in fact in progress.

The plan of the paper is the following. In Section 1 we define the studied Home Care problem in a more formal way, by introducing the used notation. In Section 2 we describe the main results from the literature on modelling and solution approaches to Home Care optimization problems, by trying to outline similarities and dissimilarities with the results in this paper. In Section 3 we present the proposed ILP models to Home Care. Valid inequalities to enhance the proposed ILP models, which exploit the peculiar cost structure of the real instances, are also presented. In Section 4 we describe some pattern generation approaches. In Section 5 we discuss the results of the computational experiments aimed at validating the proposed models on real Home Care instances. Finally, Section 6 provides conclusions and discussion about possible avenues of research.

1 The Home Care Problem: a weekly planning horizon

The Home Care Problem (HCP) under investigation is defined on a complete directed network $G = (N, A)$, having n nodes, where each node j corresponds to a patient. There is an extra node (node 0), which is used to denote the basis where the operators start their tour from and arrive in.

A *care plan* r_j is assumed to be given for each patient j. For each considered level of skill, the care plan associated with j specifies the number of visits required by j in the planning horizon W relatively to that skill. A set K of $\overline{k}$ levels of skill is assumed for both patients and operators, where skill $\overline{k}$ corresponds to the highest ability and skill 1 to the lowest one. As an example, two levels of skill may be assumed where skill 1 refers to ordinary requests, whereas skill 2 corresponds to palliative requests. This is the setting addressed in our case study, as detailed in Section 5. According to this assumption, each care plan r_j is therefore an input data with $\overline{k}$ components, such that r_{jk} with $k \in K$ gives the number of visits of skill k required by j in the planning horizon.

Let O denote the set of the (skilled) operators available in the planning horizon, and $O_d \subseteq O$ be the set of the operators available on day d, for each $d \in W$. A hierarchical structure of the skill levels is assumed so that an operator with skill k can work all the requests characterized by a skill up to k. Under this assumption, requests of the highest skill $\overline{k}$ can be operated only by the most skilled operators.

In HCP the scheduling of the patient requests in W, the operator assignment and the routing decisions are offered through a new modelling device, called *pattern*. We assume in fact that the patient requests are operated according to a set P of a priori given patterns. For example, if a patient requires three visits of a given skill a week, they can be operated according to the pattern Monday-Wednesday-Friday or Monday-Tuesday-Thursday. In particular, for each pattern $p \in P$, we define $p(d) = 0$ if no service is offered at day d, while it is $p(d) = k$ if a visit of skill k is operated according to pattern p on day d.

Given the input data above, HCP consists in assigning a pattern from P to each patient j, so scheduling the requests of j, expressed by r_j, during the planning horizon (*care plan scheduling*), in assigning operators to each patient j, for each day where a request of j has been scheduled (*operator assignment*), and in determining the tour of each operator for each scheduled day (*routing decisions*). In addressing these three groups of decisions, the skill constraints (i.e. the compatibility between the skills associated with the patient requests and the skills of the operators), the continuity of the care (i.e. at most T operators can be assigned to each patient in W, for given T) and the maximum daily workload constraints have to be taken into account. The objective function aims at balancing the overall *operator utilization*, as better specified in Section 3.

2 The literature

Planning Home Health Care services is a rather young but quickly evolving research area. As indicated before, in the state-of-the-art Home Care literature problems are usually solved in cascade: first the operators are assigned to the patients on a geographical basis so as to match the skills required by a service at patient's home with the skill owned by the operator; second, the schedule of each operator is determined, usually operator-wise. See [5] for an example of a scheduling model in Home Care where, however, routing aspects are not addressed.

In 1997, Begur et al. [3] clearly stated the importance of jointly addressing scheduling and routing decisions in Home Care. Furthermore, they firstly suggested the use of patterns in order to schedule the patient requests during the considered planning horizon. However, they used the concept of pattern as a means to decompose the overall problem into a set of daily independent routing problems, and used a heuristic approach to generate the operator tours rather than exploiting an exact optimization approach. Specifically, the heuristic proposed in [3] uses a sequential savings algorithm as well as a nearest neighbor heuristic to reoptimize each single route. The authors estimated large savings potentials for 40 patients and 7 operators per day.

By distinguishing between salaried operators (full-time nurses) and non-salaried operators (part-time nurses), in about the same period Cheng and Rich [9] studied the problem of determining an optimal schedule for each operator such that each nurse leaves from his/her home, visits a set of compatible patients within associated time windows, takes a lunch break and returns home, all within the nurse time window and the maximum duration of a shift. The objective function to be minimized is the amount of the overtime and part-time worked. Compared to the Home Care problem studied in this paper, there is no concept of skill. Pairs of compatible nurse-patient are considered instead. In addition, the planning horizon is the day. Two and three-index ILP formulations are presented. However, the problem is solved by means of a two phase algorithm that first builds parallel tours that are improved afterwards. The authors compared their heuristic results with the optimal solutions found by the solver software Cplex for some random test instances with up to 10 patients and 4 operators, and shown some heuristic results for larger instances.

A mathematical programming model for the combined vehicle routing and scheduling problem with time windows and additional temporal constraints is presented in [2]. The temporal constraints allow for imposing pairwise synchronization and pairwise temporal precedence between customer visits, independently of the vehicles. The authors describe some real world applications, among which the daily planning of Home Care staff. They propose an optimization based heuristic to solve real size instances for up to 80 visits and 16 nurses, and compare a direct usage of a commercial solver against the proposed heuristic. Again, the considered planning horizon is the day. The description of an analogous combined scheduling and routing problem, with a related decision support system to solve it, called Laps Care, can be found in [12]. The problem is formulated as a set partitioning problem and solved by repeated matching on a daily horizon. Requirements that two operators occur simultaneously or in given order are also taken into account by the decision support system. The presented case studies can deal with 86 - 123 patients and 12 - 21 nurses.

