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Technical Report

A Bilevel Programming Approach to Price Decoupling in Pay-as-Clear Markets, with Application to Day-Ahead Electricity Markets

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Abstract

The Italian and European electricity markets are experiencing a crisis caused by sharp increases in gas prices, which are reflected in dramatically higher final cost of electricity for all consumers w.r.t. historical values. This, however, is only in part motivated by the higher price of the gas actually used for electricity production: a very significant contribution is instead due to the "Pay-as-Clear" (PaC) mechanism implemented in the Day-Ahead Electricity Market (DAM), whereby all producers are remunerated at the price of the most expensive—typically, gas-fired—unit. This has led to a surge of the interest in the development of mechanisms capable of decoupling the price paid to the units whose production cost depends on that of fuel—and that therefore must be able to track that to ensure the economic compatibility of their operations—from those for which this is not the case. However, since both types of units participate in satisfying the same demand, this is technically complex. Motivated by this highly compelling application we propose the concept of *Segmented Pay-as-Clear* (SPaC) market, introducing a new family of market clearing problems—in fact, a relatively straightforward modification of standard ones—that has the potential to achieve such a decoupling without losing the crucial features of the PaC, i.e., that of providing both long- and short-term price signals. The approach is based on dynamically partitioning demand across the segmented markets, where the partitioning is endogenous, i.e., controlled by the model variables, and is chosen to minimise the total system cost. The thusly modified model belongs to the family of Bilevel Programming problems with a non-linear non convex objective function, or more generally a Mathematical Program with

*****This technical report is of an eminently scientific nature. The ideas expressed herein do not necessarily represent those of ARERA, for which the second author works.**

Complementarity Constraints; these problems have a higher computational complexity than those corresponding to the standard PaC, but in the same ballpark as the models routinely used in real-world DAMs to represent "nonstandard" requirements, e.g., the unique buying price in the Italian DAM. Thus, SPaC models should still be solvable in a time compatible with market operation with appropriate algorithmic tools. Like all market models, SPaC is not immune to *strategic bidding* techniques, but some theoretical results indicate that, under the right conditions, the effect of these could be limited. An initial experimental analysis of the proposed models, carried out through Agent Based simulations, seems to indicate a good potential for significant system cost reductions and an effective decoupling of the two markets.

Keywords: *Day-Ahead Market, Price-as-Clear, Decoupling, Bilevel Programs, Mathematical Program with Complementarity Constraints, Agent Based*

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1 Introduction, Motivations, Approach

The Price as Clear (PaC), or marginal price, is a historical market clearing model whose roots date back to the seminal works of Fred Schweppe and others of the early half of the 1980s, and that is widely used, among others, to solve the so-called electricity Day Ahead Market (DAM). In its most basic form, it stipulates that producers and consumers submit offers on the market that ultimately compose two curves: $\mathcal{S}(\pi)$, increasing with respect to the price π, for the sell offers, and $\mathcal{D}(\pi)$, decreasing with respect to the price π, for the purchase bids. Given the nature of the good of interest here—electricity—the demand curve can also be imagined as *rigid*, i.e., independent of the price, and therefore be a simple constant D with a possible cap at the price given by an estimation of the Value of Lost Load (VoLL), which in the Italian market is conventionally set at 3000 €/MWh. In reality the demand curve has a significant rigid section and a certain elasticity for the remaining quantity. The form of the price-quantity offers is diversified both in literature and in the implementation of the various DAMs in the world; in what follows we will assume the "basic" form, i.e., piecewise constant offers of the type $\{ \langle Q_i , \pi_i \rangle \}$ naturally partitioned in a certain number of sets. For instance, a pair of sell offers $\{ \langle 200 , 100 \rangle , \langle 250 , 150 \rangle \}$ indicates that the operator is willing to sell up to 200 MWh of energy at a price of no less than 100 €/MWh, and subsequently up to a further 250 MWh of energy at a price not less than 150 €/MWh. Sell offers from the same operator must be non-decreasing in price to avoid non-convex curves, i.e., that "rise and fall"; purchase bids should conversely be non-increasing in price. Different countries allow different variants to this basic concept: for instance, in the Italian market at most four offers can be submitted for each Production Unit (PU), while on other European markets so-called portfolio offers and piecewise linear offers are allowed. Catering for these most complex offers may complicate the clearing of the market, hence we will initially consider the basic form, with the impact of possible extensions discussed later on.

Under this assumption, the set S of *sell offers* $\langle sp_j , sq_j \rangle$, $j \in S$, and the set B of *purchase bids* $\langle bp_i , bq_i \rangle$, $i \in B$, mathematically define the two curves as:

$$\mathcal{S}(\pi) = \textstyle\sum_{j \in S \,:\, sp_j \leq \pi} sq_j \qquad \text{e} \qquad \mathcal{D}(\pi) = \textstyle\sum_{i \in B \,:\, bp_i \geq \pi} bq_i$$

The curves $\mathcal{S}(\pi)$ and $\mathcal{D}(\pi)$ (hopefully) meet and define a marginal price $\pi^\star$, corresponding to the cost of the *most costly* sell offer accepted, and a total quantity $d^\star$ of purchased good (energy). Crucially, in the PaC the price $\pi^\star$ is paid to *all* the sellers whose offers contribute to the total accepted quantity $d^\star$. From a mathematical point of view, what is actually solved is an optimisation problem in which the area defined by the two curves is maximised; in non-pathological situations, the optimal solution is precisely that where $S(\pi)$ and $D(\pi)$ meet. The implicit form of this *Clearing Problem* (CP) is $\pi^\star = \{ \pi : \mathcal{S}(\pi) = \mathcal{D}(\pi) \}$. As we will detail later, the explicit form of this CP is a simple *Linear Program* (LP), which is efficiently solvable. As already mentioned, changing the form of the offers/bids and/or constraints on their acceptation may change the CP, possibly making it harder to solve. Yet, all practical forms of the CP are based on the same basic underlying mechanism, to which we therefore focus.

At the time of the introduction of the PaC model, most PUs were of the fossil-fuel (or nuclear)-fired type, i.e., they almost all had a non-negligible marginal cost of production given by the cost of the fuel used, mediated by the specific thermodynamic efficiency of the cycle, start-up costs, feasible transitions for CCGT operations etc. In the monopolistic markets, the classical theory was that there should be base-load units providing basically constant uninterrupted power, some middle-merit technologies with higher (but still limited) flexibility, and then the so-called highly flexible (but also costlier) peak-load units, which could be quickly dispatched (and shut off) as the electrical load varied. Hydroelectric PUs, including pumping ones (PHS) were operated according to the classic "peak shaving" scheme where the limited total available energy was dispatched at peak load times to reduce the recourse to peak-load units. This gave rise to the classical Unit Commitment (UC) problem, an optimization model tasked with finding the least-cost dispatching of the available units (typically, one day in advance) considering all their complex technical constraints [7] and references therein.

After the end of monopolistic regimes, and thus since the opening up of electricity markets, a way had to be found to replace the UC model to obtain an efficient scheduling of the units. A common choice has been to have the PU submit sell offers, for each hour, on a (tightly regulated) DAM. Among the possible different forms that this can take, the PaC model has been widely regarded as the most appropriate, basically due to two aspects:

1. it defines a price that normally leaves technologies below the marginal price an adequate remuneration of fixed and investment costs, thus providing valuable *long-term* signals for investment (or disinvestment) in new capacity;

2. it sets an hourly price that naturally varies with the load, thus providing valuable *short-term* signals to units that have limited capacity (such as hydro reservoirs and, more recently, batteries) to use it during peak load periods, thereby performing a "peak shaving" that avoids the need for units that can quickly absorb the extra load but at a very high cost, and consequently increasing the overall efficiency of the system.

As is well known, there has been a shift—and an even stronger one is planned for the future—from predominantly fossil-based generation to electrical systems that are increasingly characterised by the presence of Renewable Energy Sources (RES), which are typically non-programmable. In addition, energy storage traditionally provided by basin-based hydroelectric PUs is increasingly complemented by other storage installations, especially of the electrochemical type.

1.1 Current situation, problems and objectives

Approximately after the summer of 2021, there was a historical increase in the price of natural gas on European spot markets also due to geopolitical tensions. Subsequently, the outbreak of the Russia-Ukraine conflict on 24 February 2022 negatively "consolidated" this natural gas price scenario. All of this generated an "anomalous" situation on the European electricity DAMs, that are coupled according to called CACM EU Regulation 2015/1222, which also provides for the implicit allocation of transmission capacity and thus cross-border flows. This is not only because the prices offered by Sources with *Non*-Negligible Marginal Costs (SNNMC), typically but not only gas-fired PUs, have unavoidably shot up. A very significant issue is that the market-accepted quantities from Sources at Negligible Marginal Costs (SNMC) are also valued at the *same marginal price*, precisely according to the PaC model, with an unprecedented overall increase in system costs.

For instance, in 2022 in the Italian DAM average monthly values were observed ranging from 201 €/MWh in January to 245 €/MWh in April, to 308 €/MWh in March, to 441 €/MWh in July to 543 €/MWh in August, with hourly peaks of over 850 €/MWh. For comparison, yearly averages in 2018 and 2019 were 61 €/MWh and 52 €/MWh respectively, even excluding the "outlier" of 38 €/MWh due to the pandemic. In the present context, overly aggressive bidding strategies can be observed to be enacted by renewables (RES) PUs that, using Formula 1 terminology, have "moved into the slipstream" and offer prices that are completely disconnected by any economic reality of the underlying operations.

This dramatic increase, which is threatening the economic stability of the entire EU block, has become a subject of intense scrutiny. By and large, two schools of thought emerged on the subject. On the one hand, those to believe that the PaC approach remains valid, identifying the gas price, rather than the functioning of the electricity market, as the fundamental problem. On the other hand, those who question what is the acceptable level of cost differentiation of the different sources that this (organised) specific marginal price market can sustain for its "proper" functioning, considering that the substitution principle underpinning the theory of the PaC approach is necessarily a medium to long-term objective. While of course the anomaly in gas prices is a root cause of the current crisis, the ability of decoupling prices of different actors/technologies that actually have different positioning and different roles in the market (say, base load vs. fast response vs. peak shaving) may be conceptually useful in general, as it allows to differentiate, over the long term, the different economic impact of investment in the different technologies, which may not necessarily be properly represented by the unique clearing price. A rethinking of the DAM can then be a valuable endeavour for the long-term efficiency of the energy system, apart from any (still, potentially highly interesting in itself) role it could play in protecting consumers from the potentially disastrous consequences of the current crisis in the short term, especially considering that it is unfortunately unclear what the actual time horizon of the crisis may be, and therefore what "short term" really means here.

The clearing model proposed in this work, which is alternative to the "pure" PaC but keeps very many of its underlying principles and mathematical structures, is aimed at contributing to the discussion about possible changes in the standard DAM model as implemented up to now. The model is based on a "principled" decoupling of the two (or more) sub-markets, i.e., it is not just intended to artificially force market prices to pre-crisis levels by imposing arbitrary price caps based on historical values (as, basically, all other proposals currently on the table). Rather, the model aims at leaving market forces as free as possible in each "segmented" market, which in fact is internally a pure PaC one. Furthermore, since all sub-markets in fact contribute to the satisfaction of the same demand, it must clearly be (and it is, according to our analysis) possible that the prices of the segmented markets influence each other. However, by explicitly recognising the peculiarities of the different PUs by segmenting them in different sub-markets, the proposed model can achieve a substantial (but, correctly, not complete) decoupling between the markets, as both our theoretical and experimental results demonstrate.

It is appropriate to immediately acknowledge that the proposed model has one obvious drawback: by modifying (although in a somewhat "simple") way the Clearing Problem (CP) of the standard PaC market it requires some changes to the existing operating procedures and software systems. Therefore, simpler procedures capable of obtaining analogous results—if any could be identified—could arguably be preferable. The most obvious idea would be setting a price cap on SNMC PUs that would allow for a fair remuneration of costs and investments without being excessive. This approach can be (and has been) criticised for weakening the long-term price signal and therefore reducing the much-needed new investments in RES, but investors know very well that the current price/remuneration levels—with or without price cap—will not be sustainable in the long run. Apart from the difficulty of determining such a price cap that would be acceptable to producers (and avoids the risk of costly and protracted litigations), such an approach would weaken, possibly to the point of eliminating it altogether, the market's function of setting an hourly price to guide the system's short-term choices. Indeed, SNMCs—even programmable ones—would have no incentive to offer their energy at peak load times, as it would presumably still be valued at the price cap at any time of day, therefore destroying dispatching efficiency. In order to mitigate this effect it would then be necessary to differentiate the price cap along the day, possibly to an hourly resolution, but the complexity in defining the right values would further dramatically increase. Other "simple" mechanisms proposed to achieve the decoupling goal, such as (mandatory) Contract for Differences (CfD) for SNMCs, have other drawbacks.

An aspect that, in our opinion, make our proposal particularly interesting is that, precisely because it is based on a (in a sense, minor) modification of the PaC approach, it is not necessarily alternative to the other proposed approaches (price caps and/or mandatory CfDs), but can be complemented and possibly strengthened by simultaneously applying it together with some of the other techniques. This aspect will be (briefly) discussed in the following. That the new approach would require some organisational and technical modifications to the existing market procedures is indeed true, but the changes are limited to the solution procedures of the Clearing Problem (CP) of the market, i.e., something of concern of a very limited number of parties: the "interface" with actors in the market (buyers and sellers) need not change. While the CP problems may become computationally harder to solve, this issue may be tackled by means of appropriate algorithmic research, exploiting a wealth of already available literature on related problems.

The—to the best of our knowledge, entirely novel—mechanism proposed here to obtain market decoupling is called the *Segmented Price-as-Clear* (SPaC) model. In this approach, a subset of the sell offers in the market are segmented away into what can be considered as a separate market, in which the offers compete only among themselves. Meanwhile, the remaining offers are treated as in the standard PaC model. In fact, the segmented offers can be further subdivided into an arbitrary number of subsets, within each of which the offers only compete with their "peers". This may actually be useful in the DAM context, say in a case where technical or political reasons may suggest to isolate certain technologies (say, RES) from others (say, nuclear), but both of them be from "mainstream" PUs (say, gas- or coal-fired ones). Both for the sake of simplicity, and because the most obvious segmentation just requires the definition of a single "SNMC market", in most of the development we will only make reference to segmented units all being treated in the same way, and therefore to having *two* markets; however, the extension to $k + 1$ markets is almost immediate and will be briefly discussing at the appropriate point.

Obtaining the stated objective of segmenting PUs so that they only compete among "peers" in the market, however, is not trivial, because the two markets must still be coordinated in order to satisfy a single demand, whether rigid or elastic. The crucial factor is indeed the amount of energy that each of the two markets must satisfy, which cannot easily be fixed *a priori*. In fact, for a market to be efficient, demand must be lower than supply, so that there is price competition: therefore, it is not trivially possible to choose for the "SNMC market" the maximum amount of demand that its PUs could serve, since in this way the PUs would be certain to see their offers accepted whatever the supply price might be. Conversely, excessively limiting the demand served by SNMCs, which typically have lower effective costs and are RES, is clearly detrimental to the overall efficiency of the system. Moreover, the total amount of demand accepted overall in the two markets is not known in advance, as it also depends on the (limited, but not absent even in the electricity case) elasticity of demand.

