UNIVERSITÀ DI PISA
DIPARTIMENTO DI INFORMATICA

TECHNICAL REPORT

Different Decomposition Strategies to Solve Stochastic Hydrothermal Unit Commitment Problems

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May 18, 2017

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Different Decomposition Strategies to Solve Stochastic Hydrothermal Unit Commitment Problems

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Abstract

Solving very-large-scale optimization problems frequently require to decompose them in smaller subproblems, that are iteratively solved to produce useful information. One such approach is the Lagrangian Relaxation (LR), a broad range technique that leads to many different decomposition schemes. The LR supplies a lower bound of the objective function and useful information for heuristics aimed at constructing feasible primal solutions. In this paper, we compare the main LR strategies used so far for Stochastic Hydrothermal Unit Commitment problems, where uncertainty mainly concerns water availability in reservoirs and demand (weather conditions). This problem is customarily modeled as a two-stage mixed-integer optimization problem. We compare different decomposition strategies (unit and scenario schemes) in terms of quality of produced lower bound and running time. The schemes are assessed with various hydrothermal systems, considering different configuration of power plants, in terms of capacity and number of units.

Keywords: *Dual Decomposition, Lagrangian Relaxation, Mixed-Integer Linear Programming, Hydrothermal power generation, Stochastic Unit Commitment.*

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I. INTRODUCTION

Most real-world problems can be mathematically represented by formulations using nonlinear functions and integer variables, which leads to hard Mixed-Integer Nonlinear Problems (MINLP). In general, the constraints of these MINLPs have a high degree of structure, encouraging the use of techniques that decomposes them. One of the most appealing methods to strategically exploit structure is the Lagrangian Decomposition or Lagrangian Relaxation (LR) [1]–[3], in which the original problem is decomposed into several independent subproblems, possibly with different sizes and mathematical nature. For further basic information of LR, as well as its applications, see references [4], [5]. The LR technique is very flexible: for the same primal problem several different decomposition schemes can be developed, with complex trade-offs between the computational cost and the quality of the results [2]. In this paper, we assess this flexibility applying the LR in electrical power systems based on real data. The main information produced by LR approaches are bounds on the optimal value of the problem and the corresponding Lagrangian multipliers that have possible uses (e.g., to estimate prices of resources). However, they also provide valuable primal information, such as the "convexified" primal solution composed of the integer solutions computed at each iteration, that is the basis of heuristic approaches for producing feasible solutions [2], [6]. The LR requires the solution of a convex nondifferentiable optimization problem, for which one of the most effective approaches is the Bundle Method (BM) [7], [8]. One of the advantages of the BM is allowing independent models (disaggregate bundles) for an objective function given by a sum of separate terms [9], as is invariably the case in Unit Commitment problems. When some of this models (components) are simple to solve (continuous and linear optimization problems) they can be treated as exact models by basically copying the corresponding constraints in the formulation of the master problem, instead of iteratively approximating them by inner linearization. This has so far been applied to different problems [9], but we will show that the approach is very useful for Stochastic Unit Commitment (SUC) problems. In SUC, the goal is finding a production schedule that satisfies the unit's and system constraints considering the uncertainties, which in our case are mainly related to water availability in reservoirs and weather conditions. This optimization model is a step of the power system planning studies, where the operator needs to determine the power plants operation for a day-ahead in a hydrothermal power system. The resulting problem is a large-scale, non-convex, *uncertain* (stochastic, robust, chance-constrained) MINLP, which is extremely challenging to solve.

In the literature, there are basically two kinds of decomposition strategies for the SUC: *Scenario Decomposition* (SD), and *Unit* (or *Space*) *Decomposition* (UD). The reference works [10], [11] in the former group separates the stochastic problem in many sets of deterministic subproblems, using methods such as Progressive Hedging [12] and Branch and Bound [13] combined with LR to solve the whole problem. The latter group [14], [15] rather decomposes the problem by power plant, using stochastic Lagrange multipliers calculated using the expected value of each scenario. The mathematical representation of the uncertainties can also be done by other techniques, for instance chance constraints [16]. The work [17] compares different approaches to represent the uncertainties in large scale problems. The review [16] presents and describes the main strategies to model and to solve the SUC.

The aim of this paper is to compare the two main decomposition strategies, SD and UD, using the same algorithm to solve the SUC. Therefore, we avoid biased results, since we are applying and setting the same solver for both. Besides, we use several different cases, considering different configuration of power plants, in terms of capacity and number of units. The results are assessed by means of the produced lower bound, quality of the solutions provided by Lagrangian heuristics, and running time.

This paper is organized as follows: in sections II and III we describe the mathematical representation of the SUC and briefly reference and comment the data for all the test cases. In section IV we present several different variants of the two main of decomposition strategies for the SUC, i.e., Unit and Scenario Decomposition. The computational comparison between the decomposition schemes is presented in section V. Finally, in section VI, we state the conclusion and some recommendations regarding the decomposition approaches.