Syncronized visits in the Home Care context are addressed also in [15]. The authors study a routing problem where some cares have to be performed by several operators and some cares cannot be performed with others. If a patient needs several cares, he/she may want to be treated by the same person. Moreover, some skills constraints and time windows have to be satisfied. Again, the planning horizon is the day. The authors showed that the studied Home Care problem is equivalent to a multiple traveling salesman problem with time windows (mTSPWT) with some specific constraints, and proposed an ILP model with some technical improvements. The proposed ILP model was not able to deal with instances of real size, but it gave rapidly solutions of good quality.

A multiple vehicle routing model with time windows and additional constraints has been proposed also in [11]. Here the concept of skill is not present, but the patients are assigned to a geographical sector depending on their home address. Whenever possible, an operator visits only patients from his/her sector, but he/she may however have to visit patients from other sectors to balance the operator workloads. Continuity of care and blood sample related constraints (the specific application context concerns in fact blood sample requests) are also taken into account. Although a mathematical model is proposed, a metaheuristic approach based on Tabu Search is indeed used to solve the specific Home Care problem under consideration. Other specific Home Care problems have been addressed in the literature, such as the planning of operations related to chemotherapy at home in [8], and the daily planning of Home Health Care in times of flood disasters in [20]. In [20], a model formulation has been presented which includes assignment constraints, working time restrictions, time windows, and mandatory break times. A feasible solution has also to consider qualification levels, language skills, and rejections due to personal reasons. The model has been implemented with the solver software Xpress 7.0 and solved for small problem instances. Real life-sized instances have been tackled with a variable neighborhood search (VNS)-based heuristic that is capable of solving even large instances covering 512 requests and 75 operators.

[10] and [18] present a Branch and Price framework for a daily Home Health Care scheduling problem in Denmark. Instances with up to 150 requests and 15 operators are presented and compared to current practice. See also [1] for a collaborative population-based metaheuristic technique to the scheduling of Home Care workers in the UK.

A daily assignment and routing Home Care problem is also addressed in [4] and in [19]. The problem in [4] considers the skill of patients and operators, and includes a variety of hard and soft constraints, plus preferences. The proposed two-phase approach interweaves two parts: finding a partition of requests to operators, and finding an optimal sequencing for each such partition. A combination of linear programming, constraint programming, and heuristics is used to develop a software tool to Home Care applications. The authors solved instances with 200-600 requests and 20-50 operators. In [19] a new insertion method is developed on the basis of an already known insertion heuristic where the assignment of visits to operators and the generation of routes is done in the same process. The solution found by the insertion heuristic is then used as the initial solution of a tabu search approach. In [19], sometimes the operators have to visit the same patient more than one time the same day. Furthermore, shared visits are allowed. However, all operators are assumed to have the same qualifications, and the visits are not characterized by skills. The instances used for parameter tuning and performance testing have 17 operators and 166.5 visits on average, and they are based on data from a municipality in Denmark.

To the best of our knowledge, the only works dealing with a weekly planning horizon (in addition to the already mentioned [3]) are [17] and [14]. [17] presents a two phase heuristic approach rather than an exact approach. In the first stage, a constraint programming heuristic guarantees the quick calculation of a feasible, applicable solution. Afterwards, an adaptive large neighborhood search (ALNS) seeks to improve the initial solution if further computational time is available. The approach has been evaluated with real-world data from Germany and the Netherlands. The results show that it is possible to solve practical instances of Home Care operations planning in reasonable time, with up to 12 operators and 95 patients.

A two-level approach is used also in [14]. The first phase constructs a masterplan which is a long-term plan. The masterplan specifies when patients are to be visited and the operators that should conduct each visit, i.e. operator assignment and scheduling decisions. This phase is implemented via a five-phase heuristic, essentially based on local search routines. The second phase is a daily planning which uses the masterplan as a starting point, but incorporates last minute changes such as employees falling in sick, ad hoc visits, and other unforeseen events. Existing software is used for daily planning, i.e. for routing decisions. Realistic instances with around 25 operators and 200 patients have been tested. An ILP formulation of the key parts of the masterplan problem is also presented, and an exact solution approach based on Branch and Price is proposed to solve the masterplan problem. Therefore, routing decisions are not incorporated into the exact approach. Anyway, the exact Branch and Price algorithm could solve only instances of limited size.

With respect to the reviewed literature, a key contribution of our study is to address a weekly planning horizon in Home Care rather than the daily planning horizon commonly investigated. In addition, with respect to the aforementioned [3], [17] and [14], that also consider such a long time horizon, here assignment, scheduling and routing decisions are addressed in a joint way, i.e. without heuristically decomposing the problem by means of a two-level approach. To this end, crucial and original is, in our opinion, the use of the pattern modelling device as a means to coordinate the diverse decision levels. Notice that the ILP models proposed in this paper strictly generalize some of the models in ([6], [7]). In fact, they incorporate the skill hierarchy structure introduced in ([6], [7]) where, however, only daily routing decisions have been addressed.

3 ILP models

In order to state HCP in a more formal way, let us denote with:

W	the planning horizon
s_t	the skill of operator t, $t \in O$
D_t	the workday length of operator t, $t \in O$
r_{jk}	the number of visits required by j in W relatively to skill k, $j \in N$ $(j \neq 0)$, $k = 1, 2$
t'_j	the service time at node j, $j \in N$ $(t'_0 = 0)$
t_{ij}	the traveling time from node i to node j, $(i, j) \in A$

Then let us define the following three families of variables in order to model the care plan scheduling, the operator assignment and the routing decisions:

$$z_{jp} = \begin{cases} 1 \text{ if pattern } p \text{ is assigned to patient } j \\ 0 \text{ otherwise} \end{cases} \qquad j \in N \ (j \neq 0),\ p \in P$$

$$u_{tj} = \begin{cases} 1 \text{ if operator } t \text{ is assigned to patient } j \\ 0 \text{ otherwise} \end{cases} \qquad t \in O,\ j \in N \ (j \neq 0)$$

$$x_{ij}^{td} = \begin{cases} 1 \text{ if operator } t \text{ travels along } (i,j) \text{ on day } d \\ 0 \text{ otherwise} \end{cases} \qquad (i,j) \in A,\, i \neq j,\, d \in W,\, t \in O_d$$

Furthermore, let:

y_{ij}^{d} auxiliary flow variable which represents the number of the patients visited after patient i by the operator moving along (i,j) on day d, $(i,j) \in A$, $d \in W$.