Our approach is based on an idea that is simple to enunciate, though not entirely trivial to implement in the CP: to make the fraction of demand served by the "SNMC market" a *decision variable*, i.e., one that can be automatically determined by the model itself. This allows a universally acceptable *objective function* to be incorporated into the decisions, i.e., the minimisation of the total system cost as perceived by *all* consumers. The approach leads to versions of the CP that are more challenging to solve than those corresponding to the standard PaC approach, but not necessarily so than the ones actually solved in practice to incorporate the variants required by the national legislations, such as the Italian Unique National Price (Prezzo Unico Nazionale, PUN). Anyway, the modified CP is likely still solvable with the necessary efficiency through the available modern approaches, be them general-purpose (e.g., [3] and references therein), or specialised (e.g., [2]).

It will be shown how the optimal and transparent splitting of the demand curve performed by SPaC over the two "partitioned" markets, each of which working according to the standard PaC model, always achieves no worse, and likely better, total system cost than the current classical PaC model. The optimal demand, the resulting accepted quantities, the *two different marginal prices for sellers* and the *unique price seen by buyers* are all the result of a single optimisation model. In fact, the model is a "minor" variation of the CP corresponding to the standard PaC. This allows to adapt the SPaC approach to many different requirements, such as network constraints and reasonably every other feature that has been so far treated in the context of PaC models.

As with all other market models, the SPaC one is not immune from strategic bidding techniques with which producers may seek to increase their profits. For this reason, in addition to presenting the details of the mathematical models for the CP, we present some theoretical results and an initial experimentation through Agent Based (AB) simulations that seek to analyse what the impact of strategic bidding techniques may be on the SPaC model. Our theoretical results show that if at least some of the PUs in the "SNMC market" can be "kept true", i.e., convinced to bid at reasonable prices, this forces the remaining ones to also "fall in line" and avoid/limit strategic bidding, or face a substantial risk of being excluded from the market even if their sell offer prices are significantly lower than these of the "SNNMC market". Our theoretical analysis indicates that this effect can be expected to be more pronounced as the relative size of the "SNMC market" grows w.r.t. that of the "SNNMC market", a trend that is expected to materialise in the future. The preliminary agent-based simulations seem to computationally confirm some of these insights.

The structure of the paper is as follows. First the SPaC approach is presented at an intuitive level by means of examples (§1.2). Then, the approach is formally introduced, starting from the simplest case of a model with rigid demand (§2.2) and gradually increasing the complexity, incorporating first demand elasticity and zonal prices (§5.2) and then more peculiar arrangements such as the Italian PUN (§5.3). In §3 a preliminary theoretical analysis is presented with regard to strategic bidding that exemplifies both the potential advantages and the limitations of the model. Finally, in §4 details of a software implementation and a preliminary set of simulations with an Agent Based approach are presented, and in §6 some preliminary conclusions are drawn and possible directions for future developments are discussed.

1.2 Intuitive aspects and examples

Figure 1 shows an example of a market cleared with a PaC model in which there are two technology clusters. We consider fixed demand and do not consider zones, network constraints between them and therefore zonal prices as implemented in the Italian market, nor complex products, e.g., block offers, Minimum Income Condition or other, as allowed in other European markets.

In the example each PU submits one price-quantity sell offer. PUs 1,2, and 3 and are of the RES type, hence of the SNMC type; PU 1 could be from a non-programmable source (e.g., PV or wind) while PUs 2 and 3 could be basin hydro and therefore programmable. PUs 4, 5 and 6 are SNNMC type, e.g., 4 could be a coal-fired PU and 5, 6 two gas-fired PUs (CCGT and OCGT). The numerical values of the $\langle\, Q \,,\, \pi \,\rangle$ offers are reported in Table 1; the quantities are deliberately small compared to the capacities of typical PUs to aid intuition in the examples. For the rigid demand $D = 23.7$ MWh, the PaC market is cleared with a price

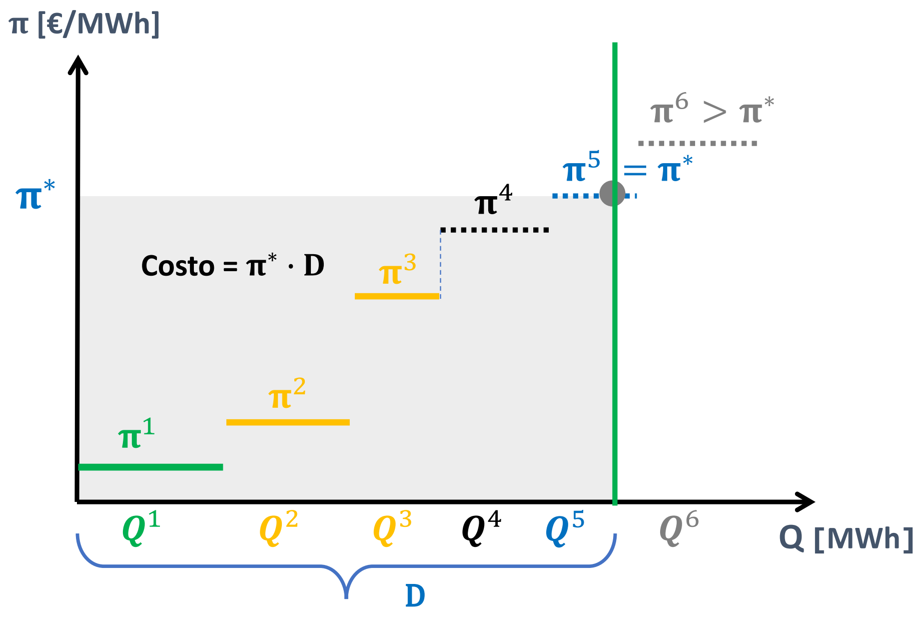

Figure 1: Classic Pay as Clear

$\pi^\star$ equal to the offer $\pi^5 = 220$ €/MWh. The quantities Q^1 to Q^4 are fully accepted for a total of 19 MWh while the marginal Q^5 is partially accepted, $Q^5 = Q^{5,\star} = (23.7 - 19)MWh = 4.7$ MWh. The classical effect is that the system cost is $\pi^\star D = 5214$ €. The need for a

	Q [MWh]	π [€/MWh]
PU_1	5	50
PU_2	5	60
PU_3	4	160
PU_4	5	190
PU_5	5	220
PU_6	7	250

Table 1: Price-Quantity offers for both PaC and PaC decoupled scenario one with hydro PU 3 out of the market and gas PU 6 in the market

decoupling of the price between SNMC and SNNMC arises precisely from the high price offered especially by gas technologies (5 and 6), which drives up the price of all energy accepted, although more than half is produced by SNMC in this simple example. The offer of PU 3 is intended to portray the "aggressiveness" of the bidding strategies of some SNMC, arising from the awareness of (not) competing with other PUs from SNNMC that will offer at even higher prices.

The SPaC approach aims at making the two (or, in principle, more) clusters of technologies compete on two different, albeit simultaneous, markets, each of which maintaining the PaC model and avoiding, at least initially, the introduction of price caps.

The following are the fundamental conceptual points of the approach:

- the SPaC approach divides the demand curve into two *flexible* components, the splitting being governed by a *variable* in the (hourly) optimisation problem that clears the market, and thus not "manually", as this would be impossible for the expected objective;
- in fact, the objective is to minimize the total system cost, given by the quantities from SNMC accepted for the marginal price of this market plus the quantities from SNNMC accepted for the *different* marginal price of this other market;
- conversely, the buyers' perceived price will be *unique*, both in the rigid demand case and in the more complex case of (partially) elastic demand;
- the actual demand does not play an "active" role in this splitting into the two different markets, as it is divided internally to the model with the stated objective, regardless of whether it is rigid or elastic; the offers segmentation, therefore, is completely transparent to buyers which continues to bid as at present;
- in the example scenarios that follow in this section, the optimal allocation of demand that attains the stated objective is just given; later sections will show how this is computed by the mathematical model.

Some scenarios are presented below, depending on offering strategies:

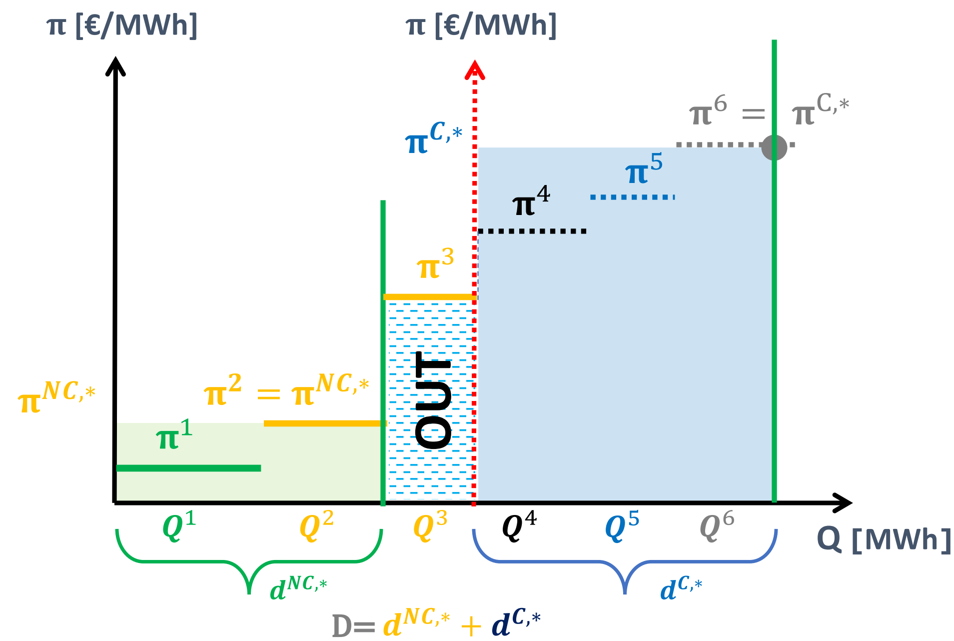

Figure 2: SPaC: scenario one

1. Scenario one. Figure 2 shows SPaC model in which, with the same offers as in the PaC model of Table 1, we modify the acceptance first seen in Figure 1 having defined two

separate sub-markets in which the two clusters of technologies can (and are exclusively enabled to) participate, this due to a particular demand allocation. The optimal allocation of demand, i.e., the one that minimises the total system cost considering the two different marginal prices, is $d^{NC,\star} = 10$ MWh and $d^{C,\star} = D - d^{NC,\star} = 13.7$ MWh. This results in the following three effects for the offers compared to the classical PaC:

(a) the *total* exclusion of the 4 MWh of basin hydro 3 in the SNMC market, which we will also call No Cost (NC): this reduces the marginal price of the total energy of SNMCs, $\pi^{NC,\star} = \pi^2 < \pi^3$;

(b) the PU 5 that was partially accepted in the classical PaC (for 4.7 MWh) is now totally accepted for 5 MWh in the corresponding SNNMC market, which we will also call Costly (C); therefore the PU 5 is no longer marginal as it was in the classical PaC market;

(c) lastly, we accept the most expensive gas offer (PU 6) which becomes marginal in market C for the residual quantity, which is equal to that previously bought by now excluded PU 3, i.e., 4 MWh, minus the additional 0.3 MWh accepted by PU 5. This increases the cost of market C, due both to the increase in the quantities accepted here and to the increase in the marginal price of the total energy of the SNNMCs, which rises from π^5 to $\pi^6 = \pi^{C,\star} > \pi^5$.

In so doing, the SPaC overall cost, given by the following sum: $Cost^{NC} = 10 \times 60 + Cost^{C} = 13.7 \times 250 = 600 + 3425 = 4025$ €, is significantly reduced compared to $23.7 \times 220 = 5214$ € cost of the classical PaC. The overall cost reduction is the result of the marginal price deltas, one increasing, $\pi^6 - \pi^5 > 0$ and one decreasing, $\pi^2 - \pi^3 < 0$, but averaged over the relative incremental and decremental quantities that "see" these prices. Thus, the reduction in total cost occurs for the interplay of both marginal prices with the *comprehensive* quantities accepted or excluded in the two markets.

2. Scenario two. The result would be even more favourable for the system if the PU 3 offered a price $\pi^3 = 100$ €/MWh as in Table 2, which is lower than in scenario one—but still comfortably high for a basin hydro PU. This would yield the opposite effect with respect to scenario one:

   (a) the acceptance of PU 3 offer, with the consequent increase of $\pi^{NC,\star}$ to π^3;

   (b) the corresponding non-acceptance of PU 6 offer, resulting in a reduction of $\pi^{C,\star}$ to π^5.

   This is due to the fact that *the optimal allocation of demand also changes* to $d^{NC,\star} = 14$ MWh and $d^{C,\star} = D - d^{NC,\star} = 9.7$ MWh, again for a play on both prices and total quantities; see Figure 3. In fact, costs now become $Cost^{NC} = 14 \times 100 = 1400$ € $+ Cost^{C} = 9.7 \times 220 = 2134$ €, $= 3534$ €, even lower than 4025 € of SPaC in scenario one, and *a fortiori* to 5214 € of classical PaC.

   All other offers being equal, one can easily find the break even value of the offer whereby PU 3 remains in the market and sets the marginal price in the corresponding market leaving the expensive gas PU 6 out of the market. With an abuse of decimals this

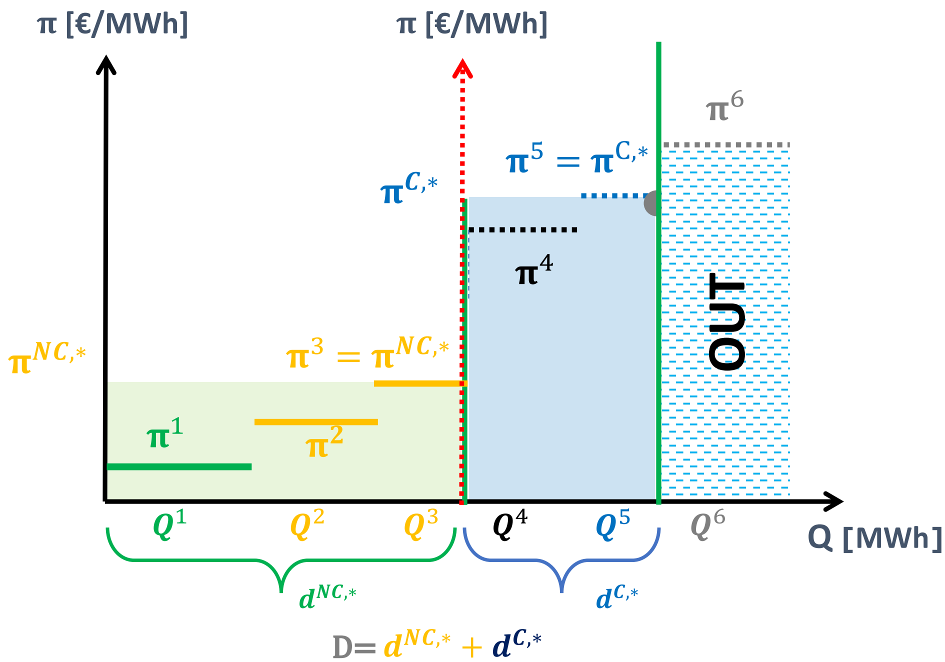

Figure 3: SPaC: scenario two

	Q [MWh]	π [€/MWh]
PU_1	5	50
PU_2	5	60
PU_3	4	100
PU_4	5	190
PU_5	5	220
PU_6	7	250

Table 2: Price-Quantity offers for SPaC, scenario two

break even value is π^3=135,071428571429 €/MWh in which case we would have a $Cost^{NC} = 1891,000$ € and $Cost^{C} = 2134,000$ €, for a total of 4025,000 €, identical to the cost of scenario one in which PU 3 is out of the market and PU 6 is accepted.