II. UNIT COMMITMENT DESCRIPTION DATA

A very common problem in the operation of electrical power systems is to determine in advance which units of a power plant will operate and their level of generation for the day ahead. This depends on the electrical energy industry regulation, i.e., if the dispatch is centralized or if power plants can offer bids for the energy production in a market framework. This paper deals with the first case, the Brazilian one, in which an Independent System Operator (ISO) executes a series of planning studies. The Unit Commitment (UC) problem is the last part of these studies, closer to the real-time operation, so more detailed mathematical models are required. In this section, we present the mathematical representation of each component of the power system, while in the following section we discuss the complete formulation for a general hydrothermal SUC problem.

For our study we use five hydrothermal test systems defined in Table I, considering different power installed capacity and number of units. The hydro power systems are based on real information of the Brazilian system, their data having been extracted from a data base of the HydroByte software [18]. The data for the thermal power plants is instead taken from the (UC) instances of [19]. Finally, the transmission system and demand for each bus were adapted from an equivalent system from the south of the Brazilian electric power system. All the data is available at http://www.di.unipi.it/optimize/Data/UC.html. We now discuss the mathematical representation and a description of the data for each test system.

TABLE I
TEST SYSTEMS FOR HYDROTHERMAL SUC PROBLEMS

	Number of power plants		Generation capacity (%)		Generation apacity (MW)	Storage capacity (hm^3)
	H	T	H	T		
A	7	14	25.0	75.0	21,297.5	5,635.1
B	7	14	75.0	25.0	9,224.0	9,309.0
C	10	10	50.0	50.0	16,132.2	10,737.5
D	14	7	76.3	23.7	16,046.5	14,944.1
E	14	7	25.2	74.8	9,671.0	5,507.2

H means hydro and T means thermoelectric power plants.

A. Thermal Power Plants

Thermal plants production cost depends on the fuel cost and varies accordingly to the power generation output, following a quadratic relationship. This nonlinear relation is approximated using the perspective cuts approach [20], resulting in linear constraints and a variable to represent the operational cost of each thermal plant. The operation of thermal units has some technical conditions that must be satisfied, such as minimum and maximum generation, minimum up and down times and ramp generation limits. All these constraints are represented by means of the mixed-integer and linear model of [21], [22] and represented by the set of constraints:

$$C^{\mathrm{T}}(pt, u, up, ud, F), \tag{1}$$

where:

pt vector of thermal power generation (MW);
u vector of commitment status;
up vector of startup status;
ud vector of shutdown status;
F vector of the production cost (R\$);

B. Hydro Power Plants

Hydro plants produce energy using the potential energy of the water in the reservoir. The energy transformation process depends on the net head, the turbine and generation efficiencies and the units' turbined outflow. This complex relationship results in a non-convex function [23] that in this paper is simplified to mixed-integer linear constraints. This simplification includes the representation of a group of identical units by a single equivalent unit and a piecewise linearization of the production function for each equivalent unit. The following figures illustrate the simplification applied. Figure 1 represents the power generation for a hydro power plant with four identical units, while Figure 2 presents the final piecewise-linear (polyhedral) model that we have used.

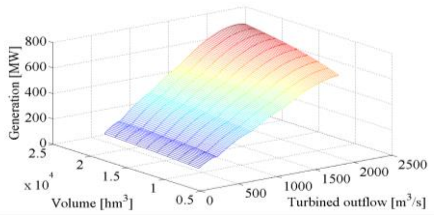

Fig. 1. Equivalent nonconvex production function.

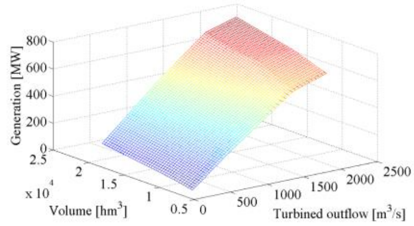

Fig. 2. Linear piecewise production function of equivalent unit.

The hydraulic connection between reservoirs (water flow balance equation) and the operational limits are taken into account as standard mixed-integer linear equations [23], [24]. Then, the set of hydro constraints is:

$$C^{\mathrm{H}}(ph, v, d, s, phg, q, z), \tag{2}$$

where:

- ph vector of hydro power generation (MW);
- v vector of water volume in the reservoirs (hm^3);
- d vector of total hydro plant outflow, i.e., sum of total turbined outflow and the spillage (m^3/s);
- s vector of spillage (m^3/s);
- phg vector of generation of the group of units (MW);
- q vector of turbined outflow by the group of units (m^3/s);
- z vector of commitment status of the group of units;

The operation of the reservoirs is coordinated with medium-term scheduling problem by means of target volumes at the end of the scheduling horizon, represented by constraints that are also included in (2).