Using the variables above, HCP can then be modeled as follows:

$$\max \quad m$$

$$\sum_{i\in N}\sum_{t} x_{ij}^{td} \leq \sum_{p:p(d)\geq 1} z_{jp} \qquad \forall j \in N\setminus\{0\}, \forall d \in W \tag{1}$$

$$\sum_{i\in N}\sum_{t:s_t\geq k} x_{ij}^{td} \geq \sum_{p:p(d)=k} z_{jp} \qquad \forall j \in N\setminus\{0\}, \forall d \in W, \forall k \in K \tag{2}$$

$$\sum_{p\in P} z_{jp} = 1 \qquad \forall j \in N\setminus\{0\} \tag{3}$$

$$\sum_{t\in O} u_{tj} \leq T \qquad \forall j \in N\setminus\{0\} \tag{4}$$

$$x_{ij}^{td} \leq u_{tj} \qquad \forall (i,j) \in A, \forall j \in N\setminus\{0\}, \forall d \in W, \forall t \in O_d \tag{5}$$

$$u_{tj} \leq \sum_{i\in N:i\neq j}\sum_{d} x_{ij}^{td} \qquad \forall j \in N\setminus\{0\}, \forall t \in O \tag{6}$$

$$D_{td} = \sum_{(i,j)\in A} (t_{ij} + t'_j)\cdot x_{ij}^{td} \leq D_t \qquad \forall d \in W, \forall t \in O_d \tag{7}$$

$$\sum_{i\in N} x_{ij}^{td} = \sum_{i\in N} x_{ji}^{td} \qquad \forall j \in N\setminus\{0\}, \forall d \in W, \forall t \in O_d \tag{8}$$

$$\sum_{j\in N} y_{0j}^{d} = \sum_{j\in N}\sum_{p:p(d)\geq 1} z_{jp} \qquad \forall d \in W \tag{9}$$

$$\sum_{i\in N} y_{ij}^{d} - \sum_{i\in N} y_{ji}^{d} = \sum_{p:p(d)\geq 1} z_{jp} \qquad \forall j \in N, \forall d \in W \tag{10}$$

$$y_{ij}^{d} \leq n \sum_{t\in O_d} x_{ij}^{td} \qquad \forall (i,j) \in A, \forall d \in W \tag{11}$$

$$\frac{\sum_{d\in W} D_{td}}{|W|\cdot D_t} \geq m \qquad \forall t \in O \tag{12}$$

Constraints (1) state that at most one operator per day can visit patient j, if a visit has been scheduled on that day for node j. Constraints (2) guarantee that, on day d, exactly one operator, of adequate skill, must visit patient j if a service of that skill has been scheduled for j on day d. In particular, the least skilled operators can perform only visits of skill 1 (case $k = 1$), whereas the most skilled operators can perform all types of visits (case $k = \overline{k}$). Constraints (3) assure that each patient is assigned exactly to a pattern.

Constraints (4) assure that at most T operators are assigned to each patient during a week, where T is assumed to be given. This is included in order to guarantee continuity of the care, an important Quality of Service requisite in Home Care. Constraints (5) guarantee that an operator can visit a patient only if he has been assigned to that patient (linking between routing and assignment variables). Furthermore, constraints (6) force variables u_{tj} to zero if technician t never visits patient j during the week. Constraints (7) assure that the workload of each operator in each day, expressed as the sum of the service times and the traveling times, does not exceed the duration of a workday. Constraints (8) are the classical flow conservation constraints on the routing variables. Constraints (9) and (10) are the flow conservation constraints on the auxiliary y variables, which are introduced to avoid subtours in the model solutions. They also guarantee the correct linking between scheduling decisions and auxiliary flow variables. Finally, constraints (11) link together routing variables and auxiliary flow variables.

Observe that a pattern variable z_{jp} can assume a value other than zero only if:

$$|\{d : p(d) = k\}| = r_{jk} \quad \forall k \in K. \tag{13}$$

Therefore, in the preprocessing phase $z_{jp} = 0$ if anyone of the $\overline{k}$ constraints (13) is not satisfied. Furthermore, in the preprocessing phase $x_{ij}^{td} = 0$ if patients i and j have only requests of skill at least k during the planning horizon, and t is an operator of skill less than k.

The objective function, to be maximized, defines the minimum operator utilization factor, expressed as the total workload of the operator during the planning horizon over the maximum possible workload (see (12)). Therefore, scheduling, assignment and routing decisions are taken in such a way as to balance the operator utilization. Note that both service time and traveling time are considered in defining the total workload of an operator.

An alternative objective function is defined in an attempt at preventing undesiderable solutions where some operators spend a lot of time moving among patient's homes, in order to increase their own utilization factor. It is defined as follows:

$$\max \quad M \cdot m - v$$

$$\frac{\sum_{d \in W} D_{td}}{|W| \cdot D_t} \geq m \qquad \forall t \in O \tag{14}$$

$$\sum_{d \in W} \sum_{(i,j) \in A} t_{ij} x_{ij}^{td} \leq v \qquad \forall t \in O, \tag{15}$$

where the auxiliary variable v defines the maximum operator's traveling time. The penalty M is introduced to give priority to the operator utilization when penalizing tours with excessive traveling time.