3. Overall aggressive SNMC scenario. Conversely, *all* the SNMC PUs may become "aggressive" in their offers, as in Table 3. The optimal demand distribution is in this case identical to that of scenario two, i.e., $d^{NC,\star} = 14$ MWh and $d^{C,\star} = D - d^{NC,\star} = 9.7$ MWh, while clearly the system cost increases because it is not convenient to accept the offer of PU 6 instead of any one of the three PUs 1/2/3, all at 200 €/MWh. In fact, not accepting (say) PU 3 would in this case have no effect because one would be forced to take PU 1 or 2 at the same price anyway. Yet, note that the SPaC costs are $Cost^{NC} = 14 \times 200 = 2800$ € $+Cost^{C} = 9.7 \times 220 = 2134$ € $= 4934$ €, still lower than the 5214 € of the classical PaC. In §3 the theoretical analysis of the strategic bidding will show under which conditions these aggressive strategy can, and cannot, be possible.

	Q [MWh]	π [€/MWh]
PU_1	5	200
PU_2	5	200
PU_3	4	200
PU_4	5	190
PU_5	5	220
PU_6	7	250

Table 3: Price-Quantity offers for SPaC, aggressive scenario

4. Indifferent scenario with respect to the PaC. It is easy to show that SPaC is not *always* better than PaC: if all PUs 1, 2 and 3 simultaneously offered like PU 5, i.e., 220 €/MWh, the total cost would be *equal* to that of the classical PaC. This scenario, however, would involve:

   (a) a *perfect* knowledge of PUs 1, 2 and 3 of the offer that PU 5 and PU 6 will make;

   (b) a contextual knowledge that this will be marginal in SNNMC market C;

   (c) that each PUs 1, 2 and 3, individually taken, are *precisely* aware of what the other two offer;

   (d) moreover, if there were any estimation error by PUs 1, 2 or 3, one would still have to assume to know the offer of PU 6 which could "shift" part of the demand to the NC market if it were less than the postulated 250 €/MWh.

5. Almost indifferent scenario with respect to the PaC. With reference to the last observation of the indifferent scenario, an interesting scenario is the one where

   (a) PUs 1, 2 and 3 wanted to bid aggressively and estimated the $\pi^\star$ of market C to make their offers as follows: $\pi_1 = 200$ €/MWh, $\pi_2 = 205$ €/MWh and $\pi_3 = 220$ €/MWh. That is, PU 3 estimated *perfectly* but the other two PUs underestimated it slightly;

   (b) at the same time PU 6 would offer slightly below w.r.t. the previous scenario, i.e., $\pi^6 = 225$ €/MWh.

   This would see PU 6 entering the market, while PU 3 would be *excluded completely*. As a consequence, $\pi^{NC,\star} = 205$ €/MWh and $Cost^{NC} = 10 \times 205 = 2050$ €, $\pi^{C,\star} = 225$ €/MWh and $Cost^{NC} = 13.7 \times 225 = 3082.5$ €, thus the total cost would be 5132 €, only slightly lower than the 5214 € of the classical PaC. The relevant point, however, is that PU 3 would be excluded from acceptance. This underlines the fact that, when bidding aggressively, market participants incur in the risk of being undercutted by other actors *in the same segment*, although they may be not by actors in the other segment. Again we refer to §3 for the theoretical analysis of the strategic bidding and to §4 for real simulations with an Agent Based approach.

Table 4 recaps all the scenarios where the price offers vary, while quantity offers remain the same, the SPaC total costs also expressed as percentage of the classic PaC one, the optimal distribution of demand in the two markets together with the various acceptances of the different PUs in the various scenarios discussed. In green the prices of the different marginal PUs in the two markets, in red the unaccepted offers.

	PaC	Sc.1	Sc.2	Sc.Aggr	Sc.BEven	Sc.Ind	Sc.A.Ind
SPAC Cost [€]	5214	4025	3534	4934	4025	5214	5132
SPAC %	100.00%	77.20%	67.78%	94.63%	77.20%	100.00%	98.43%
PU1 Pr [€/MWh]	50	50	50	200	50	220	200
PU2 Pr [€/MWh]	60	60	60	200	60	220	205
PU3 Pr [€/MWh]	160	160	100	200	135.071	220	220
PU4 Pr [€/MWh]	190	190	190	190	190	190	190
PU5 Pr [€/MWh]	220	220	220	220	220	220	220
PU6 Pr [€/MWh]	250	250	250	250	250	250	225
PU1 Acc	1	1	1	1	1	1	1
PU2 Acc	1	1	1	1	1	1	1
PU3 Acc	1	0	1	1	1	1	0
PU4 Acc	1	1	1	1	1	1	1
PU5 Acc	1	1	1	1	1	1	1
PU6 Acc	0	1	0	0	0	0	1
d^{NC} [MWh]	NA	10	14	14	14	14	10
d^{C} [MWh]	NA	13.7	9.7	9.7	9.7	9.7	13.7

Table 4: System costs, Percentage wrt PaC, offered prices, acceptance of PUs, marginal PUs and optimal allocation of demand in various scenarios for the decoupled model

Of course, these examples do not completely portray the possible variety of the (infinite) different offer scenarios and their possible outcomes. Hopefully, they do provide some initial intuition on the possible advantages (and sophistication) of the approach, as well as an inkling of its limitations.

All in all, the following preliminary remarks can be made:

1. the SPaC approach leaves the two markets "free" to express two marginal prices, hopefully different except in extreme cases, trying to decouple them as far as possible;

2. the result in terms of cost, all offers being equal, cannot be worse than the classical PaC;

3. the decoupling is operated through properly choosing demand in the two markets, SNMC and SNNMC, not in a static way but dynamically, in order to minimise, in the case with rigid demand, the overall system cost; this will be extended to the case of (partially) elastic demand, as well as to other common requests (zonal network constraints, etc.);

4. SPaC keeps providing price signals, arguably completely equivalent to these of PaC; even better, it provides one of them *for each segmented market*, which means specific long-term prices signals are delivered for each technology (mix) as opposed to just for the whole of the generation system;

5. in fact, the SPaC approach could be extended by further segmenting the PUs, e.g., by splitting SNNMCs into two further clusters; say, gas on the one hand and coal/nuclear on the other, or by differentiating gas-fired PUs into base load/middle merit (CCGT) from peak load Gas Turbines. We will not discuss this hypothesis further other than in a brief note in §5.2;

6. aggressive strategic bidding behaviour is possible in SPaC, as demonstrated by the examples and further analysed in the two lemmas in the §(3), but they are arguably riskier to implement than in PaC due to the more complex dynamics of the possible scenarios.

7. in particular, these strategic bidding behaviours appear to be more difficult to successfully implement than those that can be employed, e.g., in classical Pay as Bid (PaB) models, where the only theoretical goal of each PU is to offer (slightly) below the last accepted offer's price;

8. in the repeated game this should induce operators to be less aggressive, especially in the SNMC market but also in the SNNMC one, and to offer more in line with costs while not renouncing legitimate margin opportunities; a more detailed analysis of this central aspect will be presented in the §4.2 devoted to AB simulations;

9. the SPaC framework is complementary with, rather than alternative to, other proposed approaches to price decoupling like, say, price caps on the SNMC market. In fact, by already segmenting the market it offers more information to dynamically set such caps, e.g. by considering the cheapest offer made in the SNNMC market (which is, in principle, *not* the same as under the PaC approach), or an average of these offers. Such a cap mechanism could for instance be activate following an Australian approach, i.e., only after that several sessions have cleared at a "high" price. Furthermore, the SPaC framework is completely compatible with a share of the energy from SNMC to be sold in forward auctions of two-ways contracts for differences (CfD), as proposed in other "decoupling" approaches. These options, and their variants, could be future lines of research within the framework of the AB simulations of §4.2.

Of course, all this discussion hinges on the assumption that an *ad hoc* mathematical model for the CP of the SPaC approach can be devised and solved efficiently enough, with respect to the time constraints imposed by actual operations. This is non-trivial, and the subject of the next Section. It should be immediately noted that this mathematical modelling cannot be based on a trial and error approach or, worse, a "manual" approach. In scenario one, for instance, any value of $d^{NC,\star}$ larger than 10 MWh would not lead to the required goal precisely because beyond it would then be necessary to accept the offer of PU 3 at 160 €/MWh, thereby setting the marginal price for all energy in that market accordingly. Similarly, any value smaller then 10 MWh would not be optimal. If, for example, one were tempted to apportion "manually" the two demands, a reasonable and non-arbitrary method might be do so as the ratio of the total offers made in the two markets, i.e., (in the example) by solving the simple proportion $14 : 17 = d^{NC} : (23.7 - d^{NC})$; we would then have $d^{NC} = 10.703$ MWh, which would accept even if slightly PU 3, and $d^C = 12.997$ MWh, which would accept even if slightly PU 6. The cost would then be $10.703 \times 160 + 12.997 \times 250 = 4961.709$ €, and therefore much more than the 4025 € obtained by the model, which instead optimally allocates $d^{NC} = 10$ MWh and $d^C = 13.7$ MWh. Moreover, Lemma 1 in §3 will shown that, under reasonable conditions, the optimal set of solutions of the problem involves the complete acceptance of the critical offer that sets the price in the SNMC market.

2 Case with rigid demand

The following sub-sections formally introduce the mathematical optimisation model for the simplest of the PaC models (§2.1), and then the first formulation of the decoupled model (§2.2).

2.1 Modelling aspects: case with rigid demand and classical PaC

Let us start with the simplest case, with a fixed and rigid demand with a classical PaC model. The following are given:

- a set S of *sell offers* in the standard price, quantity form $\langle sp_j, sq_j \rangle$, $j \in S$;
- the partition $S = S^r \cup S^g$ of these offers respectively from the segmented market ("r" for reserved market, i.e., SNMC in our setting) and from all the other PUs ("g" for general market, i.e., SNNMC in our setting;
- the total demand $d > 0$, assumed to be inelastic and therefore constant.

The aim of the proposed model is to determine the optimal allocation of demand $d = d^r + d^g$, so $d^r \geq 0$ and $d^g \geq 0$ are considered as variables. At the same time, it is intended to execute two separate PaC markets on the two different sets of offers resulting in two clearing prices, one hopefully lower than the other and hence a lower overall system cost than the classical PaC. In the proposed model, the overall cost is given by the sum of the quantities sold times the corresponding clearing price.

We start by presenting the *primal* CP problem of a classical PaC corresponding to the whole set S, which is an LP:

$$\min_{s_j} \sum_{j \in S} sp_j s_j \tag{1}$$

$$0 \leq s_j \leq sq_j \qquad j \in S \mapsto (\eta_j) \tag{2}$$

$$\sum_{j \in S} s_j = d \qquad \mapsto (\pi) \tag{3}$$

Since (1)–(3), it has a *dual* problem (still an LP) that, by the theory of linear duality, is "basically the same problem" in that the optimal value is the same and pairs of optimal prima and dual solutions are tightly interlocked by means of the well-known *complementary conditions*. With the generic notation

$$a_j x \leq b_j \mapsto (\pi_j)$$

we denote that π_j are the dual variables, or shadow prices, of the constraints $a_j x \leq b_j$. For example, in (1)–(3) the demand constraint (3) has a single dual variable π which is precisely the market clearing price. The dual of (1)–(3) is easily verified to be

$$\max_{\eta_j, \pi} \; \sum_{j \in S} sq_j \eta_j \; + \; \pi d \tag{4}$$

$$\eta_j + \pi \leq sp_j \qquad j \in S \tag{5}$$

$$\eta_j \leq 0 \qquad j \in S \tag{6}$$

This pair of primal/dual problems can be shown to correctly implement the mechanism of finding the intersection between the offer curve $\mathcal{S}(\pi)$ and the demand curve $\mathcal{D}(\pi)$ alluded to in the introduction. Indeed, it is easy to verify that $\pi = \pi^\star$ in the dual optimal solution is the market price, i.e., the price of the most expensive accepted offer, and that only the least expensive offers (those whose price do not exceed $\pi^\star$) will be selected that serve to cover demand. This follows from the complementary conditions (7) and (8)

$$\eta_j(s_j - sq_j) = 0 \qquad j \in S \tag{7}$$

$$(sp_j - \eta_j - \pi)s_j = 0 \qquad j \in S \tag{8}$$

that uniquely characterise the feasible pairs of primary and dual solutions that are optimal, since:

1. from (7), $\eta_j < 0$ implies that $s_j = sq_j$, i.e., the offer j is fully accepted;
2. from (8), $s_j > 0 \implies \eta_j + \pi = sp_j$, and since $\eta_j \leq 0$ we have $sp_j \leq \pi$: i.e., for an offer even partially accepted its cost cannot be higher than the market cost;
3. in fact, if $sp_j > \pi$, since $\eta_j \leq 0$ must necessarily result $sp_j > \pi + \eta_j$: the constraint (5) is respected, but the term $sp_j - \pi$ in (8) cannot be 0. It follows that $s_j = 0$: no offer whose price is higher than π is accepted;
4. on the other hand, let $sp_j < \pi$: for (5) to be respected it must certainly be $\eta_j < 0$, and for the above $s_j = sq_j$, i.e., the offer is fully accepted.

To summarise: all offers for which $sp_j > \pi$ are fully rejected, while those for which $sp_j < \pi$ are fully accepted. If there are partially accepted offers, i.e., for which $0 < s_j < sq_j$, then $s_j > 0$ implies $\eta_j + \pi = sp_j$ while $s_j < sq_j$ implies $\eta_j = 0$, and therefore $\pi = sp_j$. Those are the "critical" offers whose price determines the market price.

