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Fair Least Core: efficient, stable and unique game-theoretic reward allocation in Energy Communities by row-generation

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***Abstract*—Energy Communities are increasingly proposed as a tool to boost renewable penetration and maximize citizen participation in energy matters. These policies enable the formation of legal entities that bring together power system members, enabling collective investment and operation of energy assets. However, designing appropriate reward schemes is crucial to fairly incentivize individuals to join, as well to ensure collaborative and stable aggregation, maximizing community benefits. Cooperative Game Theory, emphasizing coordination among members, has been extensively proposed for ECs and microgrids; however, it is still perceived as obscure and difficult to compute due to its exponential computational requirements. This study proposes a novel framework for stable fair benefit allocation, named Fair Least Core, that provides uniqueness, replicability, stability and fairness. A novel row-generation algorithm is also proposed that allows to efficiently compute the imputations for coalitions of practical size. A case study of ECs with up to 50 members demonstrates the stability, reproducibility, fairness and efficiency properties of proposed model. The results also highlight how the market power of individual users changes as the community grows larger, which can steer the development of practical reliable, robust and fair reward allocations for energy system applications.**

***Index Terms*—Energy Community, game theory, Fair Least Core, EnergyCommunity.jl, Mixed-Integer Linear Programming (MILP), coalition fairness and stability**

I. Introduction

A. Motivation

SEVERAL governments worldwide [1], [2] are promoting Energy Communities (EC) as a mean to stimulate investments in renewable assets and increase citizenship participation in energy matters. New policies enable the creation of a legal entity, called "Energy Community", that aggregates households, companies and public institutions as members. ECs are entitled to own and operate energy assets, and promote the coordination of demand and supply among the members exploiting them [2]. Hence, there is a pressing need for their optimal design, taking into account suitable reward schemes to incentivize member participation.

To maximize collective benefits [3], Cooperative Game Theory has been extensively proposed, also in the field of Energy Communities and microgrids. The Shapley Value has been widely considered the reference indicator for fairness [4], but suffers from stability concerns [5]. On the other hand, the Core and Nucleolus techniques ensure stable allocations [6], but not necessarily fair ones. Both approaches are, in general, costly from the computational viewpoint, so that their use in practice may be challenging. Recent indicators based on convex measures such as the variance, combined with stability-enforcing methods, have shown promising results to achieve fair and stable allocations; yet, computational burden is still a major concern and uniqueness is not guaranteed [6], [7].

This study proposes novel algorithmic procedures and efficient implementation techniques to plan the proper design of Energy Communities and guarantee fair and stable reward allocation within them.

B. Design of Energy Communities and aggregators

In recent times, governments worldwide have introduced supportive policies for renewable energy communities [8]. Their main target is the promotion of no-profit social, environmental, and economic targets [3], while meeting the technical challenges that the energy transition is demanding. Beyond fostering decarbonization, these initiatives have yielded broad benefits for power systems, including enhanced reserve provisions [9], reduced grid congestions [10], increased renewable penetration [11], and social welfare improvement [2]. However, to fully realize these advantages, effective coordination among assets, consumers, and prosumers is essential. This responsibility falls on aggregators, tasked with implementing efficient planning and operation, as well as defining incentive mechanisms that promote community goals and cohesion.

Traditionally, aggregators are for-profit entities that monitor and manage the energy system on behalf of consumers and prosumers. As per EU regulation, aggregators cannot be ECs themselves, given their for-profit nature [3]. However, they can have a support role in its creation, management and operation; for this reason, they can be regarded as a player in the EC and, as such, they shall be rewarded appropriately, also not to incur in the so-called agency problem [6]. Previous studies have

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primarily emphasized economically-driven techniques devoted to optimally operate the aggregate [12], using Mixed-Integer Linear Programming (MILP). Some of them have explored maximizing the aggregate's social welfare, but may have overlooked the fair distribution of profits among participants [13], [14]. In particular, the fair stable reward of aggregators—a critical but complex topic—has been rarely considered in the literature [6]. For these reasons, in this study we develop a MILP planning model able to account for the role of members, including the aggregator, and their fair and stable reward.

C. Competitive and Cooperative reward allocations

In the context of local energy markets and ECs, competitive or cooperative incentive mechanisms are commonly employed. In competitive approaches, users operate independently to maximize their individual benefits, potentially competing with each other for scarce common resources [14]. In this case, there is no guarantee that the solution maximizes the utility of the aggregate, thus competition can be detrimental. Non-cooperative strategies, such as those proposed in [15] and [16], focus on optimizing the actions of individual aggregators or users in a local energy market or network. Nash's theory is widely adopted in this context to identify the market equilibrium [17], [18]. Even if competition may suit some scenarios, cooperation can be limited, thus potentially leading to sub-optimal results, which can oppose the social goal desirable by policies, such as the EU regulation [8].