In real Home Care instances usually the set of the patients, modelled via the set of the nodes N, is clusterised. Precisely, patients located in the same municipality or small rural area form a cluster of nodes, such that the distance between any pair of nodes within the cluster is very small compared to the distance between nodes belonging to different clusters. Such intra-cluster distances can be usually assumed all equal to a certain value δ. This is the case of the Home Care dataset considered in Section 5. Hereafter these instances will be referred to as *Clusterized Home Care instances*. These clusterized instances may have a huge number of symmetric feasible solutions, due to the structure of their logistics network. This, in turn, may slow the solution of the HCP models.

To overcome these difficulties, we studied the following families of *symmetry management valid inequalities*, which are tailored to Clusterized Home Care instances:

$$\sum_{i \in (N \setminus C)} \sum_{j \in C} x_{ij}^{td} \leq 1 \qquad \forall C, d \in W, t \in O_d. \tag{16}$$

$$x_{ij}^{td} = 0 \qquad \forall d \in W, t \in O_d, i, j \in C (i > j). \tag{17}$$

The valid inequalities (16) impose that, for each cluster of patients C, and in each day of the planning horizon, each available operator enters C at most once. On the other hand, inequalities (17) break some symmetries by forcing an ordered visit of the patients within each cluster. As shown in Section 5, these valid inequalities proved to be crucial either to halve the optimum percentage gap, or to find feasible solutions in the most critical cases.

4 Pattern generation policies

Three policies to generate the patterns have been analyzed. Specifically, we designed the following policies:

1. a heuristic procedure based on the frequency of request types; this is a greedy algorithm which firstly orders the requests according to their numbers of requirements of skill 1 and 2 and then, by scanning the ordered list, generates patterns that can accomplish with the frequency of such requests;

2. a procedure based on the extraction of pattern information from the solution actually implemented at the Home Care provider;

3. a flow based model described in the following in details.

It is worth saying that the problems addressed by the provider currently disregard both the continuity of care constraints and the routing decisions.

Flow based patterns As a third approach to generate a subset of a priori patterns, let us consider an auxiliary layered network $G_W = (N_W, A_W)$ with $|N_W| = n_w$, having one layer L_d for each considered day d in the planning horizon W, plus a source node and a destination node. For matter of convenience the source node will be denoted by index 1 and the destination by n_w. Each layer is composed of $\overline{k} + 1$ nodes: node 0, which indicates that no visit is scheduled in the day corresponding to the layer; and node k where $k \in K$, which represents the scheduling of a visit of skill k in the day corresponding to the layer. In G_W there exists a directed arc from the source node to the nodes in first layer, a directed arc from each node in the layer corresponding to the last day of the planning horizon to the destination node, and there exists a directed arc from each node in layer L_d to each node in the next layer, $\forall d \in W$.

Observe that any directed source-destination path in G_W corresponds to a potential pattern. Therefore, let us introduce a binary commodity for each patient j, having node 1 as the origin node and having n_w as its destination, and state the following auxiliary multicommodity flow problem on G_W as a tool to generate a set of feasible patterns:

$$\min \sum_{(h,i)\in A_W} q_{hi}$$

$$\sum_{(h,i)\in A_W} f^j_{hi} - \sum_{(i,h)\in A_W} f^j_{ih} = \begin{cases} -1, & \text{if } i = 1, \\ 1, & \text{if } i = nw, \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in N_W, \forall j \in N \setminus \{0\} \tag{18}$$

$$\sum_{d\in W} \sum_{(h,k):k\in L_d} f^j_{hk} = r_{jk} \qquad \forall j \in N \setminus \{0\}, \forall k \in K \tag{19}$$

$$\sum_{j\in N\setminus\{0\}} t'_j \sum_{k'\geq k} \sum_{(h,k'):k'\in L_d} f^j_{hk'} \leq \sum_{t\in O_d:s_t\geq k} D_t \qquad \forall d \in W, \forall k \in K \tag{20}$$

$$\sum_{j\in N\setminus\{0\}} f^j_{hi} \leq n q_{hi} \qquad \forall (h,i) \in A_W \tag{21}$$

$$f^j_{hi} \in \{0,1\} \qquad \forall (h,i) \in A_W, \forall j \in N \setminus \{0\} \tag{22}$$

$$q_{hi} \in \{0,1\} \qquad \forall (h,i) \in A_W \tag{23}$$

For each patient j, the multicommodity flow variables $\{f^j_{hi}\}$ model a directed path in the layered graph, which starts from the source node 1 and ends to n_w (see constraints (18)). These variables satisfy conditions (13), thanks to constraints (19), and therefore they model a feasible pattern for j, that is a pattern which is compatible with the care plan of j. Constraints (20) take into account, skill by skill, the operators availability in each day of the planning horizon. In fact, they impose that the total service time of scheduled visits of skill k per day does not exceed the daily availability in that day of the set of the operators of skill at least k. Finally, constraints (21) link together flow variables f^j_{hi} with desing variables q_{hi} and they guarantee that if arc (h, i) is used, i.e. $q_{hi} = 1$, it can be crossed by any number of patients, whereas if that arc is not used, i.e. $q_{hi} = 0$, then it cannot be traversed.

The auxiliary variables $\{q_{hi}\}$ are introduced to discover what arcs are used to design the patterns: by minimizing the total number of used arcs, the model thus tends to minimize, in an implicit way, the number of generated patterns.

It is worth saying that, in the experimental campaign, this flow based policy has been used in combination with a parameter that reduces the operators availability. In fact, since the flow based model neglects the traveling times, it may occur that the patterns thus provided generate an infeasible solution when the routing issue is also considered. A reduction of the operators availability is then used as a means to prevent some undesirable infeasibilities. Specifically, in the experimental campaign three values of the aforesaid parameter are considered: 0.5 which halves the operators availability, 0.75 which reduces the availability by 25%, and 1 which maintains the real availabilities.