2.2 Modelling aspects: case with rigid demand and decoupled PaC

The CP of the SPaC paradigm is then written in an "obvious" manner as the *bilevel* problem in dual space. The objective function is given by the sum of the two costs, i.e., that of the

"r-market" and that of the "g-market", expressed as the product of the corresponding prices and quantities. Given a certain allocation of demand, d^r and d^g, the prices are the result of the two markets at PaC; hence, a formulation is

$$\min \pi^r d^r + \pi^g d^g \tag{9}$$
$$d^r + d^g = d \ , \ d^r \geq 0 \ , \ d^g \geq 0 \tag{10}$$
$$\pi^r \in \operatorname{argmin} \textstyle\sum_{j \in S^r} sq_j \eta_j \ + \ \pi^r d^r \tag{11}$$
$$\eta_j + \pi^r \leq sp_j \qquad j \in S^r \tag{12}$$
$$\eta_j \leq 0 \qquad j \in S^r \tag{13}$$
$$\pi^g \in \operatorname{argmin} \textstyle\sum_{j \in S^g} sq_j \eta_j \ + \ \pi^g d^g \tag{14}$$
$$\eta_j + \pi^g \leq sp_j \qquad j \in S^g \tag{15}$$
$$\eta_j \leq 0 \qquad j \in S^g \tag{16}$$

Such a problem can be represented as a Mathematical Program with Complementary Constraints (MPCC) by explicating both the primary and dual variables and the complementary slackness conditions that characterise optimality:

$$\min \pi^r d^r + \pi^g d^g \tag*{(9)}$$
$$d^r + d^g = d \ , \ d^r \geq 0 \ , \ d^g \geq 0 \tag*{(10)}$$
$$0 \leq s_j \leq sq_j \qquad j \in S \tag*{(2)}$$
$$\textstyle\sum_{j \in S^r} s_j = d^r \tag{17}$$
$$\eta_j + \pi^r \leq sp_j \ , \ \eta_j \leq 0 \qquad j \in S^r \tag*{(12)}$$
$$\textstyle\sum_{j \in S^g} s_j = d^g \tag{18}$$
$$\eta_j + \pi^g \leq sp_j \ , \ \eta_j \leq 0 \qquad j \in S^g \tag*{(15)}$$
$$\eta_j (s_j - sq_j) = 0 \qquad j \in S \tag{19}$$
$$(sp_j - \eta_j - \pi^r) s_j = 0 \qquad j \in S^r \tag{20}$$
$$(sp_j - \eta_j - \pi^g) s_j = 0 \qquad j \in S^g \tag{21}$$

The bilevel problem is nonlinear and non-convex for two reasons: the complementarity (slackness) constraints (19)–(21), and the objective function (9). For the former, it is possible to introduce binary variables to encode the complementary conditions, resulting in a mixed-integer-linear problem (MILP). This, however, does not apply to the objective function. That having said, some modern MILP solvers are also able to optimise non-convex bilinear functions such as (9), and thus (in principle) solve the problem.

In fact, a slightly different analysis is possible that simplifies and clarifies the arguments. Instead of considering two separate markets, we consider only one, whereby we place a limitation on the maximum amount of energy that can be purchased from one of the two subsets of offers. In other words, we consider the modified market model

$$\min_{s_j} \; \sum_{j \in S} sp_j s_j \tag{1}$$

$$0 \leq s_j \leq sq_j \qquad j \in S \mapsto (\eta_j) \tag{2}$$

$$\sum_{j \in S^r} s_j \leq d^r \qquad \mapsto (\pi^r \leq 0) \tag{22}$$

$$\sum_{j \in S} s_j = d \qquad \mapsto (\pi \geq 0) \tag{3}$$

whose dual formulation reads

$$\max_{\eta_j, \pi, \pi^r} \; \sum_{j \in S} sq_j \eta_j \; + \; \pi d \; + \; \pi^r d^r \tag{23}$$

$$\eta_j + \pi \leq sp_j \qquad j \in S^g \tag{24}$$

$$\eta_j + \pi + \pi^r \leq sp_j \qquad j \in S^r \tag{25}$$

$$\eta_j \leq 0 \qquad j \in S \tag{6}$$

$$\pi^r \leq 0 \tag{26}$$

Replicating the previous analysis, we obtain that the market has two different equilibrium prices: π for offers in S^g (the g-market) and $\pi + \pi^r$ for those in S^r (the r-market). Crucially, $\pi + \pi^r \leq \pi$ due to the constraint (26) which requires π^r to be less than or equal to 0. Introducing the constraint (22) on the maximum quantity acceptable by the offers in S^r thus decreases its value, at least if the constraint is active in the optimal solution s^*, i.e., $\sum_{j \in S^r} s^*_j = d^r$; otherwise, for the complementary conditions $\pi^r = 0$ and the two prices, $\pi + \pi^r$ and π, are equal. Therefore, by acting on the quantity d^r (which, note, is a *parameter* in this model, not a variable) one can obtain a decrease in the market equilibrium price paid for such offers.

Let then $DSM(d^r)$ be the previous dual problem (23)–(26): the problem to be solved is naturally written as a bilevel problem in the dual space, in which d^r becomes a variable. Since the demand accepted in the r-market, $\sum_{j \in S^r} s_j$, is valued at the price $\pi + \pi^r$, while the demand accepted in the g-market is the remaining $d - \sum_{j \in S^r} s_j$ and is valued at the price π, the total system cost reads:

$$(\pi + \pi^r) \sum_{j \in S^r} s_j + \pi(d - \sum_{j \in S^r} s_j) = \pi d + \pi^r \sum_{j \in S^r} s_j$$

In general, one wants to choose d^r so that $\sum_{j \in S^r} s_j = d^r$ holds, since otherwise $\pi^r = 0$ and no benefit is surely obtained. Under this condition the objective function of (27) can be written as

$$\pi d + \pi^r d^r \qquad \text{or} \qquad (\pi + \pi^r) d^r + \pi(d - d^r).$$

In fact, it is easy to prove that the objective functions $\pi d + \pi^r d^r$ and $\pi d + \pi^r \sum_{j \in S^r} s_j$ are equivalent. This is evident for the first product, which is identical. For the second, if $\pi^r = 0$ then the value is still 0, whatever the value of the other term of the product. But by the complementary conditions $\pi^r < 0$ implies $d^r = \sum_{j \in S^r} s_j$, and so the product is still equal. Therefore the overall problem becomes

$$\min \left(\pi d + \pi^r d^r \right) : \; (\pi, \pi^r) \in \operatorname{argmax} DSM(d^r) \tag{27}$$

The bilevel problem (27) says:

- the objective of the *leader problem* is to minimise the total cost given by $(\pi d + \pi^r d^r)$, which function depends on the unique variable d^r (controlled by the "leader") but also on the marginal prices (π, π^r);

- given a certain d^r, these marginal prices (π, π^r) are variables in the (unique) *follower problem* $DSM(d^r)$ that solves the two markets (at once).

Schematically, one has a "iterative" situation as depicted below:

$$\begin{tikzcd} \min_{d^r}(\pi d + \pi^r d^r) \arrow[r] & d^r \arrow[r] & DSM_{(\pi,\pi^r)}(d^r) \arrow[dl, bend left] \\ & (\pi, \pi^r) \arrow[ul, bend left] & \end{tikzcd}$$

From a taxonomy point of view, these types of optimisation problems are referred to as bilevel or leader-follower type problems, in which the "leader" decision maker wants to optimise his objective function, which however depends on the optimal response of the "follower" decision maker. These are optimisation problems considered "difficult" because of their inherent non-convexity. For them an extensive algorithmic literature exists, see, e.g., the book [3] and the recent survey [6] and references therein.

3 Modelling and gaming analysis

The optimal solution of the SPaC CP problem (27) generates an integral cost that is not higher than the one obtained with the classical PaC market: in fact, the feasible solution obtained by setting $d^r = d$ yields the market price corresponding to the PaC model. It is, however, important to note that the cost reduction in the worst case is closely related to the relative size of the two fractions of demand satisfied by the two different sets of offers. Let us begin the analysis with the following:

Lemma 1 *Let $d^r < d$ and $\pi^r < 0$ in the optimal solution of (27): then, the offer of S^r that generates the equilibrium cost $(\pi + \pi^r)$ in the r-market is accepted in its entirety.*

Proof. Assume by contradiction that for the "critical" offer of S^r, i.e. $j^r \in S^r$ such that $sp_{j^r} = \pi + \pi^r$, one has $0 < s^*_{j^r} < sq_{j^r}$. Note that "critical" offers cannot have $s^*_j = 0$, since an offer with $s^*_j = 0$ cannot have cost equal to the equilibrium cost unless there is another offer with the same cost and $s^*_j > 0$ that defines it (and in this case that is the "critical" offer).

Let then $j^g \in S^g$ be the "critical" offer of S^g, i.e., such that $sp_{j^g} = \pi$; for the discussion above $0 < s^*_{j^g}$ must hold. But then, by changing d^r to $d^r + \varepsilon$ for an arbitrarily small $\varepsilon > 0$ one could satisfy the constraint (3) by increasing by ε the value of $s^*_{j^r}$ and decreasing by ε the value of $s^*_{j^g}$, without changing anything about the rest of the solution, either primal or dual. In particular, the equilibrium costs $(\pi + \pi^r)$ and π for the two markets would not change. This would lead to an increase in the total market cost S^r by $(\pi + \pi^r)\varepsilon$ and a decrease in the total market cost S^g by $\pi\varepsilon$. Since by hypothesis $(\pi + \pi^r) < \pi$, the decrease would be greater than the increase and therefore the new solution $d^r + \varepsilon$ would have a lower objective function value, which contradicts the fact that d^r was the optimal solution. ■

The previous lemma has a potentially useful corollary: the set of possible optimal solutions of the problem (27) (27) is limited to the points at which the "critical" offer of S^r (27) is totally accepted. This could serve to decrease the set of points to be considered for the solution. However, the result is also useful to continue the analysis as follows:

Lemma 2 *Consider an optimal solution of the problem (27), with the following data:* $d^r < d$, $\bar{\pi} = \pi + \pi^r < \pi$, j^r *the "critical" offer in* S^r *(such that* $\bar{\pi} = sp_{j^r}$*) and* j^g *that in* S^g *(such that* $\pi = sp_{j^g}$*). Suppose further that* sq_{j^r} *is "small" with respect to* d^r *and* d*. Then, the relation (28) approximately holds*

$$(\bar{\pi} - \bar{\pi}') \leq \frac{d - d^r}{d^r}(\pi' - \pi) \tag{28}$$

where $\bar{\pi} \leq \bar{\pi}$ *is the cost of an offer in* S^r *costing immediately less than the "critical" cost (necessarily fully purchased) and* $\pi \geq \pi$ *is the cost of an offer in* S^g *costing more than "critical" (necessarily not accepted, even partially).*

Proof. The hypothesis $\pi^r < 0$ necessarily implies $\sum_{j \in S^r} s^*_j = d^r$. For the previous lemma, we assume that the offer j^r "critical" for S^r is completely accepted. Since the solution is optimal for the problem, it must be better than any alternative solution. Consider the alternative solution in which d^r is reduced by the ("small") quantity $\varepsilon = sq_{j^r}$, such that the "critical" offer in S^r becomes that of cost $\bar{\pi}'$. Correspondingly, the quantity accepted by the offers in S^g is increased, and so the "critical" offer in S^g potentially (but not necessarily) changes, and with it the equilibrium cost which becomes $\bar{\pi}'$. Since this solution is not better than the optimal one, the result is

$$(d - d^r)\pi + d^r\bar{\pi} \leq (d - d^r + \varepsilon)\pi' + (d^r - \varepsilon)\bar{\pi}' \ .$$

For the hypothesis on ε we can approximate $d - d^r + \varepsilon$ by $d - d^r$ and $d^r - \varepsilon$ by d^r, which, rearranging the terms, gives (28). ■

The relationship (28) provides some insight on the complex dynamics governing the effect of the second market, and of the choice of d^r, on the total cost of the system. These relations depend partly on the relationship between the quantities traded in the two segments of the market, i.e., d^r and $d - d^r$, but also partly on the "distribution" of offers, i.e., on the fact *that there is an adequate variability of price offered in the two markets.*

The latter point is illustrated by the following example: let us assume that *all* the units in the r-market bid at exactly $\pi^\star - \varepsilon$, where π^* is the equilibrium price of the classical PaC market, while all units in the g-market maintain their offers. In this case, an optimal solution of (27) clearly involves accepting all offers from the r-market, and then the same offers from the g-market that would have been accepted by the PaC market, causing an overall negligible decrease $d^r\varepsilon^*$ of the system cost.

Therefore, the proposed SPaC market is not immune to strategic bidding, which however requires the ability of *all* the participants in the r-market to *accurately* estimate the "classical" equilibrium price set in the g-market and *simultaneously* all bid at the same price. This would immediately rise flags about collusive behaviour, and would impose to each r-market participant a significant risk of not being accepted, with the corresponding loss of revenue (especially if the PU is of a non-programmable type), in case the marked does not go exactly as planned, as illustrated in the examples section.

If, on the other hand, *not* all participants (particularly those from the r-market) decide to engage in aggressive strategic bidding, then the model could instead lead to significant reductions in system costs, mainly driven by the *ratio of quantities traded* in the two markets.

Consider the following example: in the formula (28) let $\bar{\pi} = 0$, corresponding to the "optimal" case in which all units in the r-market (say, non-programmable RES that do not want to risk being excluded from the market), offer zero, except the one corresponding to the critical bid j^r which exercises strategic bidding. For symmetry we instead take the "bad" case on the other side in which $\pi' = \pi^*$, i.e., rejecting the offer j^r immediately brings the market price on S^g back to the PaC one. Furthermore, still in the "bad" case assume that $\bar{\pi} = \pi$ ($\leq$ must surely hold). Applying these assumptions in (28) and simplifying we obtain

$$\bar{\pi} \leq \left(\frac{d - d^r}{2d^r - d} \right) \pi^* \tag{29}$$

The relation (29) shows that the PU carrying out strategic bidding may not be able to avoid a potentially substantial drop in the price of the r-market with respect to the PaC one, and thus a substantial overall gain for the system, but only if the amount of energy offered in the r-market is predominant. In fact, the factor in the formula is less than 1 only if $d^r \geq (2/3)d$. The factor tends to 0 when d^r tends to d: for example, if $d^r = (4/5)d$ (80%) then the factor is 1/3, which corresponds to the fact that the unit performing strategic bidding cannot in any case command a price higher than 33% of the PaC market price. Crucially, this positive result is linked to the fact that the g-market represents a "minority" share of the energy, and of course again to the fact that not all units in S^r engage in aggressive strategic bidding at the same time.

In practice, individual players in the r-market wanting to offer aggressively will likely end up with estimates $\pi^\star$ with different errors. In other words, each of the k participants in the r-market would offer strategically high but with offers like

$$\pi^\star - \varepsilon_k > \ldots > \pi^\star - \varepsilon_h > \ldots > \pi^\star - \varepsilon_2 > \pi^\star - \varepsilon_1 \quad .$$

It could then happen that some offer in the g-market, that would be excluded in the classical PaC, substitute (end thereby exclude) a number of those $\pi^\star - \varepsilon_i$ with i between $k \geq i \geq h$, i.e., among the highest in the r-market; see the "almost indifferent" example §1.2. Furthermore, some operator in the r-market might err by excess and offer $\pi^\star + \varepsilon_i$. While rejecting these offers may not have a significant impact on the system cost in the specific session, it could induce, in the repeated game, less aggressive offering in the r-market which in turn should translate in appreciable reductions in system cost. We will evaluate these and other aspects in the following §4.2.

4 Implementation and simulations Agent-based like

In §4.1 we briefly describe our prototypal implementation of the SPaC CP model in the Julia language with the JuMP and BilevelJuMP packages. Then, §4.2 describes an approach for evaluating bidding strategies in the repeated game by means of Agent Based (AB) like simulations, whose results are presented and discussed in §4.3.

4.1 Implementation

The proposed bilevel model of the SPaC CP problem, as well as that of the classical PaC, has been implemented using the language Julia version 1.7 [1], an open source programming language used for scientific applications. For modelling the optimisation problems, the package JuMP [4] belonging to the Julia ecosystem was used. In particular, a subpackage of JuMP called BilevelJuMP is used for the bilevel model. The optimisation solver used is Gurobi version 9.5.1 [5], capable of handling bilinear components such as those found in the objective functions of the model (27) and later.