C. Transmission network

In this paper, the electrical network constraints are represented by DC power flow equations and spinning reserve requirements. In Brazil, only hydro power plants provide spinning reserve. All the transmission network constraints are represented by the set:

$$C^{\mathrm{D}}(pt, ph) \tag{3}$$

III. TWO-STAGE UNIT COMMITMENT MODEL

The hydrothermal SUC aims at finding the optimal generation schedule while meeting operational and system wide constraints at a minimum expected cost. The latter takes into account a level of uncertainty due to the high dependence of the hydro production and the demand on the weather conditions. The uncertain data are the system load requirements (set C^{D}) and the water inflows (set C^{H}) represented by scenario trees. Figure 3 illustrates the combination of the uncertain data: two realizations of inflow and two of load profile result in four possible realizations for the second stage. In the illustration, each stage refers to 24 hours, representing each one the operation of one day. As a consequence, the corresponding scenario tree has a total of 120 nodes.

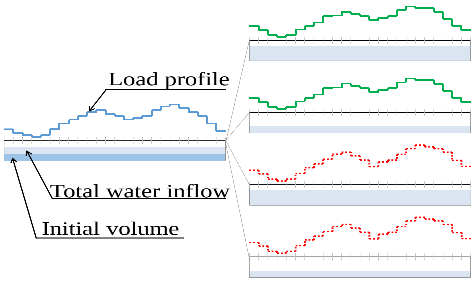

Fig. 3. Illustration of the uncertain data.

All in all, the model for the hydrothermal SUC problem is given by:

$$\min = \sum_{n=1}^{\mathrm{N}} p_n \cdot f_n\,(up_{in}, F_{in}) \tag{4}$$

$$\begin{aligned} \text{s.t.:}\quad & C_n^{\mathrm{H}}(ph_{rn}, v_{rn}, d_{rn}, s_{rn}, phg_{jrn}, q_{jrn}, z_{jrn}), && \forall n \\ & C_{in}^{\mathrm{T}}(pt_{in}, u_{in}, up_{in}, ud_{in}, F_{in}), && \forall n, i \\ & C_n^{\mathrm{D}}(pt_{in}, ph_{rn}), && \forall n \end{aligned} \tag{5}$$

where:

- n index of nodes in the scenario tree (n = 1, N);
- N number of nodes of the scenario tree;
- p_n discrete probability of node n;
- f_n operational cost function, given by the thermal generation and start up costs;
- i index of thermal plants (i = 1, I);
- I number of thermal plants;
- r index of hydro plants (r = 1, R);
- R number of hydro plants;
- j index of the group of units in the plant r (j = 1, J_r);
- J_r number of groups in the reservoir r.

In the compact formulation (4)-(5), each variable is related to a specific node of the scenario tree. For instance, variable ph_{rn} represents the power of hydro plant r and node n.

IV. Decomposition Strategies

Applying LR to problem (4)-(5) can be done in different ways. The most common approaches are the unit decomposition (UD) and the scenario decomposition (SD). In the former, the whole problem is splitted by its physical characteristics, typically a subproblem for each power plant. On the other hand, the SD separates the stochastic problem in many deterministic UC subproblems, each one related with a specific scenario. The different strategies are illustrated in the Figure 4, which are described in the next sections.

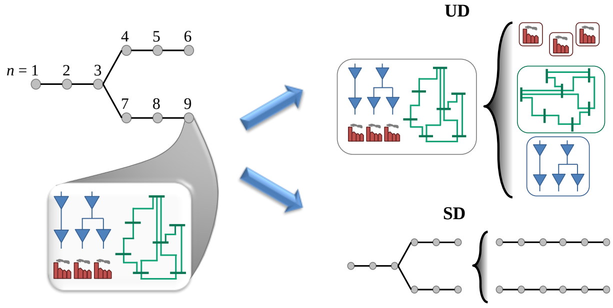

Fig. 4. Illustration of unit and scenario decomposition schemes.

A. Unit Decomposition

Considering the coupling constraints of problem (4)-(5), the most logical is to separate the problem by it characteristics: a set of subproblems for thermal power plants, another set for all the hydro power plants, and yet another set for the transmission network (which is not a unit). Further, each set of subproblem can be divided even more. Given the predominance of hydropower generation as energy source in Brazil, we propose three different schemes for the UD. These schemes are illustrated in Figure 5 for a problem with three time periods in each stage, two scenarios, five hydro power plants (located in two cascades), three thermal power plants and an electrical network with seven buses and ten transmission lines. The continuous line represents the time coupling between each node (in this paper, one-hour period), and dotted line indicates that there is not time coupling.

The first scheme (UD1) splits problem (4)-(5) in:

a) many Linear Programing (LP) subproblems, representing the electrical network constraints;
b) many Mixed-Integer Linear Programming (MILP) subproblems, each one representing a thermal plant operation;
c) a MILP subproblem, coupled in time and space, representing the operation of all the hydro power plants.

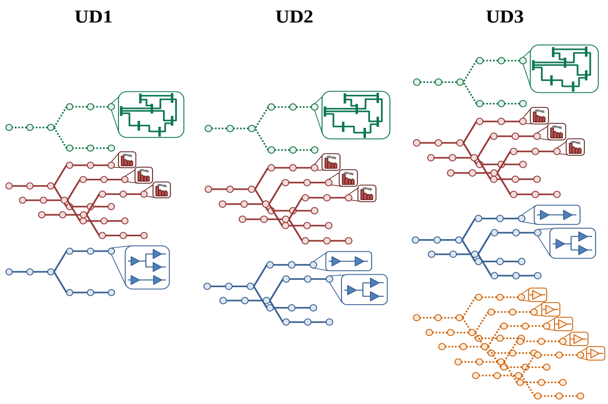

Fig. 5. Illustration of different kinds of unit decomposition.