5 Computational results

We generated a set of Home Care instances starting from real data, and analyzed the behavior of the proposed ILP models depending on:

- pattern characteristics
- kind of objective function selected
- symmetry management.

We also analyzed the quality of the solutions obtained in terms of operator utilization factor and travel time. In addition, we investigated the impact of:

- increasing number of requests of skill 2
- management of care continuity constraints.

The main achievements that are shown can be summarized as follows: (*i*) determining a good but limited set of patterns is crucial for efficiently solving the LP relaxation, and for finding good quality solutions to the ILP models; in particular, the flow based procedure, and specifically the one halving the operators availability, proved to be very effective in selecting a small number of ad-hoc patterns; (*ii*) the symmetry management allowed either to halve the optimum percentage gap, or to find feasible solutions in the most critical cases; however, it was not an issue for the flow based pattern generation policy, which proved to be very efficient independently of the symmetry management tool (*iii*) the flow based pattern generation procedure allowed, in almost all the cases, to compute very good quality solutions, with an optimum percentage gap within 0.17% for the distance unaware objective function, and within 0.32% for the distance aware objective function.

In addition, the two objective functions provided solutions of quite similar quality in terms of operator utilization factor and travel time, thus allowing to conclude that the undesirable effect of operators spending a lot of time in traveling between municipalities to increase their utilization factor is not crucial, at least on this data set. Also, the care continuity constraints seem not to increase the difficulty of solving HCP, at least for the considered instances, although the computed solutions may be different. Finally, the percentage of more specialized visits influences the resolution method efficiency, and an increase of such a percentage can counterbalance the increase of computational time due to an increased number of selected patterns.

5.1 Data set

The real data have been provided by one of the largest Italian public medical care unit, and they have been already used in [16]. An instance of HCP is characterized by: (*i*) the geographical area where the service is provided; (*ii*) a time period; (*iii*) the set of patients; (*iv*) the set of operators; (*v*) the set of patterns. The considered provider operates in the north of Italy and its services cover a region of about 800 km^2 that is organized in three divisions. In turn, each division is organized in districts. The instances we used consider the largest district of the Merate area which corresponds to the largest division in the considered area and comprises 10 municipalities. As already indicated, two skills are considered for operators and patient requests: *ordinary*, corresponding to skill 1 in the proposed models, and *palliative*, corresponding to skill 2. In regards to the patients, we had access to the care profile of 4123 patients in the time period [2004 - 2008] and we selected two weeks, i.e. a week in January 2006 (hereafter denoted by *January 2006*) and a week in April 2007 (hereafter denoted by *April 2007*). Patient demands had been computed by looking at the scheduling implemented by the provider in the period considered: specifically, for each skill, the requested number of visits in our instances is set equal to the real number of visits performed by operators of that particular skill. This choice is supported by the fact that the provider never used operators with skill different from the skill required by a visit. In instances related to January 2006 all the 10 municipalities are considered, whereas in instances related to April 2007 there were no requests regarding two of the 10 municipalities. The district under consideration is characterized by 11 operators, 8 of which of skill 1 and 3 of skill 2. The workday durations (D_t) are 4, 6 or 8 hours.

In all the generated instances, the traveling times t_{ij} have been computed via Google Maps for the inter-municipalities distances, while they have been set equal to 3 minutes for the intra-municipalities distances, consistently with the provider indications. Furthermore, according to the medical care unit indications the service time has been fixed to 30 minutes (i.e. $t'_j = 30$ min), while the maximum number of operators per patient has been initially set to the value 2 (i.e. $T = 2$). Summary information of the data set are given in Table 1.

In regards to the patterns, which are a peculiarity of our approach, three generation policies have been used as described in Section 4. Hereafter the heuristic selection policy will be referred to as *Heur*, the selection policy based on the real solution implemented by the provider will be referred to as *ImplSol* and, finally, the generation policy based on the flow model provided in Section 4 will be referred to as *FlowBased* or *FB*. Specifically, *FB-50*, *FB-75* and *FB-100* indicate the setting which halves the operators availability, the one which reduces the availability by 25%, and the one which maintains the real availabilities, respectively. The three policies may produce a number of patterns very different the one from each other; these values are reported in Table 2 and commented hereafter.

In addition to the patterns selection, the other two features that characterize our approach to solve HCP are the selection of the objective function and the management of symmetries. In regards to the objective function, the first one aims at maximizing the minimum operators utilization factor while the second one selects, among all the solutions for which the minimum operators utilization factor is maximum, the one that limits the travel time consumed in the routing. The two objectives will be referred to as *Distance Unaware* and *Distance Aware*, respectively.

Summarizing, the experimental campaign analyzes the behavior of the models on the considered HCP instances both in terms of efficiency and quality of the solutions provided when all of the pattern generation policies (5 choices) are combined with the two alternative objective functions and the symmetries are managed or not by means of constraints (16) and (17), thus resulting in 40 runs.

The experiments have been performed on a AMD Opteron(tm) Dual Core Processor 246 (CPU MHz 1991.060). The solver is CPLEX 12.4 with a time limit of 12 hours and a memory limit for the branch and bound tree of 1 GB. In the following the computational times are

Table 1: Instances features

Week	Patients	Operators	Municipalities
January 2006	129	11	10
April 2007	163	11	8

Table 2: Number of patterns used

Week	Heur	ImplSol	FB-50	FB-75	FB-100
January 2006	20	29	13	11	11
April 2007	27	33	17	14	14

expressed in seconds of CPU time.