4.2 Agent Based like simulations

Again using the Julia language, we implemented an Agent-Based-like simulation based on the standard probabilistic approach that employs several random components to evaluate the dynamics of the offer strategies in the two different SPaC and PaC paradigms. In this context, the SNMC-type PUs have been divided into two further subsets, the programmable (`SubType=P`) and the non-programmable (`SubType=NP`) ones; on the other hand, it is assumed that all the SNNMC-type PUs are of type P, hence programmable. This differentiation is considered to be relevant in defining the different offering strategies, in particular in the reduction of offers as they are not accepted, and thus the related dynamics of the SNMC market.

Starting from a set of offers at iteration $k = 1$, and the relative acceptances given by the corresponding variable s_i resulting from the DAM model used, one can define the offering strategies of the generic operator/agent at iteration $2, 3, \dots, k$ with a standard AB machinery as exemplified in the Algorithm 1. In the algorithm there are simply three macro occurrences at iteration k, i.e., the offer of PU was Not Accepted by the corresponding market (NA, $s^k = 0$), it was Partially Accepted (PA, $0 < s^k < Q$), or it was totally accepted (TA, $s^k = Q$). Depending on the occurrence, the type of PU and other aspects detailed in the algorithm, the rational operator makes choices for the offer at the next iteration as detailed in the sub-algorithms 2, 3 and 4.

We assume that:

1. The minimum probabilities that trigger the change of the offer respect the inequalities $0 \leq \alpha << \beta < \gamma \leq 1$. This means that it is much more likely for the AB machine to decrease the offered price in the NA case than to increase it in the TA one. Increasing the offered price in the PA case has the lowest probability. Dually, it is likely that the AB machine confirms the previously made offer, unless in a case discussed below;

2. The three different probabilities $ProbD$, $ProbI$ and $ProbIM$, which trigger the offer change (or not) when compared with the fixed α, β and γ, are also chosen randomly at each iteration k.

3. The multiplicative increase and decrease percentages of the offers are chosen randomly in appropriate intervals at each iteration k.

4. In the NA case, when reducing the price offered, this cannot under any circumstances go below a fixed (sort of) marginal cost. For the SNMC PUs recent literature-estimated values of the Levelized Cost of Electricity (LCOE) have been used.

5. Again in the NA case, the || (logical OR) condition requires to decrease its offer if the number of times it has not been accepted exceeds a predetermined value τ regardless of the trigger given by $ProbD$; this aspect provides a "memory" to the unsuccessful strategy of the last iterations, and clearly requires a counter.

6. The increase in the offered price is not limited above; it would have been possible to envision some sort of cap, but recent developments in the market go a long way towards proving that prices can increase manyfold. The effect of this lack of cap is seen when the demand approaches maximum capacity as we will see in §(4.3).

The simulations using Algorithm AB 1 were ran in parallel for both the SPaC and the classical PaC model, since obviously the dynamics of acceptance, and consequent operator strategy, are different along k iterations of the simulation, even starting from an identical set of offers.

The parameters used in the simulations of §4.3 are $a^- = 0.8$, $b^- = 0.9$, $a^+ = 1.05$, $b^+ = 1.07$, $a^{++} = 1.03$, $b^{++} = 1.05$, $\alpha = 0.20$, $\beta = 0.90$, $\gamma = 0.95$, $\tau = 2$. For instance, the multiplicative factor Δ^+ may vary uniformly at random in the range $[a^+, b^+] \equiv [1.05, 1.07]$. Also, in the NA case if the randomly generated probability exceeds $\alpha = 0.20$, then the offer reduction algorithm is triggered.

Algorithm 1: AB algorithm for updating offers of generic PU with decoupled model

Data: $(\pi^1, Q^1), (\alpha, \beta, \gamma), (a^-, a^+, a^{++}, b^-, b^+, b^{++}), \tau, MCost$

Result: $\left(\pi^k, Q^k\right), s^k, \pi^{k,r,\star}, \pi^{k,g,\star}$ for all $k \in [1, N]$

sZero=0

for $k = 1$ **to** N **do**

- $\Delta^- = rand(a^-, b^-)$, $\Delta^+ = rand(a^+, b^+)$, $\Delta^{++} = rand(a^{++}, b^{++})$
- $ProbD = rand()$, $ProbIM = rand()$, $ProbI = rand()$
- $\left(s^k, \pi^{k,r,\star}, \pi^{k,g,\star}\right)$=`SolveBil` $(\pi^k, Q^k, \cdots)$ `// Prob.27`
- **if** $s^k = 0$ **then** `// Offer not accepted`
  - $\pi^{k+1} \leftarrow$ `UpNonAccepted`(...) `// SubAlg.(2)`
- **else if** $0 < s^k < Q$ **then** `// Offer partly accepted`
  - $\pi^{k+1} \leftarrow$ `UpPartAccepted`(...) `// SubAlg.(3)`
- **else if** $s^k == Q$ **then** `// Offer totally accepted`
  - $\pi^{k+1} \leftarrow$ `UpTotAccepted`(...) `// SubAlg.(4)`

Algorithm 2: Function `UpNonAccepted` to update non accepted offers

Data: $\left(\pi^k, Q^k\right), MCost, \pi^{k,r/g,\star}, \Delta^-, \Delta^+, \alpha, \tau, sZero$

Result: $\left(\pi^{k+1}, Q^{k+1}\right), sZero$

$sZero++$

$ProbD = rand()$

if $ProbD \geq \alpha \;\; || \; sZero \geq \tau$ **then**

- **if** $PU.Type$==$SNMC$ **then**
  - **if** $PU.SubType$==NP **then**
    - $\pi^{k+1} \leftarrow \Delta^+ \cdot MCost$
  - **else if** $PU.SubType$==P **then**
    - $\pi^{k+1} \leftarrow \dfrac{MCost + \pi^{k,r,\star}}{2}$
- **else if** $PU.Type$==$SNNMC$ **then**
  - $\pi^{k+1} \leftarrow \max(MCost, \Delta^- \cdot \pi^{k,g,\star})$

else if $ProbD < \alpha$ **then**

- $\pi^{k+1} \leftarrow \pi^k$

Algorithm 3: Function `UpPartAccepted` to update partly accepted offers

Data: $\left(\pi^k, Q^k\right), \Delta^+, \gamma, sZero$
Result: $\left(\pi^{k+1}, Q^{k+1}\right), sZero$

```
ProbIM = rand()
if ProbIM ≥ γ then
    π^{k+1} ← Δ^+ · π^k
else if ProbIM < γ then
    π^{k+1} ← π^k
sZero = 0
```

Algorithm 4: Function `UpTotAccepted` to update totally accepted offers

Data: $\left(\pi^k, Q^k\right), \Delta^{++}, \beta, sZero$
Result: $\left(\pi^{k+1}, Q^{k+1}\right), sZero$

```
ProbI = rand()
if ProbI ≥ β then
    π^{k+1} ← Δ^{++} · π^k
else if ProbI < β then
    π^{k+1} ← π^k
sZero = 0
```

4.3 Simulations results

We now report on the results of several simulations using the previously described AB approach. We let the strategy evolve according to the AB logic for 300 iterations and we report aggregate statistics along all of them, plus a few "anecdotal" graphs showing the evolution of relevant system parameters to better illustrate the dynamics of the system. Demand remains fixed for during these consecutive iterations, then it is changed by choosing it between 40% and 85% of the maximum capacity offered (DMax) at steps of 5% of DMax itself so we have 10 different demand levels; the ratio 40% – 85% being more or less the typical range of low and high, non-extreme, electrical load versus installed capacity. Therefore, keeping demand fixed, 300 iterations of market clearing is performed modifying the offering strategy accordingly with the AB logic of each PU. These 300 iterations define a single simulation. All these 300 iterations for each of the 10 demand levels where performed over four different test cases with 6, 30, 50 and 100 PU, each one issuing a single price-quantity offer that varies over the iterations: the 6 PU test case is identical to that of the first scenario presented in Table 1, while the other three are more realistic, where the "marginal cost" (lower bound on price of sell offers) and the offered prices have been randomly generated in convenient ranges reflecting the offers observed at the time of writing on the Italian DAM. In particular, the offers in the initial iterations were chosen to represent "aggressive bidding" where the prices of SNMCs PU are aligned with those of the SNNMCs, so as to highlight the cost dynamics towards an equilibrium, if any.

Table 5 shows as an example the initial data of the 30-PU test case. We stress that different operators may offer a portfolio of different PUs but each UP follows single logic and no complex - per operator - strategies is envisioned.

As for the synthetic performance indicators, the following 19 quantities (averages along the iterations and/or percentages when it comes to ratios) are given in the various results tables for each demand level:

1. DMax: the maximum capacity offered [MWh];
2. D: the actual demand [MWh];
3. D/DMax: the ratio between demand and maximum capacity offered;
4. TC_SNMC: the average costs of the SNMC market in the decoupled market [€];
5. TC_SNNMC: the average costs of the SNNMC market in the decoupled market [€];
6. QDeCTotSNMC: the total quantities accepted in the decoupled SNMC market [MWh];
7. QDeCTotSNNMC: the total quantities accepted in the market decoupled from SNNMC [MWh];
8. QPaCTotSNMC: the total quantities accepted in the PaC market by SNMC [MWh];
9. QPaCTotSNNMC: the total quantities accepted in the PaC market by SNNMC [MWh];
10. π^r: the marginal prices of the SNMC market in the decoupled market [€/MWh];

11. π^g: the marginal prices of the SNNMC market in the decoupled market [€/MWh];

12. π^{PaC}: the marginal prices in the PaC market [€/MWh];

13. TC_SNMC/TC_SNNMC: The ratio between the two average costs of SNMC and SNNMC in the decoupled market;

14. TC_Dec: The average total costs in the decoupled market [€];

15. TC_PaC: the total average costs in the PaC market [€];

16. TC_Dec/TC_PaC: the ratio of total average costs in the decoupled market to the classic PaC market;

17. Min(TC_Dec/TC_PaC): the minimum value of the previous ratio along the iterations;

18. Max(TC_Dec/TC_PaC): the maximum value of the previous ratio along the iterations;

19. Std(TC_Dec/TC_PaC): the standard deviation of the previous ratio along the iterations;

UP	Type	SubType	Mcost	π	Q
UP_1	FCMT	NP	50.000	230.041	250
UP_2	FCMT	NP	50.000	201.355	250
UP_3	FCMT	NP	50.000	243.518	250
UP_4	FCMT	NP	50.000	237.584	250
UP_5	FCMT	NP	50.000	220.102	250
UP_6	FCMT	NP	50.000	215.913	250
UP_7	FCMT	NP	50.000	201.971	250
UP_8	FCMT	NP	50.000	217.574	250
UP_9	FCMT	P	50.000	227.195	200
UP_10	FCMT	P	40.000	221.112	200
UP_11	FCMT	P	40.000	206.056	200
UP_12	FCMT	P	40.000	244.575	200
UP_13	FCMT	P	40.000	211.135	250
UP_14	FCMT	P	40.000	234.345	250
UP_15	FCMT	P	40.000	212.774	350
UP_16	FCMT	P	40.000	242.364	250
UP_17	FCMT	P	40.000	228.902	250
UP_18	FCMT	P	40.000	243.346	350
UP_19	FCMNT	P	133.678	200.45	250
UP_20	FCMNT	P	157.268	225.175	250
UP_21	FCMNT	P	137.743	223.615	350
UP_22	FCMNT	P	144.4	210.236	250
UP_23	FCMNT	P	157.085	210.381	250
UP_24	FCMNT	P	155.957	230.748	350
UP_25	FCMNT	P	170.254	242.247	250
UP_26	FCMNT	P	179.021	249.545	250
UP_27	FCMNT	P	145.62	210.212	350
UP_28	FCMNT	P	161.904	228.641	250
UP_29	FCMNT	P	168.761	236.562	250
UP_30	FCMNT	P	175.944	238.795	350

Table 5: Initial data for 30 PU test case

Tables 6, 7, 8 and 9 show the results of the test case with 6, 30, 50 and 100 PU, respectively. For the 30-PU test case, Fig. 4 and 5 show charts for the total costs and marginal prices along the 300 iterations of a single simulation for an intermediate fixed demand value D equal to 60% of the DMax.

The tables and charts show the following global trends:

DMax	31	31	31	31	31	31	31	31	31	31
D	12.4	13.95	15.5	17.05	18.6	20.15	21.7	23.25	24.8	26.35
D/DMax	40.00%	45.00%	50.00%	55.00%	60.00%	65.00%	70.00%	75.00%	80.00%	85.00%
TC_SNMC	1053.984	1129.711	1273.813	1214.256	1180.718	1282.037	1200.798	1231.509	1832.372	2733.33
TC_SNNMC	6.92	9.45	244.47	479.44	711.31	944.38	1178.34	1415.89	3923.92	5661.73
QDeCTotSNMC	3706.8	4167.15	4164	4160	4163	4168	4173	4164	4182	4196
QDeCTotSNNMC	13.2	17.85	486	955	1417	1877	2337	2811	3258	3709
QPaCTotSNMC	3706	4163.25	4170	4161	4172.6	4174	4169.7	4163.5	4191	4196
QPaCTotSNNMC	14	21.75	480	954	1407.4	1871	2340.3	2811.5	3249	3709
π^r	85.26	81.3	91.7	87.56	85.07	92.22	86.25	88.7	131.4	195.3
π^g	150.15	150.14	150.35	150.33	150.41	150.74	151.11	150.98	361.25	458.17
π^{PaC}	126.76	127.36	150.15	150.16	150.17	150.39	150.64	150.66	361.25	458.17
TC_SNMC/TC_SNNMC	152.31	119.525	5.211	2.533	1.66	1.358	1.019	0.87	0.467	0.483
TC_Dec	1060.9	1139.16	1518.28	1693.69	1892.03	2226.42	2379.14	2647.4	5756.29	8395.06
TC_PaC	1571.84	1776.6	2327.27	2560.19	2793.1	3030.27	3268.87	3502.84	8959.01	12072.78
TC_Dec/TC_PaC	67.49%	64.12%	65.24%	66.16%	67.74%	73.47%	72.78%	75.58%	64.25%	69.54%
Min(TC_Dec/TC_PaC)	40.00%	40.00%	45.60%	50.70%	56.30%	55.10%	56.90%	62.20%	55.00%	60.40%
Max(TC_Dec/TC_PaC)	110.30%	106.40%	87.30%	85.60%	83.50%	95.90%	91.70%	93.40%	79.60%	73.00%
Std(TC_Dec/TC_PaC)	14.40%	16.30%	9.80%	7.70%	7.10%	9.90%	8.50%	7.60%	7.50%	2.80%