In typical ECs, users are less likely to individually perform active trading, and therefore cooperative approaches are particularly relevant [6], [11]. In these approaches users cooperate towards the best outcome for the entire community and distribute rewards according to each individual contribution [14], with no detrimental effect on the global benefit. The Shapley Value is generally considered the reference indicator for fair reward sharing [4], [19]. However, it suffers from stability issues, i.e., there is no guarantee that no subset of users is better off from leaving the community [20], [21], as proven in [6] in the context of ECs. The set of reward allocations (a.k.a., imputations) that guarantee stability is named "Core", and is typically not a singleton. However, imputations within the Core may be marginally stable, i.e., a subset may be equally better off inside or outside the community. For these reasons, the stricter formulations of the "Least Core" [22] and "Nucleolus" [23] have been proposed: the former maximizes the benefit of the coalition that is most likely to exit the community, whereas the latter iteratively applies the same concept to each most likely coalition to leave. Conversely to Core and Least Core, Nucleolus is proven to be unique [24], which is a desirable property.

D. Computational challenges of game-theoretic allocations

Despite its benefits, the combinatorial nature of cooperative game theory is a significant barrier to its practical use. In case studies involving a small number of members, their enumerative formulation can be used [6], [25], but with communities exceeding 20-30 members the computational requirements quickly become prohibitive. In [26] an approximation for the Shapley Value has been proposed that reduces the combinations by about 99%; yet, concerns on stability still apply. Nucleolus and Least Core have been used in various studies but only for system operation [21], [27], with no application to ECs. One of the few exceptions is [6], but the computational approach used there does not scale efficiently with size. A decomposition algorithm of Nucleolus based on Benders' decomposition is proposed in [21], but it is not applicable in the EC field given the intrinsic binary nature of membership of each user to the EC. To overcome that, a simplification of Nucleolus has been proposed using a pure Variance equivalence [7], but stability concerns were overlooked. For these reasons, in [6] a methodology is proposed to stabilize imputation; yet, the approach is still combinatorial and limited to few members. An alternative solution is offered by the Owen sharing method [22] that distributes the reward based on the equivalent market prices created by the dual solution of the optimization problem for the bidding of wind generators. However, while being simple to calculate, the Owen solution may not achieve desirable properties such as these of Least Core, Nucleolus or Shapley Value [28]. Row-generation has been shown to be a promising approach for decomposing Nucleolus-like formulations [29], among other problems [30], but it has not been applied to EC. For these reasons, it is considered in this study and combined with Core, Least Core and Variance mechanisms.

E. Contributions and organization of the paper

The main contributions of our work are as follows:

1) definition of generalized reward allocation schemes, named Fair Core and Fair Least Core, that aim at maximizing fairness and stability of reward allocations;
2) uniqueness for Fair Core and Fair Least Core;
3) novel algorithm to efficiently calculate reward allocation methods for ECs, including row-generation and smart decomposition of the EC planning problem;
4) application and validation of the algorithm to several reward allocation mechanisms and comparison with existing methodologies, e.g. Shapley Value and Nucleolus;
5) evaluation of the impact of EC size into the fair reward allocation to provide policy recommendations;
6) open-source implementation of the methodologies in EnergyCommunity.jl [31] and TheoryOfGames.jl [32].

The remainder of the paper is organized as follows. Section II describes the EC and its mathematical optimization problem. Section III reviews the literature about reward allocation by game theory. Section IV details the general fair stable reward allocation proposed in this study, whose efficient computation is detailed in Section V and Section VI. The case study and results are reported in Section VII and VIII, respectively. Finally, conclusions are drawn.

II. The Energy Community planning

A. Business model

According to the literature [6] and the European Union regulation [1], an EC operates as a non-profit entity, sharing