5.2 Assessing the impact of pattern selection

Tables 3 and 4 report, respectively for the two alternative objective functions, the performance of the models in terms of time required to solve the Linear Programming relaxation (LPTime) and in terms of LP objective function (LPValue). As already indicated, the model equipped with objective function (12) is said "distance unaware" since it does not prevent the operators to commute from one municipality to the other in order to raise their utilization factor. On the contrary, the model equipped with the alternative objective function is said "distance aware". For both weeks the impact of the pattern generation policies (see the last 5 columns of Tables 3 and 4) is evaluated against the symmetry management (see the rows).

It is worth observing that the LPTime required for policies Heur and ImplSol can be quite high and the solver may even exceed the time limit without reporting a continuous solution (character T in the presented tables). On the contrary the LPTime is much more shorter for the flow based policies which are characterized by a much smaller number of generated patterns with respect to the other policies (see Table 2). Determining a good and limited set of patterns seems thus to be crucial in our study. Particular attention should be given to the LPValue: observe that the LPValue is the same (when available) for policies Heur, ImplSol and FlowBased with the parameter set to 0.5, thus suggesting that the selected patterns are sufficient to obtain almost the same solution quality (but, as observed, with very different computational times).

On the contrary, it may happen that equal sized sets of patterns provide different continuous solution values. This is the case for example of the selected week in April 2007, where the FlowBased policies with parameter 0.75 and 1 both return a set of 11 patterns. However, while the patterns obtained with policy FlowBased 1 provides an infeasible solution regardless of the symmetries management (string *inf* in the presented tables), all of the other policies give the same solution value.

In Tables 5 and 7, respectively for the two alternative objective functions, the results

Table 3: DistanceUnaware - LP results

			Heur	ImplSol	FlowBased 0.50	0.75	1.00
January 2006	NoSymmMan	LPTime	9044.38	17805.47	1337.04	219.17	266.21
		LPValue	0.3552	0.3552	0.3552	0.2922	0.2922
	SymmMan	LPTime	12787.30	14918.92	1154.87	379.84	312.94
		LPValue	0.3552	0.3552	0.3552	0.2922	0.2922
April 2007	NoSymmMan	LPTime	17450.49	17689.80	1942.01	1702.29	1785.68
		LPValue	0.5393	0.5393	0.5393	0.5393	inf
	SymmMan	LPTime	13655.99	13949.60	3613.89	1256.49	547.60
		LPValue	0.5393	0.5393	0.5393	0.5393	inf

Table 4: DistanceAware - LP results $\star$ Note that LPValue is the objective function value (14)-(15), and it does not represent the operator utilization factor

			Heur	ImplSol	FlowBased 0.50	0.75	1.00
January 2006	NoSymmMan	LPTime	13227.69	5720.60	2988.47	681.53	309.20
		LPValue$\star$	350.0006	350.0006	350.0006	289.4151	289.4151
	SymmMan	LPTime	23468.02	22474.06	735.80	379.65	637.7
		LPValue$\star$	350.0006	350.0006	350.0006	289.4151	289.4151
April 2007	NoSymmMan	LPTime	T	14779.83	5934.24	1194.70	1176.43
		LPValue$\star$	n.a.	533.0884	533.0884	533.0884	inf
	SymmMan	LPTime	T	9136.70	2775.73	1224.46	798.92
		LPValue$\star$	n.a.	533.0884	533.0884	533.0884	inf

Table 5: DistanceUnaware - IP results

		Heur		ImplSol		FlowBased 0.50		FlowBased 0.75		FlowBased 1.0	
		%Gap	Stop	%Gap	Stop	%Gap	Stop	%Gap	Stop	%Gap	Stop
January 2006	NoSymmMan	1.02	T	0.66	T	0.07	T	n.a.	T	n.a.	T
	SymmMan	0.54	T	0.30	T	0.18	M	n.a.	T	n.a.	T
April 2007	NoSymmMan	0.48	T	0.41	T	0.17	M	0.17	T	n.a.	inf
	SymmMan	0.25	T	0.41	T	0.17	T	0.48	M	n.a.	inf

obtained when the integer problem is solved are reported for all the combinations of pattern selection policies and symmetries management in terms of percentage relative gap computed with respect to the best upper bound obtained in the branch and bound tree (string "n.a." is used to point out that no integer solution is found). The stopping criterion that determines the termination of the algorithm is also given in columns Stop, where the character T is used to indicate that the time limit has been exceeded while a M is used to indicate an out of memory condition.

The Distance Unaware objective function performs better than the Distance Aware one at least by looking at the gaps, since it is able to find feasible solutions in a greater number of approach combinations and often it provides solutions of better quality. A more in-depth analysis of the solutions returned by the two objective functions in terms of percentage of operators travel time with respect the their total workload will be the subject of Section 5.4.

Tables 5 and 7 also reveal that symmetry management is a very effective tool to reduce the optimality gap and even to find feasible solutions for the most critical approach combinations. Indeed, the symmetry management used in combination with Heur and ImplSol policies allows to halve the gap almost everywhere when the Distance Unaware objective function is considered; moreover, when the Distance Aware objective function is used, the symmetries management allows to obtain good quality solutions especially on the week related to January 2006 (gap equal to 0.35%).

However, it is worth observing that the symmetry management is not an issue when the LP problem can be solved efficiently, as it happens with the Flow Based policies. In addition, whereas the Flow Based policy with the parameter set to 0.5 provides very good results (without the help of the symmetry management tool), and thus seems to be a viable option to address also the most difficult instances, the Flow Based policy fails almost everywhere to find feasible solutions when the number of patterns is too small (parameter set to 0.75 and 1). For this reason the last two policies will not be commented further in the following.

Tables 6 and 8, again respectively for the two alternative objective functions, report for each approach the node of the branch and bound tree where the best integer solution is found with respect to the total number of nodes explored: as an example, 4/30 reports that 30 nodes have been overall explored and the best solution has been found at node number 4. These information allow to measure the efficiency of the approach: the bigger the number of explored nodes the faster the computing at each branch and bound node; likewise, the smaller the node where the best solution is found the bigger the capability of a truncated branch and bound approach, based on the proposed ILP models, to provide solutions of good quality.