Table 6: Results for the 6-PU test case

DMax	7900	7900	7900	7900	7900	7900	7900	7900	7900	7900
D	3160	3555	3950	4345	4740	5135	5530	5925	6320	6715
D/DMax	40.00%	45.00%	50.00%	55.00%	60.00%	65.00%	70.00%	75.00%	80.00%	85.00%
TC_SNMC	221082.3	251654.6	285023.1	376765.3	416160.3	437897	448722.9	464453.6	486163.7	523021.4
TC_SNNMC	1276.92	2169.03	3225.8	8805.2	46803.54	104200.4	164002.4	223368.5	295289.1	380941.8
QDeCTotSNMC	946060	1062960	1179700	1285577	1320279	1322150	1319780	1322925	1321990	1327465
QDeCTotSNNMC	1940	3540	5300	17922.85	101720.7	218350	339220	454575	574010	687035
QPaCTotSNMC	946650	1064640	1182100	1290660	1307330	1320845	1323160	1325900	1325740	1325960
QPaCTotSNNMC	1350	1860	2900	12840	114670	219655	335840	451600	570260	688540
π^r	70.26	71.22	72.72	88.04	94.65	99.53	102.18	105.51	110.52	118.37
π^g	130.78	130.92	129.45	131.29	134.12	142.03	144.36	146.75	154.01	166.13
π^{PaC}	70.82	72.06	74.16	128.95	136.54	142.4	144.21	146.83	153.61	166.16
TC_SNMC/TC_SNNMC	173.137	116.022	88.357	42.789	8.892	4.202	2.736	2.079	1.646	1.373
TC_Dec	222359.2	253823.6	288248.9	385570.5	462963.8	542097.4	612725.3	687822	781452.8	903963.2
TC_PaC	223786.3	256176.6	292931.6	560282.1	647205.6	731230	797471.4	869948	970789.1	1115772
TC_Dec/TC_PaC	99.36%	99.08%	98.40%	68.82%	71.53%	74.14%	76.83%	79.07%	80.50%	81.02%
Min(TC_Dec/TC_PaC)	72.80%	69.50%	63.50%	49.90%	50.50%	66.20%	68.70%	70.10%	72.70%	72.00%
Max(TC_Dec/TC_PaC)	101.20%	102.20%	104.90%	100.20%	99.20%	99.10%	100.00%	100.00%	100.40%	99.60%
Std(TC_Dec/TC_PaC)	2.70%	2.70%	4.80%	4.40%	5.00%	4.30%	3.60%	4.00%	4.20%	4.00%

Table 7: Results for the 30-PU test case

DMax	11990	11990	11990	11990	11990	11990	11990	11990	11990	11990
D	4796	5395.5	5995	6594.5	7194	7793.5	8393	8992.5	9592	10191.5
D/DMax	40.00%	45.00%	50.00%	55.00%	60.00%	65.00%	70.00%	75.00%	80.00%	85.00%
TC_SNMC	325917.7	371795.6	429128.4	561316.9	601414.6	612698.8	630456.4	622968.8	650067.4	711947.6
TC_SNNMC	2373.74	3213.33	7215.82	38794.88	119821.7	208701.2	298083.9	396621.2	503522.2	626780
QDeCTotSNMC	1434720	1613351	1783830	1892128	1895326	1899827	1903263	1900650	1902124	1906266
QDeCTotSNNMC	4080	5299.5	14670	86222	262874	438223	614637	797100	975476	1151184
QPaCTotSNMC	1435544	1615045	1793415	1889804	1898122	1913925	1910489	1914113	1908358	1910299
QPaCTotSNNMC	3256	3605.5	5085	88546	260078	424125.5	607411	783637.5	969242	1147151
π^r	68.16	69.16	72.38	89.17	95.36	96.89	99.54	98.52	102.7	112.2
π^g	125.18	125.19	124.64	129	135.08	142.07	144.96	149	154.6	163.15
π^{PaC}	68.92	70.53	78.28	129.95	135.04	142.11	144.75	148.75	154.69	163.45
TC_SNMC/TC_SNNMC	137.302	115.704	59.47	14.469	5.019	2.936	2.115	1.571	1.291	1.136
TC_Dec	328291.5	375009	436344.2	600111.8	721236.3	821400	928540.3	1019590	1153590	1338728
TC_PaC	330534.9	380531.2	469288	856983.3	971450.3	1107499	1214891	1337661	1483745	1665769
TC_Dec/TC_PaC	99.32%	98.55%	92.98%	70.03%	74.24%	74.17%	76.43%	76.22%	77.75%	80.37%
Min(TC_Dec/TC_PaC)	60.60%	55.50%	58.50%	57.60%	64.00%	67.50%	68.20%	70.80%	71.70%	73.60%
Max(TC_Dec/TC_PaC)	101.00%	103.20%	102.80%	99.40%	98.00%	98.10%	100.10%	99.40%	99.50%	99.50%
Std(TC_Dec/TC_PaC)	3.60%	5.30%	7.00%	4.60%	4.00%	3.60%	3.70%	3.70%	4.00%	4.40%

Table 8: Results for the 50-PU test case

DMax	23870	23870	23870	23870	23870	23870	23870	23870	23870	23870
D	9548	10741.5	11935	13128.5	14322	15515.5	16709	17902.5	19096	20289.5
D/DMax	40.00%	45.00%	50.00%	55.00%	60.00%	65.00%	70.00%	75.00%	80.00%	85.00%
TC_SNMC	694794.8	893497.7	1027317	1036275	1069590	1076248	1121054	1133663	1190198	1184430
TC_SNNMC	7127.03	28592.26	169967.5	344267.5	526871.4	713777.4	905394	1104893	1324085	1573771
QDeCTotSNMC	2851004	3158449	3199388	3204197	3210730	3207942	3214510	3209161	3219240	3215163
QDeCTotSNNMC	13396	64001.5	381112	734353.5	1085870	1446709	1798190	2161589	2509560	2871688
QPaCTotSNMC	2857290	3191983	3221508	3226758	3233937	3234659	3235354	3235654	3232925	3238399
QPaCTotSNNMC	7110	30467	358992	711792	1062663	1419992	1777346	2135096	2495875	2848451
π^r	73.29	85.12	96.53	97.19	100.09	100.77	104.77	106.16	111.06	110.69
π^g	122.57	123.43	132.05	139.85	145	147.66	150.83	153.19	158.14	164.31
π^{PaC}	74.74	122.22	131.67	139.47	144.79	147.43	150.7	152.98	157.79	164.02
TC_SNMC/TC_SNNMC	97.487	31.25	6.044	3.01	2.03	1.508	1.238	1.026	0.899	0.753
TC_Dec	701921.8	922089.9	1197284	1380542	1596462	1790026	2026448	2238555	2514284	2758201
TC_PaC	713626.5	1312827	1571513	1831063	2073668	2287389	2517995	2738770	3013189	3327834
TC_Dec/TC_PaC	98.36%	70.24%	76.19%	75.40%	76.99%	78.26%	80.48%	81.74%	83.44%	82.88%
Min(TC_Dec/TC_PaC)	74.00%	66.40%	71.00%	70.80%	72.70%	74.90%	77.00%	78.20%	79.00%	78.20%
Max(TC_Dec/TC_PaC)	101.50%	100.70%	99.70%	99.30%	98.70%	99.60%	100.40%	100.00%	99.10%	100.30%
Std(TC_Dec/TC_PaC)	3.70%	3.30%	2.70%	2.70%	2.60%	2.30%	2.90%	2.90%	2.90%	3.30%

Table 9: Results for the 100-PU test case

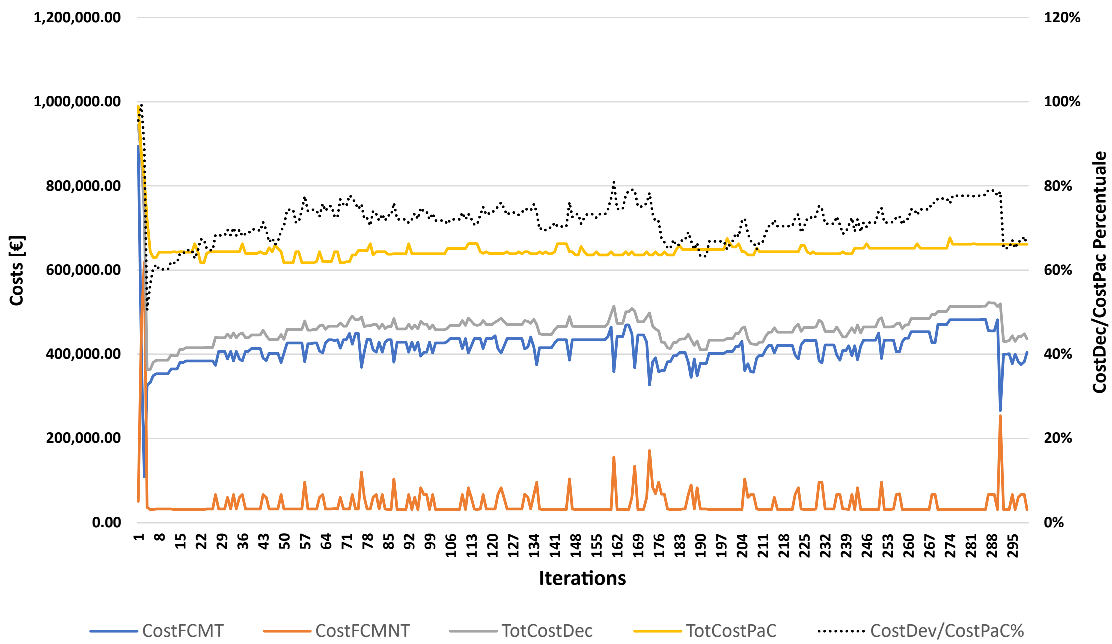

Figure 4: System costs for the 30-PU test case with D = 60% DMax

- SPaC does indeed succeed in generating competition, both between SNMCs and SNNMCs and internally to those (which have lower production costs/LCOE from business plans than the current prices offered by gas-fired SNNMCs) in the SNMC market: consequently, system costs are lower. Eliminating the extreme cases to be discussed shortly, the average ratios between the two SPaC VS PaC costs, row TC_Dec/TC_PaC, are approximately in the range 64%-75% for the 6-PU test case, in the range 68%-81% for the 30-PU test case, in the range 70%-80% for the 50-PU test case, and in the range 70%-83% for the 100-PU test case. Naturally, the complement to 100% of these percentages gives the estimated system cost reduction. Furthermore, the standard deviations of this ratio are relatively low. Maximum values are observed in the first

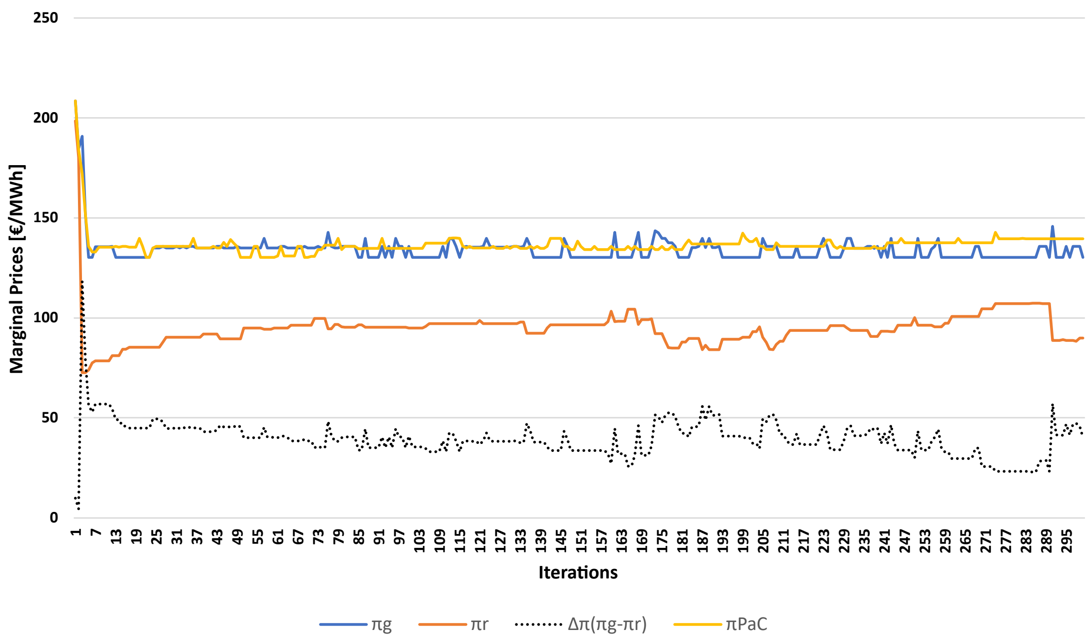

Figure 5: Marginal prices for the 30 PU test case with D = 60% DMax

iterations of each simulation.

- The few cases in which this ratio is close to 100% (test cases 50- and 100-PU) are due to low demand levels where even the PaC almost exclusively accepts SNMCs offers. In fact in these cases the π^{PaC} is aligned to the π^r so the cost is "low", even if obviously influenced by the offers of the SNNMC PUs, and there is no possibility of further reduction.

- In the SPaC market, the SNMCs, both those of the `SubType=P` and `SubType=NP` types, do not give up opportunities, as can be seen from the trend of prices π^r which are very often around 100 €/MWh. This is a direct effect of how the opportunity cost is articulated in the two markets and how the AB algorithm is designed.

- The graph of the 30-PU test case with D = 60% DMax in Fig. 4 show an immediate descent with respect to the initial offers, which—being very aggressive—immediately excluded several PUs in the SNMC market, forcing them to reposition. Thereafter, a certain stability of costs is observed even if some fibrillation is observed. In other test cases and for other demand values, a more lively dynamic is observed in the overall prices of both (the two sub-markets of) SPaC and PaC.

- The graph in Fig. 5 basically confirms the observations just made from a marginal price point of view, with the expected alignment of the π^g values of the SNNMC market with those of the π^{PaC}. This is also the effect of the use of the floor on the $MCost$ which was left in the AB algorithm. Removing it from the AB algorithm would make the market for SNNMCs in the decoupled model even more competitive. At the same time, and most importantly, the π^r values of the SNMC market are always lower.

- Since no cap has been placed on the offers, the model evolves naturally and, when demand approaches the maximum capacity offered, the prices π^{PaC} and π^g have an (abrupt, in the 6-PU test case) upward shift. This on the one hand confirms that the opportunity-dependent price signal is preserved, thus maintaining one of the main useful features of PaC. At the same time, the instants when demand is highest are exactly those when the two markets SNMC and SNNMC differ the most. From the point of view of AB simulations, inserting an arbitrary, global price cap would limit this classic exercise of PU power in both PaC and SPaC markets.

Indeed, the theoretical analysis of the §3 indicates how having a reasonable fraction of PU SNMCs that "offer low" is a potentially decisive aspect for system cost reduction. One could therefore envisage, for instance, a price cap on SNMCs of type `SubType=NP`, either unique or possibly differentiated by technology, i.e., solar, wind etc. Such a price cap would not have the effect of nullifying the price signal during the various relevant periods of the market, because such PUs in fact "do not participate strongly in the market", in the sense that being non-programmable they have no interest in having their energy not accepted at certain times rather than at others; instead, the strategy should reasonably be to ensure that offers are always accepted, trusting that the marginal price will be "reasonable" because of the offers of the programmable PUs. This type of PU, therefore, typically does not have an important effect on the marginal price and thus on the corresponding price signal, apart from the obvious effect on volumes. Furthermore, some of these types of PU are either incentivised (at least in Italy but also elsewhere) or have a large part of their energy hedged against price risk with (purely financial) two-way contracts.