Figure 6 shows coupling structure of UD1.

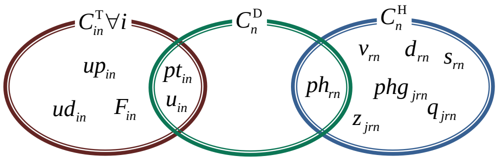

Fig. 6. Illustration of the relationship between variables for UD1.

To decompose the problem, one can apply the variable splitting technique [25], resulting in the following dual problem.

$$\Phi^{\mathrm{UD1}}(\lambda pt, \lambda ph) = \min f_n(up_{in}, F_{in}) + \sum_{n=1}^{\mathrm{N}} \left(\sum_{i=1}^{\mathrm{I}} \lambda pt_{in} \cdot (pta_{in} - pt_{in} - u_{in} \cdot pt_i^{\min}) + \sum_{r=1}^{\mathrm{R}} \lambda ph_{rn} \cdot (pha_{rn} - ph_{rn}) \right) \quad (6)$$

$$\begin{aligned} \text{s.t.:} \quad & C_n^{\mathrm{H}}\left(ph_{rn}, v_{rn}, d_{rn}, s_{rn}, phg_{jrn}, q_{jrn}, z_{jrn}\right), && \forall n \\ & C_{in}^{\mathrm{T}}\left(pt_{in}, u_{in}, up_{in}, ud_{in}, F_{in}\right), && \forall n, i \\ & C_n^{\mathrm{D}}\left(pta_{in}, pha_{rn}\right), && \forall n \end{aligned} \quad (7)$$

where:

λpt vector of dual variables related to thermal power generation;
λph vector of dual variables related to hydro power generation.

Then the dual function (6) can be evaluated by means of (1+I+N) independent subproblems.

$$\Phi^{\mathrm{UD1}} = \Phi^{\mathrm{H}} + \sum_{i=1}^{\mathrm{I}} \Phi_i^{\mathrm{T}} + \sum_{n=1}^{\mathrm{N}} \Phi_n^{\mathrm{D}} \quad (8)$$

where:

Φ^{D} set of subproblems a), one for each node;
Φ^{T} set of subproblems b), one for each thermal plant;
Φ^{H} a single subproblem c) concerning all hydro plants.

The second scheme (UD2), which derives from UD1, is obtained applying the same decomposition and relaxing the spinning reserve constraints. The set of subproblem a) and b) are the same, but the subproblem c) changes as follows:

d) a set of MILP subproblems, representing the operation of the hydro power plants for each cascade.

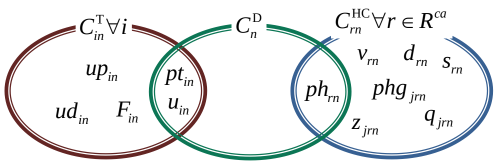

Fig. 7. Illustration of the relationship between variables for UD2.

In this case, besides the strategy used in UD1, the spinning reserve constraint is relaxed applying the classical LR technique. Despite this constraint formally belongs to the set C^{D}, it just couples the operation of hydro plants. The new set for hydro constraints C^{HC} in Figure 7 is defined for each cascade separately, as shown in (9)-(10).

$$\Phi^{\mathrm{UD2}}(\lambda pt,\lambda ph,\lambda res)=\min f_n\,(up_{in},F_{in})+\sum_{n=1}^{\mathrm{N}}\left(\sum_{i=1}^{\mathrm{I}}\lambda pt_{in}\cdot(pta_{in}-pt_{in}-u_{in}\cdot pt_i^{\min})+\right.$$

$$\left.\sum_{r=1}^{\mathrm{R}}\lambda ph_{rn}\cdot(pha_{rn}-ph_{rn})+\lambda res_n\cdot\left(\sum_{r=1}^{\mathrm{R}}\left(\sum_{j=1}^{\mathrm{J}_r}phg_{jr}^{\max}\cdot z_{jrn}-ph_{rn}\right)-Res_n\right)\right)$$

$$\text{s.t.: } C_{rn}^{\mathrm{HC}}(ph_{rn},v_{rn},d_{rn},s_{rn},phg_{jrn},q_{jrn},z_{jrn}), \qquad \forall n,r\in R^{\mathrm{CA}} \tag{9}$$

$$C_{in}^{\mathrm{T}}(pt_{in},u_{in},up_{in},ud_{in},F_{in}), \qquad \forall n,i$$

$$C_n^{\mathrm{D}}(pta_{in},pha_{rn}), \qquad \forall n$$

where:

λres vector of dual variables related to spinning reserve constraint.