Table 6: DistanceUnaware - a measure of efficiency

		Heur	ImplSol	FlowBased 0.50	FlowBased 0.75
January 2006	NoSymmMan	4/30	0/3	821/63973	n.a.
	SymmMan	714/1677	308/785	1554/38922	n.a.
April 2007	NoSymmMan	467/887	260/260	824/5633	16083/16083
	SymmMan	809/810	766/901	3105/8539	2139/7556

Table 7: DistanceAware - IP results

						FlowBased					
		Heur		ImplSol		0.50		0.75		1.0	
		%Gap	Stop	%Gap	Stop	%Gap	Stop	%Gap	Stop	%Gap	Stop
January 2006	NoSymmMan	n.a.	T	n.a.	T	0.12	T	n.a.	T	n.a.	T
	SymmMan	0.35	T	0.35	T	0.32	M	n.a.	T	n.a.	T
April 2007	NoSymmMan	n.a.	n.a.	0.75	T	0.39	T	0.32	T	n.a.	n.a.
	SymmMan	n.a.	n.a.	1.62	T	0.46	M	0.40	T	n.a.	n.a.

Table 8: DistanceAware - a measure of efficiency

		Heur	ImplSol	FlowBased 0.50	FlowBased 0.75
January 2006	NoSymmMan	n.a.	n.a.	12030/17173	n.a.
	SymmMan	878/1005	1736/3372	775/34735	n.a.
April 2007	NoSymmMan	n.a.	404/821	830/4895	4301/9347
	SymmMan	n.a.	696/847	4027/4723	4479/10074

Table 9: Impact of symmetry management

		Heur		ImplSol		FlowBased 0.50	
		LPRows	LPCols	LPRows	LPCols	LPRows	LPCols
January 2006	NoSymmMan	85042	712375	90304	743486	39813	304141
	SymmMan	81067	669563	85697	696822	37878	285417
April 2007	NoSymmMan	139962	1236919	142368	1214617	71353	593663
	SymmMan	123609	1061379	125938	1044417	63572	510963

5.3 Assessing the impact of symmetry management

Comments about the impact of symmetry management on the efficiency and efficacy of the approach, i.e. in terms of computational time and quality of the returned solutions, have been already provided in Section 5.2. Here we want to provide further comments and explanations in terms of dimension of the solved problems. To this end, Table 9 presents the dimension of the LP problems in terms of constraints (columns LPRows) and variables (columns LPCols) for the three policies Heur, ImplSol and Flow Based 0.50 when the Distance Unaware objective function is used. The data refer to the LP dimension reported by CPLEX after the preprocessing phase. Similar results hold for the alternative objective function. As in the previous tables, results relative to the symmetry management are given in rows labeled "SymmMan" while the rows "NoSymmMan" report data relative to the base model. The symmetry management allows to reduce the LP dimension and the three policies exhibit a quite similar behavior in terms of reduction. In fact, for the selected week in January 2006 the average reduction of the number of constraints when the model is equipped with the symmetry management is equal to 4.88% while the percentage reduction of the variables is equal to 6.15. For April 2007 the figures are respectively 11.38 for the rows and 14.04 for the columns. Despite the reduction in terms of constraints and variables, the time required to solve the LP may increase when the symmetries are treated. However Tables 5 and 7 shows that the symmetry management is crucial either to halve the IP gap (see the "Distance Unaware" case, policies Heur and ImplSol) or to find feasible solutions in the most critical cases (see the "Distance Aware" case for the scenario January 2006).

5.4 Assessing the solution quality

Table 10 provides some information on the quality of the solutions obtained in terms of operator utilization factor and travel time. Specifically, we considered the solutions obtained with the Flow Based 0.50 policy and the symmetries management disabled; this choice is motivated by the results above reported, showing that such a policy exhibits the best performance among the ones proposed, and that the symmetries management is not an issue when the LP problem can be solved efficiently. The columns of Table 10 report respectively the minimum, the maximum, the average operator utilization factor and the average fraction of travel time with respect to maximum weekly workload, computed over the available operators. The utilization factor is computed as the weekly time an operator globally spends in

Table 10: Quality analisys

		min UF	max UF	avg UF	avg TT
January 2006	DistUnaware	0.3550	0.4633	0.3697	0.1316
	DistAware	0.3550	0.4767	0.3712	0.1334
April 2007	DistUnaware	0.5383	0.5489	0.5400	0.1580
	DistAware	0.5375	0.5467	0.5404	0.1587

servicing the patients and traveling among them over the weekly availability time (as defined in the r.h.s. of constraints (12)).

We can observe that the two objective functions provide solutions of quite similar quality in terms of operator utilization factor and travel time, thus allowing to conclude that the undesirable effect of operators spending a lot of time in traveling between municipalities to increase their utilization factor is not crucial, at least on this data set and for the considered pattern generation policy.

As indicated before, we also investigated the impact of the number of requests of skill 2 (i.e. the ones requiring the greater ability), and of the care continuity constraints. This will be the subject of the next two sections.

5.5 Trend for increasing percentage of palliative requests

Starting from the scenario January 2006, which is the most difficult one in the test bed, we randomly generated three further instances characterized by a percentage of visits of skill 2 equal respectively to the 10%, 20% and 30% of the total number of visits. Notice that in the selected week of January 2006 the percentage of visits of skill 2 with respect to the total number of visits is equal to 23.91%. For these additional three instances the patterns have been computed by means of the heuristic policy; moreover, since the number of patterns is quite small (respectively 13, 15 and 16 for the three instances), the symmetries management has been disabled. Table 11 compares the two objective functions, and reports for each of them the LPTime, the optimality gap, the stopping criterion and the node where the best solution is found over the total number of explored branch and bound nodes.