Therefore, ensuring that they do not engage in extreme opportunistic behaviour should not have a negative effect on the validity of the price signal; for the price and quantity game on the "SNNC market", however, this could have the effect of significantly reducing the ability of `SubType=P` SNMCs to successfully engage in aggressive strategic bidding techniques, with a net positive effect on the whole market. This is all the more relevant in the medium/long-term perspective, where a significant growth of RES (typically SNMCs of the `SubType=NP` type) is expected to make this effect even greater, as discussed in §3. Many nontivial issues would remain to be settled for a practical use of the SPaC approach, such as if and how also set a (possibly, differentiated by technology) price cap acceptable to operators, or any other approach that complements and strengthens the decoupling obtained by SPaC like forcing some classes of PU to enter into (mandatory) two-way long term contracts, perhaps by means of some organized tender. These aspects are clearly outside of the scope of this treatment and are left for future research.

5 Case with network constraints, elastic demand and PUN

In this and the next sections, we will generalise the modelling analysis to introduce elasticity of demand and network constraints, both of which leave the PaC CP an LP (even though a larger one), thereby making the SPaC CP a significantly more difficult problem. We will then discuss further generalise the approach to features like the Unique National Price (PUN) as seen in the Italian DAM, for which the PaC CP is already a hard problem, so that the SPaC approach does not radically change the complexity.

5.1 The PaC case with network constraints and elastic demand

Let us now discuss the case in which zonal network constraints, and corresponding potential zonal prices, are imposed as envisaged e.g. by the current Italian model of the DAM, and demand is (partly) elastic, i.e., the purchase bids are also to be considered. The problem data are thus:

- a set S of *sell offers* $\langle sp_j, sq_j \rangle$, $j \in S$;
- the partition $S = S^r \cup S^g$ of offers in the r-market and g-market;
- a set B of *purchase bids* in the standard price, quantity form $\langle bp_i, bq_i \rangle$ $i \in B$;
- k the zone index and $\mathcal{K}$ the related set;
- $I(k)$ and $J(k)$ respectively the sets of purchase bids and sell offers relating to withdrawal points/PUs located in the zone k, $k \in \mathcal{K}$;
- $k(i)$ and $k(j)$ respectively the zones to which the purchase bid i/sales offer j refer;
- l the index of (equivalent) transmission branches between the zones and $\mathcal{L}$ the relative set;
- S_k^l the sensitivity coefficient of the active power transit on link l with respect to the net injection (accepted sell offers minus accepted purchase bids), into zone k;
- m_l and M_l the minimum and maximum feasible active power flow along (equivalent) transmission branch l.

The Primary Constrained Market Clearing Problem (PPMCV) corresponding to the entire set of offers S in the classical PaC is

$$
\begin{aligned}
\max\ & \textstyle\sum_{i\in B} bp_i b_i \ - \ \sum_{j\in S} sp_j s_j && && (30)\\
& 0 \le b_i \le bq_i && i \in B && (31)\\
& 0 \le s_j \le sq_j && j \in S && (32)\\
& \textstyle\sum_{i\in B} b_i \ = \ \sum_{j\in S} s_j && && (33)\\
& m_l \le \textstyle\sum_{k\in\mathcal{K}} S_l^k \left(\sum_{i\in I(k)} b_i - \sum_{j\in J(k)} s_j \right) \le M_l && l \in \mathcal{L} && (34)
\end{aligned}
$$

and its dual is

$$
\begin{aligned}
\min\ & \textstyle\sum_{i\in B} bq_i \mu_i \ + \ \sum_{j\in S} sq_j \eta_j \ + \ \sum_{l\in\mathcal{L}} (M_l \lambda_l^+ - m_l \lambda_l^-) && && (35)\\
& \mu_i + \pi + \textstyle\sum_{l\in\mathcal{L}} S_l^{k(i)} (\lambda_l^+ - \lambda_l^-) \ge bp_i \ , \ \ \mu_i \ge 0 && i \in B && (36)\\
& \eta_j - \pi - \textstyle\sum_{l\in\mathcal{L}} S_l^{k(j)} (\lambda_l^+ - \lambda_l^-) \ge -sp_j \ , \ \ \eta_j \ge 0 && j \in S && (37)\\
& \lambda_l^+, \lambda_l^- \ge 0 && l \in \mathcal{L} && (38)
\end{aligned}
$$

The meaning of this Dual Problem of Constrained Market Clearing (PDMCV) becomes clearer by rewriting it as

$$\begin{aligned}
&\min \textstyle\sum_{i\in B} bq_i\mu_i \ + \ \sum_{j\in S} sq_j\eta_j \ + \ \sum_{l\in\mathcal{L}}(M_l\lambda_l^+ - m_l\lambda_l^-) && && (35)\\
&\quad \pi^k = \pi + \textstyle\sum_{l\in\mathcal{L}} S_l^k(\lambda_l^+ - \lambda_l^-) && k\in\mathcal{K} && (39)\\
&\quad \mu_i + \pi^{k(i)} \geq bp_i \ \ , \ \ \mu_i \geq 0 && i\in B && (40)\\
&\quad \eta_j - \pi^{k(j)} \geq -sp_j \ \ , \ \ \eta_j \geq 0 && j\in S && (41)\\
&\quad \lambda_l^+, \lambda_l^- \geq 0 && l\in\mathcal{L} && (38)
\end{aligned}$$

This model defines, by analogy with the simple case, *zonal* prices π^k that determine the acceptance of offers and bids. Furthermore, the amount of energy purchased is not fixed but depends on the meeting of the demand and offer curves, expressed through the constraint (33).

It is useful to discuss the problem of the *economic equilibrium* of the market relative to the prices that are determined. It is in fact obvious that, in the case where all multipliers $\lambda_l^\pm$ are 0, i.e., the network constraints are not active in the solution, from (33) we immediately have:

$$\pi \textstyle\sum_{i\in B} b_i \ = \ \pi \sum_{j\in S} s_j \ \ .$$

Hence, the total price paid by the buyers corresponds to the total revenue received by the sellers, i.e., the market is in equilibrium in terms of both the quantity of energy and its total economic value. However, the condition is no longer true in the case where some of the $\lambda_l^\pm$ is strictly positive. In order to examine this case and its impact on the economic equilibrium, we must explicitly state the complementary conditions of the pair of problems, namely

$$\begin{aligned}
&\eta_j(sq_j - s_j) = 0 && j\in S && (42)\\
&\mu_i(bq_i - b_i) = 0 && i\in B && (43)\\
&(\eta_j - \pi^{k(j)} + sp_j)s_j = 0 && j\in S && (44)\\
&(\mu_i + \pi^{k(i)} - bp_i)b_i = 0 && i\in B && (45)
\end{aligned}$$

We omit those relating to (34) as they are not relevant to the analysis. From these descend immediately, by simple algebraic transformations the following:

$$\eta_j sq_j = \eta_j s_j \ , \ \mu_i bq_i = \mu_i b_i \ , \ (\eta_j + sp_j)s_j = \pi^{k(j)}s_j \ , \ \pi^{k(i)}b_i = (bp_i - \mu_i)b_i \qquad (46)$$

Now, the optimality of the primal and dual solution corresponds to the fact that the value of the two objective functions is equal, i.e.,

$$\textstyle\sum_{i\in B} bp_i b_i \ - \ \sum_{j\in S} sp_j s_j = \sum_{i\in B} bq_i\mu_i \ + \ \sum_{j\in S} sq_j\eta_j \ + \ \sum_{l\in\mathcal{L}}(M_l\lambda_l^+ - m_l\lambda_l^-) \ \ .$$

Rearranging and using the first two in (46) gives

$$\textstyle\sum_{i\in B}(b_i - \mu_i)b_i \ - \ \sum_{j\in S}(sp_j + \eta_j)s_j = \sum_{l\in\mathcal{L}}(M_l\lambda_l^+ - m_l\lambda_l^-) \ \ .$$

Now the second two in (46) can be used to obtain

$$\textstyle\sum_{i\in B} \pi^{k(i)}b_i \ - \ \sum_{j\in S} \pi^{k(j)}s_j = \sum_{l\in\mathcal{L}}(M_l\lambda_l^+ - m_l\lambda_l^-) \ \ .$$

Since $M_l \geq 0$, $m_l \leq 0$, and (38) holds, the term on the right hand side is ≥ 0; therefore the market is still in economic equilibrium, in the sense that

$$\sum_{i\in B} \pi^{k(i)} b_i \geq \sum_{j\in S} \pi^{k(j)} s_j \quad .$$

That is, the total price paid by the buyers is at least equal to the total revenue received by the sellers, who can thus receive what was agreed upon. The difference is strictly positive if there exists some $\lambda_l^{\pm} > 0$, in which case this is a price signal for the TSO indicating that there would be a convenience to invest in increasing the transmission capacity of the link l, also quantifying this convenience. In fact, the term $\sum_{l\in\mathcal{L}}(M_l\lambda_l^+ - m_l\lambda_l^-)$ in (35) is normally referred to as the "congestion rent" i.e., the fee that the network operator (the TSO) receives and which is intended to be used to fund the betterment of the interconnection network (in particular along the congested lines) and thus to reduce congestion and the resulting price differentiation. The fundamental point remains that zonal prices define a market that can reasonably be considered to be in "economic equilibrium".

5.2 The SPaC case with network constraints and elastic demand

Let us now introduce, in analogy with the simple treatment, the basic step to define the SPaC model, i.e., the market problem with elastic demand and network constraints, but artificially segmented demand:

$$\begin{aligned}
&\max \ \textstyle\sum_{i\in B} bp_i b_i \ - \ \sum_{j\in S} sp_j s_j && && (30)\\
&\quad 0 \leq s_j \leq sq_j && j \in S && (32)\\
&\quad \textstyle\sum_{j\in S^r} s_j \leq d^r && && (17)\\
&\quad 0 \leq b_i \leq bq_i && i \in B && (31)\\
&\quad \textstyle\sum_{j\in S} s_j = \sum_{i\in B} b_i && && (33)\\
&\quad m_l \leq \textstyle\sum_{k\in\mathcal{K}} S_l^k \left(\sum_{i\in I(k)} b_i - \sum_{j\in J(k)} s_j \right) \leq M_l && l \in \mathcal{L} && (34)
\end{aligned}$$

Note that the accepted quantity of supply and demand is not rigid but subject to the standard equilibrium constraint (33). The only modification with respect to the standard model is the constraint (17), which imposes an "arbitrary" cap on the maximum amount of energy purchased from the r-market. Note that, again, the cap is on energy and not on price, but it is obvious that the appropriate choice of d^r has an impact on the marginal price of the r-market. Again, in the context of the problem (30)–(34) d^r is a *parameter*, not a variable, i.e., fixed as far as this model is concerned. As done before, we are then interested in making it "flexible" i.e. a variable.

The dual reads as follows:

$$\min\ d^r\pi^r + \textstyle\sum_{i\in B} bq_i\mu_i\ +\ \sum_{j\in S} sq_j\eta_j + \sum_{l\in\mathcal{L}}(M_l\lambda_l^+ - m_l\lambda_l^-) \tag{47}$$

$$\mu_i + \pi + \textstyle\sum_{l\in\mathcal{L}} S_l^{k(i)}(\lambda_l^+ - \lambda_l^-) \geq bp_i\ \ ,\ \ \mu_i \geq 0 \qquad i \in B \tag{36}$$

$$\eta_j + \pi^r - \pi - \textstyle\sum_{l\in\mathcal{L}} S_l^{k(j)}(\lambda_l^+ - \lambda_l^-) \geq -sp_j\ \ ,\ \ \eta_j \geq 0 \qquad j \in S^r \tag{48}$$

$$\eta_j - \pi - \textstyle\sum_{l\in\mathcal{L}} S_l^{k(j)}(\lambda_l^+ - \lambda_l^-) \geq -sp_j\ \ ,\ \ \eta_j \geq 0 \qquad j \in S^g \tag{49}$$

$$\lambda_l^+, \lambda_l^- \geq 0 \qquad l \in \mathcal{L} \tag{38}$$

$$\pi^r \geq 0 \tag{50}$$

or

$$\min\ d^r\pi^r + \textstyle\sum_{i\in B} bq_i\mu_i\ +\ \sum_{j\in S} sq_j\eta_j + \sum_{l\in\mathcal{L}}(M_l\lambda_l^+ - m_l\lambda_l^-) \tag{47}$$

$$\pi^k = \pi + \textstyle\sum_{l\in\mathcal{L}} S_l^k(\lambda_l^+ - \lambda_l^-) \qquad k \in \mathcal{K} \tag{39}$$

$$\mu_i + \pi^{k(i)} \geq bp_i\ \ ,\ \ \mu_i \geq 0 \qquad i \in B \tag{40}$$

$$\eta_j + \pi^r - \pi^{k(j)} \geq -sp_j\ \ ,\ \ \eta_j \geq 0 \qquad j \in S^r \tag{51}$$

$$\eta_j - \pi^{k(j)} \geq -sp_j\ \ ,\ \ \eta_j \geq 0 \qquad j \in S^g \tag{52}$$

$$\lambda_l^+, \lambda_l^- \geq 0 \qquad l \in \mathcal{L} \tag{38}$$

$$\pi^r \geq 0 \tag{50}$$

Apart from the different choices of sign—here we used the one more prevalent in the literature, whereas in the previous sections the CP was written as a minimum problem as this made the analysis more intuitive—similar interpretations obviously apply. The sales are all made at the zonal price π^k of the corresponding zone, as are the purchases on the g-market. On the other hand, purchases in the r-market are made at the price $\pi^k - \pi^r$, which is surely not higher than π^k because of (50). In particular, for the complementary conditions we will have $\pi^r = 0$, i.e., the price will be equal, if d^r is choosen "too large" so that it does not reduce the quantity purchased in the r-market; conversely, the price will be strictly less than π^k (the desired result) if the constraint (17) is active at the optimal solution.

Repeating the analysis carried out in the previous section, *mutatis mutandis*, we obtain the following relation describing the economic equilibrium of the market:

$$\textstyle\sum_{i\in B} \pi^{k(i)} b_i - \sum_{j\in S^g} \pi^{k(j)} s_j + \sum_{j\in S^r}(\pi^{k(j)} - \pi^r)s_j = d^r\pi^r + \sum_{l\in\mathcal{L}}(M_l\lambda_l^+ - m_l\lambda_l^-)$$

Since d^r is understood to be greater than or equal to zero and the (50) is valid, the term on the right-hand side is still greater than or equal to zero, and therefore the market is still in economic equilibrium in the sense that the total price paid by the buyers is at least equal to the total revenue received by the sellers.

Qualitatively, however, it should be noted that in this case the difference between the two quantities is positive not by "by chance", in correspondence with zonal network constraints activations that may not occur; rather, it is precisely the sought-after effect that the term d^r be positive, and not necessarily "small". Indeed, d^r (multiplied by the positive quantity π^r) is minimised in (47), but clearly in the overall problem a "large" d^r will be sought after, as this corresponds to having more energy in the r-market and hence at a lower price. In this case, the allocation of the extra revenue resulting from the difference between the price paid by the buyers and that returned to the producers must be explicitly considered in the regulation of the proposed market, since it is not insignificant. In particular, this extra revenue, which corresponds exactly to the savings due to market segmentation, should be used to reduce the final cost of energy to buyers. This, however, creates a peculiar aspect: *the (zonal) price π^k at which bids are accepted does not correspond to the actual price paid by buyers*, which is lower. This difference should be explicitly accounted for, since one might find oneself in the case where certain bids (purchase) are not accepted because of a price lower than the "official" market price π^k, when finally the actual price paid by consumers might in fact be lower than that. The savings due to the above term should therefore be explicitly accounted for as some kind of "discount" from the market price, so as to formally justify the non-acceptance of such offers. This mechanism does not appear to be excessively onerous, but must nevertheless be considered as one of the effects of the proposed mechanisms. Note, however, that the problem is somewhat "automatically" solved when considering the Unique National Price in the Italian DAM, as discussed in §5.3.