Then the dual function in (9) can be evaluated by means of many (CA+I+N) independent subproblems, as follows:

$$\Phi^{\mathrm{UD2}}=\sum_{ca=1}^{\mathrm{CA}}\Phi_{ca}^{\mathrm{HC}}+\sum_{i=1}^{\mathrm{I}}\Phi_i^{\mathrm{T}}+\sum_{n=1}^{\mathrm{N}}\Phi_n^{\mathrm{D}}-\sum_{n=1}^{\mathrm{N}}\lambda res_n\cdot Res_n \tag{10}$$

where:

Φ^{HC} set of subproblems d);

CA number of cascades.

The third scheme (UD3 – Figure 8), which derives from UD2, separates even more the hydro set of subproblems. In this decomposition, the set of subproblems a) and b) are the same, but the set of subproblems d) in UD2 changes to:

e) a set of LP subproblems, representing the constraints between the reservoirs for each cascade;

f) a set of MILP subproblems, each one representing the operation of a hydro plant.

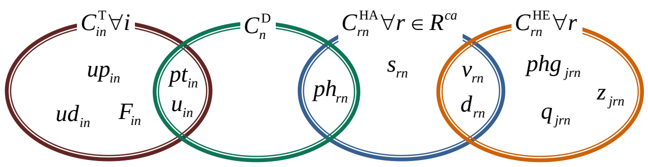

Fig. 8. Illustration of the relationship between variables for UD3.

To achieve UD3 we use the same relaxation strategy used in UD2, but splitting two more variables: *v* (volumes in the reservoirs) and *d* (total water discharged). The new sets for hydro constraints C^{HA} and C^{HE}, shown in Figure 8, derive from C^{H} without the spinning reserve constraint. The first represents hydraulic constraints of the reservoirs, defined for each cascade, and C^{HE} represent the constraints of the operation for each hydro plant. Mathematically, UD3 is given by:

$$
\begin{aligned}
\Phi^{\mathrm{UD3}}(\lambda pt,\lambda ph,\lambda res,\lambda v,\lambda d) = \min f_n\,(up_{in},F_{in}) + \sum_{n=1}^{\mathrm{N}}\Bigg(&\sum_{i=1}^{\mathrm{I}}\lambda pt_{in}\cdot(pta_{in}-pt_{in}-u_{in}\cdot pt_i^{\min})+\\
&\sum_{r=1}^{R}\Big(\lambda ph_{rn}\cdot(pha_{rn}-ph_{rn})+\lambda v_{rn}\cdot(va_{rn}-v_{rn})+\lambda d_{rn}\cdot(da_{rn}-d_{rn})+\\
&\lambda res_n\cdot\Bigg(\sum_{r=1}^{\mathrm{R}}\Bigg(\sum_{j=1}^{\mathrm{J}_r} phg_{jr}^{\max}\cdot z_{jrn}-ph_{rn}\Bigg)-Res_n\Bigg)\Big)\Bigg)\\
\text{s.t.: } & C_{rn}^{\mathrm{HA}}(va_{rn},da_{rn},s_{rn}), && \forall n,r\in R^{\mathrm{CA}} \qquad (11)\\
& C_{rn}^{\mathrm{HE}}(ph_{rn},v_{rn},d_{rn},s_{rn},phg_{jrn},q_{jrn},z_{jrn}) && \forall n,r\\
& C_{in}^{\mathrm{T}}(pt_{in},u_{in},up_{in},ud_{in},F_{in}), && \forall n,i\\
& C_n^{\mathrm{D}}(pta_{in},pha_{rn}), && \forall n
\end{aligned}
$$

where:

λv vector of dual variables related to water volume;

λd vector of dual variables related to water discharged;

Then the dual function (11) can be evaluated by means of many (CA+N·R+I+N) independent subproblems.

$$
\Phi^{\mathrm{UD3}}=\sum_{ca=1}^{\mathrm{CA}}\Phi_{ca}^{\mathrm{HA}}+\sum_{n=1}^{\mathrm{N}}\sum_{r=1}^{\mathrm{R}}\Phi_{rn}^{\mathrm{HE}}+\sum_{i=1}^{\mathrm{I}}\Phi_i^{\mathrm{T}}+\sum_{n=1}^{\mathrm{N}}\Phi_n^{\mathrm{D}}-\sum_{n=1}^{\mathrm{N}}\lambda res_n Res_n \qquad (12)
$$

where

Φ^{HA} set of subproblems e);

Φ^{HE} set of subproblems f).

B. Scenario Decomposition

This strategy separates the stochastic problem in single-scenario deterministic subproblems, applying the variable splitting technique for the linking variables of the first stage; in other words, the *non-anticipativity constraints* are relaxed. In this case, the constraints are rearranged and the sets are separated by scenario. For instance, all the constraints belonging to nodes of the scenario one, set of nodes $N(1)$, make up the set $\mathrm{C}_1^{\mathrm{C}}$, which is derived from C^{T}, C^{H} and C^{D} regarding the nodes of scenario one. Figure 9 illustrates the sets of constraints for the problem of Figure 4, representing the variables that couples the subproblems. The set N_x represents all the nodes of period x.