As already shown in [7] for the Skill VRP case, the difficulty in solving the routing problem decreases as the percentage of visits requiring the most specialized skill increases. In this experimental campaign such a behavior is however mitigated by the presence of patterns (and so, by the scheduling decisions) as explained in the following. The computational time required to solve the LP when the percentage of visits of skill 2 increases from 10% to 20% decreases significantly despite the increase in the number of patterns, although such an increase is shown to have a great impact on the computational times. On the other side, when the percentage of visits of skill 2 increases from 20% to 30% the computational time required to solve the LP grows as well. In fact, the computational time is greatly influenced by the number of patterns. However the computational time required to solve the LP with a percentage of more specialized requests equal to 30 is comparable (see Distance Unaware)

Table 11: January 2006 - impact of the percentage of palliative requests

	DistUnaware				DistAware			
%skill 2	LPTime	%Gap	Stop	F/N	LPTime	%Gap	Stop	F/N
10%	9951.34	0.71	T	864/871	41584.11	3.97	T	0/0
20%	5807.84	1.05	T	456/1816	3958.22	1.38	T	860/975
30%	10753.24	0.69	T	1705/1705	10246.57	2.40	T	348/428

or still much smaller (see Distance Aware) than the time required to solve the LP with a percentage equal to 10. The increase in the percentage of more specialized visits seems thus to offset the increase of the computational times arising when the number of patterns increases of a very few units.

5.6 Impact of care continuity constraints

Tables 12 and 13, respectively for the Distance Unaware and for the Distance Aware objective functions, show the effect of the care continuity constraints on the efficiency of the resolution method and on the solution quality for the pattern generation policies Heur, ImplSol and Flow Based 0.50 when the symmetries management is done. In fact, the computational results shown in previous sections reveal that the management of symmetries allows to find a greater number of feasible solutions, at least for the policies Heur and ImplSol. For this set of additional 12 runs, the care continuity constraints have been removed thus allowing that a patient is visited by up to 5 operators each week. First of all we report that the elimination of care continuity constraints had no effect on the LPValue, at least on this data set. Furthermore, by looking at the upper bound value, there was no potential increase of the operators utilization factor in increasing the number of operators that can service a client beyond two. Columns in Tables 12 and 13 report, for each pattern generation policy, respectively the LPTime, the optimality gap, the termination condition and the node where the best integer solution is found with respect to the total number of nodes explored. We observed no reduction of the LPTime for the most critical instances and no valuable variations in gaps and capability of finding feasible solutions with respect to the case where the care continuity is ensured. We can thus conclude that, at lest on this data set, the care continuity does not increase the difficulty of solving HCP. The computed solutions are however different in terms of operator utilization factor and travel time. This is shown in Table 14, where columns report respectively the minimum, the maximum, the average operator utilization factor and the average travel time computed over the available operators for both scenarios (i.e. January 2006 and April 2007). See also Table 15 for an analysis of the number of operators per patient in both scenarios, under the care continuity constraints (CareCont) and their removal (NoCareCont).

6 Conclusions

In this paper we addressed a weekly planning horizon Home Care Problem (HCP). With respect to [3], [14] and [17] that, to best of our knowledge, are the only papers which extend

Table 12: Distance Unaware - Impact of care continuity

	Heur				ImplSol				FlowBased 0.50			
	LPTime	%Gap	Stop	F/N	LPTime	%Gap	Stop	F/N	LPTime	%Gap	Stop	F/N
January 2006	8242.36	0.30	T	734/783	15861.31	0.18	T	981/1212	958.60	0.30	M	1611/11748
April 2007	13439.94	1.11	T	52/835	14120.15	n.a.	T	n.a.	2087.20	0.25	M	6730/8138

Table 13: Distance Aware - Impact of care continuity

	Heur				ImplSol				FlowBased 0.50			
	LPTime	%Gap	Stop	F/N	LPTime	%Gap	Stop	F/N	LPTime	%Gap	Stop	F/N
January 2006	25088.51	1.13	T	4/94	24138.13	0.71	T	0/4	833.92	0.24	M	16722/16722
April 2007	35202.56	2.56	T	43/153	25858.75	1.31	T	360/360	3106.74	0.48	M	3062/4504

Table 14: Care continuity: quality analisys

		min UF	max UF	avg UF	avg TT
January 2006	NoCareCont	0.3546	0.4342	0.3676	0.1301
	CareCont	0.3542	0.4442	0.3701	0.1328
April 2007	NoCareCont	0.5383	0.5525	0.5406	0.1578
	CareCont	0.5379	0.5494	0.5403	0.1589

Table 15: Frequency analisys

	January 2006		April 2007	
	CC	NoCC	CC	NoCC
1	81.25	79.69	61.73	64.81
2	18.75	14.84	38.27	24.07
3	0.00	3.13	0.00	8.02
4	0.00	2.34	0.00	3.09
5	0.00	0.00	0.00	0.00

the daily planning horizon usually investigated in Home Care, here assignment, scheduling and routing decisions are addressed in a joint way, i.e. without heuristically decomposing the problem by means of a two-level approach. To this end, crucial and original is, in our opinion, the use of the pattern modelling device as a means to coordinate the diverse decision levels. In fact, we proposed a new Integer Linear Programming (ILP) formulation, with some variants, which combines the three levels of decisions by means of a set of a priori given patterns. Relevant QoS constraints such as continuity of care and maximum daily workload were incorporated into the models. Moreover, valid inequalities breaking the symmetries which may characterize the HCP instances have been proposed to enhance the ILP models.

The computational experiments performed on large real instances reveal, in our opinion, the potentiality of the proposed ILP models in successfully addressing large Home Care instances. Crucial, to this end, is the selection of a limited set of patterns of good quality, such as the ones produced by the flow based pattern generation approach, which is another contribution of this paper. The design of decomposition methods based on the proposed ILP models, and tailored to solve even larger Home Care instances, is in in progress.

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