Let then $PDSM(d^r)$ be the preceding pair of primal and dual problems: the problem to be solved is naturally written as a bilevel problem, but this time in both primal and dual space:

$$\min\ (\pi - \pi^r) \textstyle\sum_{j\in S^r} s_j + \pi \sum_{j\in S^g} s_j \tag{53}$$

$$0 \le d^r \le \textstyle\sum_{i\in B} sq_i \tag{54}$$

$$(s, \pi, \pi^r) \in \text{argmax/argmin } PDSM(d^r) \tag{55}$$

The objective function (53) can also be written as

$$(\pi - \pi^r) \textstyle\sum_{j\in S^r} s_j + \pi \sum_{j\in S^g} s_j = \pi \sum_{i\in B} b_i - \pi^r d^r \tag{56}$$

since for the first term we have $\sum_{j\ inS^r} s_j + \sum_{j\ inS^g} s_j = \sum_{j\ inS} s_j$ and for the budget constraint (33) this sum must be equal to $\sum_{i\ inB} b_i$, while for the second term the reasoning at the end of §2.2 remains true. We may decide to transform the first term if $card(S) < card(B)$, i.e., if the sell offers were numerically lower than the purchasing bids, so that there would be fewer bilinear terms and this may make the solution of the problem somewhat less onerous.

We also observe that the same objective function (56) could be written differently as the maximum of social welfare i.e. as in (57)

$$\max\ \Big(\textstyle\sum_{i\in B} bp_i b_i - \sum_{j\in S} sp_j s_j\Big) - \big(\pi \sum_{i\in B} b_i - \pi^r d^r\big) \tag{57}$$

Similarly to the "simple" case, the left and right-hand side of the constraint (54) can be improved, but now this requires also considering the elasticity of demand and hence more complex formulae. Unlike the "simple" case, in which the quantity terms in the objective function used the rigid demand d in addition to d^r (a variable of the "leader" level of the problem), in this case because of the elasticity of demand it is necessary to use explicitly the quantity variables s of the "follower" level of the problem. However, this does not change the complexity of the problem by much. In fact, as we shall see below, the classical reformulations still rewrite $PDSM(d^r)$ as explicit variables subject to (reformulations of) the complementary conditions; in such reformulations the variables s are as "accessible" as the variables π and π^r.

We observe that the objective function (53) is non-convex, and that it has a larger number of bilinear terms than in the simple case. This can only make the problem more complex to solve, but how relevant this is in practice can only be determined through computational experiments. On the other hand, the reformulation of the objective function from (53) to (56) is believed to generate a favourable computational impact.

Whatever the chosen formulation, it is possible to solve (53)–(55) by means of standard methodologies based on the use of Mixed-Integer Programming (MIP) general-purpose solvers.

Finally, it is time to discuss the announced result that the SPaC approach can be extend to partitioning the market into more than two distinct sub-markets. For example, if we wanted to introduce a third market S^c disjoint from the two previous ones ($S^r \cap S^c = \emptyset$, $S^g \cap S^c = \emptyset$), it would suffice to introduce in the market problem with elastic demand and network constraints a double segmentation of the offer: that is, in addition to the constraint (17), also one of the type:

$$\textstyle\sum_{j \in S^c} s_j \leq d^c$$

where again d^c is a parameter, i.e., a variable in the "leader" problem. This would simply lead to the creation of a dual variable $\pi^c \geq 0$ which would enter the constraints on the selling price of offers in S^c in a completely analogous way to that already seen. It would also suffice to modify the objective function as a sum of three terms. Of course the increase of variables in the "leader" level may lead to an increase in computational cost, but a "small" number of markets should have a cost comparable to the case under consideration.

In the next section we are ready to present the last extension: keeping the two markets S^r and S^g, we introduce the treatment of the PUN (Unique National Price). This is possible with the techniques already substantially described, at the cost, however, of developing the formulation of the $PDSM(d^r)$.

5.3 The SPaC case with network constraints, elastic demand and PUN

Based on the ideas developed above, it is therefore possible to propose an overall MPCC (Mathematical Program with Complementary Constraints) formulation for the problem, including the PUN:

$$
\begin{array}{lrr}
\min \pi \sum_{i\in B} b_i - \pi^r d^r & & (56) \\
\quad 0 \le d^r \le \sum_{i\in B} sq_i & & (54) \\
\quad 0 \le s_j \le sq_j & j \in S & (32) \\
\quad \sum_{j\in S^r} s_j \le d^r & & (17) \\
\quad 0 \le b_i \le bq_i & i \in B & (31) \\
\quad \sum_{j\in S} s_j = \sum_{i\in B} b_i & & (33) \\
\quad m_l \le \sum_{k\in\mathcal{K}} S_l^k \left(\sum_{i\in I(k)} b_i - \sum_{j\in J(k)} s_j\right) \le M_l & l \in \mathcal{L} & (34) \\
\quad \pi^k = \pi + \sum_{l\in\mathcal{L}} S_l^k(\lambda_l^+ - \lambda_l^-) & k \in \mathcal{K} & (39) \\
\quad \mu_i + \pi^{PUN} \ge bp_i \ , \ \ \mu_i \ge 0 & i \in B & (58) \\
\quad \eta_j + \pi^r - \pi^{k(j)} \ge -sp_j \ , \ \ \eta_j \ge 0 & j \in S^r & (51) \\
\quad \eta_j - \pi^{k(j)} \ge -sp_j \ , \ \ \eta_j \ge 0 & j \in S^g & (52) \\
\quad \sum_{i\in B}(bp_i b_i - \mu_i bq_i) - \sum_{j\in S}(\eta_j sq_j + sp_j s_j) \ge \sum_{l\in\mathcal{L}}(M_l\lambda_l^+ - m_l\lambda_l^-) & & (59) \\
\quad \lambda_l^+, \lambda_l^- \ge 0 & l \in \mathcal{L} & (38) \\
\quad \pi^r(d^r - \sum_{j\in S^r} s_j) = 0 & & (60) \\
\quad \pi^r \ge 0 & & (50) \\
\quad \eta_j(sq_j - s_j) = 0 & j \in S & (42) \\
\quad \mu_i(bq_j - b_j) = 0 & j \in S & (43) \\
\quad (\eta_j + \pi^r - \pi^{k(j)} + sp_j)s_j = 0 & j \in S^r & (61) \\
\quad (\eta_j - \pi^{k(j)} + sp_j)s_j = 0 & j \in S^g & (62) \\
\quad (\mu_i + \pi^{PUN} - bp_i)b_i = 0 & i \in B & (63) \\
\quad \lambda_l^- \left(\sum_{k\in\mathcal{K}} S_l^k \left(\sum_{i\in I(k)} b_i - \sum_{j\in J(k)} s_j\right) - m_l\right) = 0 & l \in \mathcal{L} & (64) \\
\quad \lambda_l^+ \left(M_l - \sum_{k\in\mathcal{K}} S_l^k \left(\sum_{i\in I(k)} b_i - \sum_{j\in J(k)} s_j\right)\right) = 0 & l \in \mathcal{L} & (65) \\
\quad \lambda_l^+, \lambda_l^- \ge 0 & l \in \mathcal{L} & (38)
\end{array}
$$

The model combines in an "obvious" manner all of the elements outlined above, including the crucial (17) market segmentation and network constraints. The substantial change concerns purchases, which are all made at a unique price, π^{PUN}, irrespective of the zone in which they take place, as required by the Italian regulation. This brings the need for the constraint (59) concerning the economic equilibrium of the market, which deserves explicit justification.

We define for simplicity of notation π^* the price at which a purchase or sale is made: in particular we have:

$$
\begin{array}{ll}
\pi_j^* = \pi^{PUN} & j \in B \\
\pi_j^* = \pi^{k(j)} & j \in S^g \\
\pi_j^* = \pi^{k(j)} - \pi^r & j \in S^r
\end{array}
$$

As in the analysis above, the problem's constraints on the complementary conditions give

$$\eta_j sq_j = \eta_j s_j \ , \ \mu_i bq_i = \mu_i b_i \ , \ (\eta_j + sp_j) s_j = \pi_j^* s_j \ , \ \pi_i^* b_i = (bp_i - \mu_i) b_i \tag{66}$$

Using the last two in (66) with an appropriate algebraic sum we obtain

$$\textstyle\sum_{i \in B} \pi_i^* b_i \ - \ \sum_{j \in S} \pi_j^* s_j = \sum_{i \in B} (bp_i - \mu_i) b_i \ - \ \sum_{j \in S} (\eta_j + sp_j) s_j \ \ .$$

Using the first two in (66) we can obtain

$$\textstyle\sum_{i \in B} \pi_i^* b_i \ - \ \sum_{j \in S} \pi_j^* s_j = \sum_{i \in B} (bp_i b_i - \mu_i bq_i) \ - \ \sum_{j \in S} (\eta_j sq_j + sp_j s_j)$$

which justifies (59). In fact, that implies that the right-hand side in the above equation is greater than or equal to zero, which obviously implies that the left-hand side is too, which is exactly the difference between the total price paid by the buyers and the total revenue received by the sellers. Therefore, the system is in economic equilibrium as defined above. In fact, it is conceptually possible to choose between different forms of the constraint (59), among which the most "lax" is simply

$$\textstyle\sum_{i \in B} (bp_i b_i - \mu_i bq_i) - \sum_{j \in S} (\eta_j sq_j + sp_j s_j) \geq 0$$

i.e., exactly the minimum requirement for the market to be in economic equilibrium. In (59) it is chosen instead to keep the term $\sum_{l \in \mathcal{L}} (M_l \lambda_l^+ - m_l \lambda_l^-)$ relating to the TSO's remuneration for uniformity with the current situation. It would also be possible to add the term $\pi^r d^r$ to the right-hand side similarly to what was done in §5.2, but this would entail the problem of differentiating the official market purchase price from the actual price due to a redistribution proportional to the factor $\pi^r d^r$, as discussed in §5.2. Intuitively, this does not seem reasonable. Rather, it would be appropriate to adapt this solution to the case of the market without PUN, if this were the one to be implemented. An important advantage of the chosen form (59) is that it is a simple *linear* constraint, whereas the natural form would contain products of variables and would thus be both non-linear and non-convex.

Apart from this, all the other components of the problem are justified by the analysis above. Therefore, this problem allows the definition of market prices—divided by zones and segmented into two buy-side markets, one sell-side market—that fulfil all the required conditions and for which the choice of the segmentation parameter/variable d^r minimises the system cost (53), possibly in its reformulation (56).

For the rest, (56)–(38) is an MPCC that can be addressed by special techniques, including the rewriting of complementarity constraints in the generic form

$$xy = 0 \qquad \text{with} \qquad 0 \leq x \leq M \ , \ 0 \leq y \leq N$$

with linear Big-M reformulations as follows

$$0 \leq x \leq Mu \ , \ 0 \leq y \leq N(1-u) \ , \ u \in \{0,1\}$$

thus obtaining a MILP reformulation of the constraints. From a computational point of view it is critical to define as small values as possible for M and N, which is often difficult especially for dual variables, but should always be possible in our case with a suitable analysis.

The objective function (56) still remains bilinear and non-convex, but nevertheless the reformulated problem remains within reach of modern general-purpose solvers of mixed integer problems. We again refer to, e.g., the book [3] and recent survey [6], and references therein, for the several modelling and algorithmic approaches available for solving optimization problems of the kind. Finally, the analysis given here can also be extended to an arbitrary but "small" number, at least for computational reasons, of disjoint segmented markets.

6 Conclusions and future developments

Our analysis allow us to draw the following conclusions:

1. We have proposed the innovative SPaC (Segmented Price-as-Clear) general concept that allows to decouple sellers (and likely buyers as well, with basically the same approach, although we have not formalised this yet) in a market into an arbitrary number of disjoint subsets. Therefore sellers in the same subset "primarily compete only with their peers" while still contributing to satisfying the global demand. This is done by means of an optimal and transparent segmentation of the demand curve into the separate but simultaneous markets, which are always cleared according to the standard PaC model.

2. We have proposed various formulations with increasing complexity of the corresponding Clearing Problems, showing that the SPaC approach can be easily applied to many of the standard constructs found in practical DAM, such as elastic demand, zonal constraints and the PUN as seen by the Italian market. Since the SPaC approach basically relies on introducing *one single new constraint* (for each segmented market) in the PaC CP, and then writing a bilevel program in which the right-hand side of that constraint is a "leader" variable, the approach can clearly be applied to many other constructs for which the PaC CP can currently be solver by standard MILP approaches, such as complex products (e.g., block offers or MICs) in multi-period formulations as envisaged by other European markets.

3. The proposed CP are more onerous to solve than the simple LPs corresponding to the classical PaC market, but likely still within reach of even current general-purpose MIP tools, even more so of the plethora of specialised approaches developed for Bilevel Programs and Mathematical Programs with Complementary Constraints. In fact, the SPaC CP is not fundamentally more complex of other CP that are currently in practical use already, such as those taking into account the PUN concept.

4. The theoretical analysis of §3 an provides important insights regard to the possibility of sellers employing strategic bidding techniques that highlights both the potential advantages and the possible limits of the proposed model.

5. An Agent Based algorithm was designed and implemented to simulate price dynamics with reasonable logic. Numerical simulations confirm that the SPaC model can indeed provide significant savings w.r.t. the classical PaC one, to the tune of average reductions of system cost between about 20% and 30%, except in "extreme" conditions where

demand is low (and prices are therefore low already). This also directly confirm that the SPaC approach preserves the fundamental short-term price signal whereby the price dynamically reacts to changes in demand, unlike most approaches based on price caps. However, the SPaC approach can easily be combined with, perhaps flexible, price caps, which may yield further reductions in system cost.

Possible future developments are as follows:

1. On the modelling side, studying preprocessing algorithms to define correct and tighter values to be used in defining, e.g., big-M formulations of the complementary constraints involving the π and π^r variables. This may allow to substantially reduce computing times, that for large test cases with thousands of offers could be non-negligible.

2. On the simulation side, improve or modify the Agent-Based logic to provide more accurate simulation of actual market participants, e.g., by:

    - incorporating a profit-risk component on the part of the offering agents, possibly by defining different agents with different risk aversion;
    - introducing a more detailed memory for the offering agent regarding acceptances and rejections of offers, possibly employing Q-learning (average learning) approaches often used in simulations for electricity markets;

3. Study the impact on the total cost savings of "hybridising" the SPaC approach with other proposed "decoupling-oriented" techniques, like

    - adding (both in SPaC and PaC) caps to the SNMC offers, possibly dynamically set depending on the offers in the SNNMC market;
    - running (both SPaC and PaC) after a preliminary (mandatory) auction for two-ways contracts for differences (CfD) is ran on SNMC, possibly with different types of products.

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