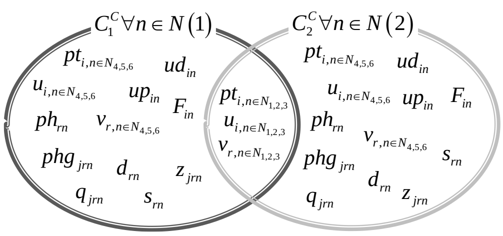

Fig. 9. Illustration of the relationship between variables for SD.

Relaxing the non-anticipatively constraints results in the following dual problem:

$$\Phi^{\mathrm{SD}}\left(\gamma pt,\gamma u,\gamma v\right)=\min\ f_n(up_{in},F_{in})+\sum_{\omega=1}^{\Omega}\left(\sum_{n\in N(\omega)}\left(\sum_{i=1}^{I}\left(\gamma pt_{in}\cdot\left(pt_{in}-\sum_{\sigma=1}^{\Omega}p_\sigma\cdot pt_{i,m\in N_t\cap N(\sigma)}\right)+\right.\right.\right.$$
$$\left.\left.\left.\gamma u_{in}\cdot\left(u_{in}-\sum_{\sigma=1}^{\Omega}p_\sigma\cdot u_{i,m\in N_t\cap N(\sigma)}\right)\right)+\sum_{r=1}^{R}\gamma v_{rn}\cdot\left(v_{rn}-\sum_{\sigma=1}^{\Omega}p_\sigma\cdot v_{r,m\in N_t\cap N(\sigma)}\right)\right)\right) \quad (13)$$
$$\text{s.t.: } C_\omega^{\mathrm{C}}\begin{pmatrix} pt_{in},u_{in},up_{in},ud_{in},F_{rn},ph_{rn},v_{rn},d_{rn},s_{rn}, \\ phg_{jrn},q_{jrn},z_{jrn}\end{pmatrix},$$
$$\forall i,r,b,n\in N\left(\omega\right),\omega=1,...,\Omega$$

where:

γpt vector of dual variables related to thermal power generation;
γph vector of dual variables related to hydro power generation;
γv vector of dual variables related to water volume;
ω index of scenarios ($\omega = 1, \Omega$);
Ω number of scenarios;
$N(\omega)$ set of all the nodes of scenario ω.

Then the dual function (13) can be evaluated by means of as many independent subproblems (Ω) as scenarios, as follows:

$$\Phi^{\mathrm{SD}}=\sum_{\omega=1}^{\Omega}\Phi_\omega^{\mathrm{C}} \quad (14)$$

where:

Φ_ω^{C} UC subproblem associated with scenario ω.

C. *Algorithm*

The Lagrangian duals corresponding to all of the strategies described in the previous section are solved by means of a Bundle-type method [26]. In particular, it is a “generalized” Bundle method [27], in that the Master Problem (MP) does not necessarily need to be a Quadratic Program. For this work, we used the linear (box) stabilizing term, which is known to significantly reduce master problem time w.r.t. the usual quadratic term [9]. Solving the MP provides an estimate of the optimal Lagrangian multipliers that are fed to the subproblems, which in turn provide dual function values and subgradients that are used to update the MP, iterating the process until convergence is attained.

The problem solved in this work has a dual function with a disaggregated structure, i.e., the dual function is a sum of many components, as can be seen in (8), (10), (12) and (14), and in similar works [28], [29]. Both the MP and the subproblems are solved by means of a general optimization solver. Table II presents, for the different strategies, the size of the dual problem and associated subproblems.

TABLE II
CHARACTERISTIC OF THE DECOMPOSITION STRATEGIES

Decomposition	Number of dual variables	Number of subproblems	
		LP	**MILP**
UD1	N·(I+R)	N	1+I
UD2	N·(I+R+1)	N	CA+I
UD3	N·(I+1+3·R)	N+CA	N·R+I
SD	T_1·Ω·(2·I+R)	0	Ω

T_1 is the number of nodes in stage 1.

The disaggregated model allows to use the "easy component" technique [9], whereby some of the components (subproblems), the "easy" ones, are included into the MP. This increases its size, but provides an exact description of the easy components, instead of an approximated iteratively refined by means of cuts. All the LP subproblems are candidate to be treated as easy components.

V. RESULTS

All the results have been obtained on an Intel Xeon CPU X5690 (3.47 GHz) computer with 32.0 GigaBytes of RAM. The LP and MILP subproblems are solved using the 6.0.5 Gurobi optimization solver. In the tests, we have various hydrothermal systems, different initial conditions and distinct scenarios trees to produce a wide range of results. The results are compared mainly using the performance profile technique [30]. It allows defining a distribution function for some metric (number of iteration, processing time, quality of the objective function, etc.) comparing algorithms by means of this metric. The performance profiles are cumulative distribution functions for a metric, as follows:

$$\Phi_X(mf) = \frac{\text{number of cases which } \eta_X(p) \le mf \cdot \eta^*(p)}{\text{total of evaluated cases}}, \tag{15}$$

where:

Φ_X	probability for method *X*;
mf	multiplying factor;
$\eta_X(p)$	performance metric for method *X* to solve case *p*;
$\eta^*(p)$	best performance metric found with whatever method;
p	a case of the problem, where *p* belongs to a representative set of cases.

The results of the tests are presented in the next sections using these performance profiles and the usual statistical metrics (average values and standard deviation).

A. *Easy components*

The goal of this section is to explore the effect of applying the easy component technique. We consider 30 cases: 15 deterministic cases and 15 stochastic ones. The 15 are different regarding the kind of system (five hydrothermal systems) and the initial conditions (three initial volume conditions). For this test, we compare the two kinds of decomposition where the use of easy components is possible and cause more impact: UD2 and UD3. Figure 10 report the performance profile for the number of iterations, where "ccs" is the variant with easy component, while "scs" (dashed) is the one without it. Figure 10 shows that the use of easy component does not have a major impact in the resolution of UD2, although on average it results 18% less iterations. The impact for UD3 is much more significant: in about 85% of the cases there is less iterations, and more importantly, the average reduction of iterations in UD3 is around 600%. In terms of computational times, using easy components on average reduces them by 22% and 2,544%, for UD2 and UD3, respectively. Since the technique is clearly beneficial, in all the tests presented in the following sections it is applied.

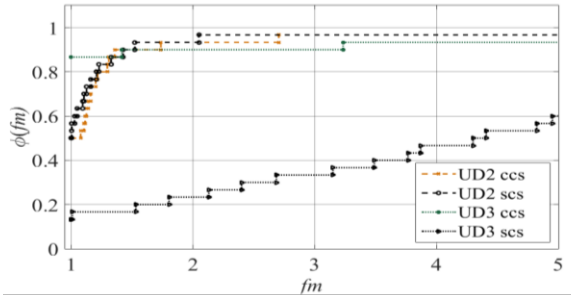

Fig. 10. Performance profile for UD2 and UD3 (iterations count).

B. *Deterministic instances*

In this section we present a comparison between the decomposition strategies using deterministic data for the five test systems (Table I) with three initial conditions, resulting in 15 cases. We present the average values (and standard deviation, in brackets) in Table III. All the unit decomposition schemes are compared by means of the duality gap (gap_1) and gap for the continuous relaxation solution (gap_2), given by:

$$gap_1 = \frac{UB - LB}{LB}, \quad gap_2 = \frac{LB - CR}{CR}, \qquad (16)$$

where the upper bound *UB* is best known solution for the problem, the lower bound *LB* is the optimum of the corresponding Lagrangian Dual, and *CR* is the lower bound provided by the continuous relaxation. We also present the total processing time, the oracle time and the number of iterations for all the schemes.

TABLE III
RESULTS FOR DETERMINISTIC CASES

Strategy	gap_1 [%]	gap_2 [%]	Time*	Oracle time [%]	Iterations*
UD1	1 (2)	139 (292)	9 (17)	88 (10)	4 (5)
UD2	2 (4)	137 (290)	10 (9)	86 (16)	2 (2)
UD3	5 (5)	122 (259)	1 (1)	39 (12)	1 (1)

*The time and the number of iterations are presented with regard to the smaller values, which are the ones obtained by UD3.

The table shows that strategy UD3 is the preferable one, considering the very significant difference in the processing time and in the number of iterations, while the difference in the gaps is comparatively much smaller. The main reason for this large advantage is the difficulty of the subproblem that represents the hydro constraints. In UD1 and UD2 this subproblem is rather complex and consumes almost all the time. Indeed, Table III shows that for these strategies the fraction of time spent in solving the subproblems is much larger than for the UD3.

C. Stochastic instances

In this section, we compare all the strategies for stochastic data. We consider two sizes of scenario tree, with four and nine scenarios, for the five test systems with three initial conditions, resulting in 30 cases. Figures 11 and 12 show the performance profiles for the number of iterations and the processing times, respectively.

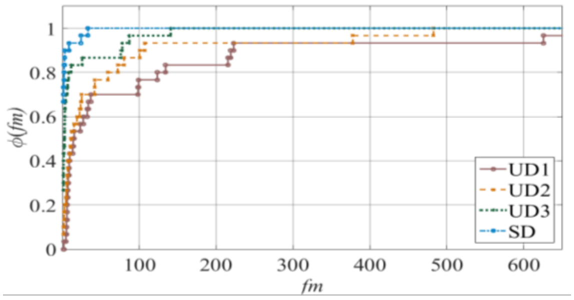

Fig. 11. Performance profile for number of iterations – stochastic cases.

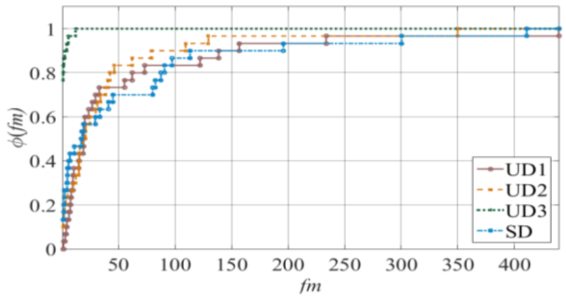

Fig. 12. Performance profile for processing time – stochastic cases.

Figure 11 shows that the SD converges in less iteration than the other methods in about 90% of the cases. However, its running time is one of the worst, as shown in Figure 12. This is due to the fact that it has to solve rather hard subproblems, which is where most of the time is spent. UD3 is still the best choice in terms of running times, with the trade-off again being its optimal dual function, which is not so good, as it can be seen in Figure 13. Conversely, SD results in the best lower bound in most cases. However, as we can see in Table IV (which presents the results by means of statistical metrics, similarly to Table III), the bounds found by the four strategies are close.

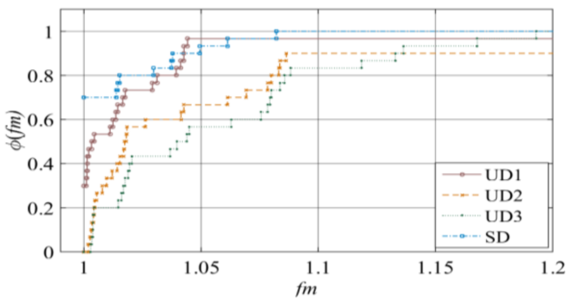

Fig. 13. Performance profile for dual function – stochastic cases.

As the table IV shows, two strategies stand out. The SD presents the smaller values for gap_1, with a processing time that is close to that of UD1. On the other hand, the UD3 presents by far the smaller processing times, with a gap_1, slightly bigger than the others strategies, but still reasonable. We finish this section by presenting more detailed results regarding the distribution of power plants for each test system presented in Table I. Table V shows the values of gap_1, in %, considering the deterministic and stochastic cases (but for the SD that is not applicable for deterministic instances).

TABLE IV

RESULTS FOR STOCHASTIC CASES

Strategy	Gap1 [%]	Gap2 [%]	Time*	Oracle time [%]	Iterations*
4 SCENARIOS					
UD1	2 (2)	169 (518)	21 (14)	70 (13)	32 (31)
UD2	10 (21)	142 (287)	11 (6)	80 (21)	12 (14)
UD3	7 (7)	171 (370)	1 (1)	24 (8)	4 (6)
SD	2 (2)	198 (425)	10 (5)	100 (1)	1 (1)
9 SCENARIOS					
UD1	4 (3)	20 (47)	13 (20)	49 (21)	8 (10)
UD2	4 (4)	24 (44)	27 (28)	62 (26)	9 (13)
UD3	7 (7)	123 (378)	1 (1)	16 (6)	1 (1)
SD	2 (2)	143 (436)	16 (10)	99 (1)	1 (1)

The value within brackets represents the standard deviation.

*The time and the number of iterations are presented with regard to the smaller values, i.e., UD3 for the time and SD for the iterations.

TABLE V

RESULTS FOR DIFFERENT SYSTEMS

System	UD1	UD2	UD3	SD
A	0.3 (0.2)	0.4 (0.2)	0.4 (0.2)	0.2 (0.1)
B	10.9 (28.0)	10.4 (20.4)	12.8 (7.6)	4.1 (3.2)
C	4.6 (3.0)	8.2 (5.4)	7.3 (3.4)	1.6 (1.4)
D	2.4 (2.0)	9.8 (22.4)	9.7 (5.0)	2.0 (1.0)
E	2.1 (1.1)	2.4 (0.8)	2.1 (0.8)	0.8 (0.4)

The value within brackets represents the standard deviation.

As the Table V shows, systems with predominance of hydro plants (systems B, C and D) present larger values for the gap_1, and, in general, take more time to converge. Test system B results in the largest values of the gap_1 for all the strategies. On the other hand, the system A, which has a larger participation of thermal plants presents the smallest values of gap_1.

VI. CONCLUSIONS

Lagrangian Decomposition is a fundamental technique for solving very-large-scale, hard optimization problems like SUC and others [9]. It exploits the problem structure, splitting it in many subproblems. However, applications like SHUC have actually more than one forms of exploitable structure, such as unit and scenarios, each with possibly different variants. Although some theoretical guidelines exist [2], [16], choosing the best variant is never obvious, as complex trade-offs between bounds and iterations have to be taken into account. Although there are studies comparing different kinds of decomposition for the SUC problem [11], [15], to the best of our knowledge no one has compared the use of scenario and unit decomposition to the same UC problem, in particular with the three different variants of the Unit (Space) Decomposition and the use of “easy components”. Our results show that

Scenario Decomposition, although providing the best duality gap, is not competitive in terms of computational burden. On the other hand, UD, and in particular UD3 (using “easy components”) has worse gaps, but only slightly so, while being much more efficient computationally. Furthermore, we have found that solution difficulty of dual problem depends on the amount of hydro in the system. This work provides solid foundations for a subsequent one, in which we will analyze the performance of the different decompositions schemes in the primal recovery phase, i.e., either the inexact augmented Lagrangian or the Lagrangian Heuristics that are used to construct the actual feasible solution required by the users.

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