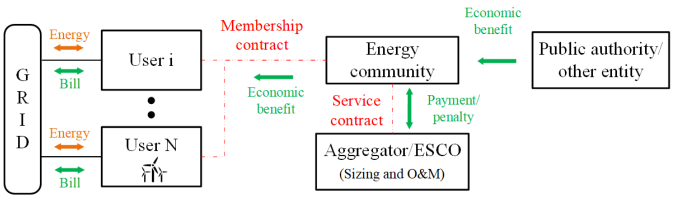

Fig. 1. Business model of the Energy Community

all revenues among its members after fulfilling its obligations; technical support by third-parties, e.g. aggregators, is admitted. Accordingly, this study focuses on the business model depicted in Fig. 1, where members create the legal entity Energy Community and can engage with an aggregator to maximize the overall benefits. An EC coordinated by an aggregator is denoted with "CO". Without the aggregator, the community is still able to create an EC, referred to as Aggregated Non-Cooperative (ANC), but it cannot coordinate consumption and production nor the investments to achieve the maximum economic performance. The EC is awarded an economic benefit for every unit of energy that is produced by a user and virtually consumed by another user in the same time step [33]. Each user can invest in renewable assets or storage and owns such devices. In the CO configuration, users collaborate to maximize the overall benefit measured with Net Present Value (NPV) [34], and they shall be remunerated fairly. Finally, in cases where no EC is established, referred to as the Non-Cooperative (NC) configuration, users invest in local decentralized resources to maximize their own profits. This scenario serves as the baseline for the analysis.

We now present reasonably complete, although somewhat stylised, mathematical models for EC planning and operations that underpin our development and will be used in the final computational study. We remark that more sophisticated EC models (including, e.g., other generation units with more complex operational constraints, multi-energy aspects, and even explicit representation of stochastic aspects) could be used without significantly impacting the proposed approach.

B. Users' objective

When no EC is established, the objective of each user j is to maximize its own NPV, reported in (1), composed of the net profit for selling/buying electricity to/from the market (R_y) for each year $y \in Y$, the investment costs (I_y) that are non-null only at the first year ($y = 0$), the operating charges due to peak tariffs and maintenance charges (OP_y), the replacement costs of the assets (RP_y) and the recovery value (RV_y), which is non-null only at the end of the project. Net economic flows with the energy market are modelled in (2), accounting for selling price (π_t^+), buying price (π_t^-), including grid tariffs and taxes, and excise (π_t^{ex}) for each time step $t \in T$ having weight m_t^T, which accounts for granularity and number of representative days; $P_t^{U\pm}$ denotes the power exchanged at the Point of Delivery (POD), where positive apex stands for injection into the distribution grid; P_t^L is the consumer demand. According to (4), for every tariff horizon $w \in W$ (for instance in Italy corresponding to a month), the peak charges OP_y are dynamically accounted for considering the peak tariff c_w^P and the actual maximum power exchanged P_w^{Umax} at the POD. Yearly maintenance costs OP_y, represented by the second term of (4), are proportional to the investment capacity x^a of each asset a of the set of assets A_j of user j, according to a coefficient $c^{a,M}$. Replacement charges RP_y detailed in (5) apply when an asset reaches its end of life $N^{Y,a}$, while the residual value of assets is recovered as described in (6). r represents the discount rate.

$$NPV_j = \sum_{y \in Y} \frac{R_{j,y} - I_{j,y} - OP_{j,y} - RP_{j,y} + RV_{j,y}}{(1+r)^y} \quad (1)$$

$$R_{j,y} = \sum_t m_t^T \left(\pi_{j,t}^+ P_{j,t}^{U+} - \pi_{j,t}^- P_{j,t}^{U-} - \pi_{j,t}^{ex} P_{j,t}^L\right) \quad (2)$$

$$I_{j,0} = \sum_a c_j^{a,I} x_j^a \quad (3)$$

$$OP_{j,y} = \sum_w m_w^W c_{j,w}^P P_{j,w}^{Umax} + \sum_{a \in A_j} x_j^a c_j^{a,M} \quad (4)$$

$$RP_{j,y} = \begin{cases} \sum_{a \in A_j} x_j^a c_j^{a,I} & \text{if} \quad \bmod (y, N_j^{Y,a}) = 0 \\ 0 & \text{else} \end{cases} \quad (5)$$

$$RV_{j,|Y|} = \sum_{a \in A_j} x_j^a c_j^{a,I} \frac{N_j^{Y,a} - \bmod (|Y|-1, N_j^{Y,a})}{N_j^{Y,a}} \quad (6)$$

C. Constraints

This section details the major technical constraints of each user. The power balance within each internal system is ensured through (7), where $P_{j,t}^{U\pm}$ denotes the power dispatch at the user's POD, $P_{j,t}^{c\pm}$ represents the power dispatch of the battery converter (with $+$ indicating supply and $-$ indicating absorption), $P_{j,t}^R$ corresponds to the renewable production, and $P_{j,t}^L$ is the demand. A_j^C denotes the converters of user j.

$$P_{j,t}^{U+} - P_{j,t}^{U-} + \sum_{c \in A_j^C} \left[P_{j,t}^{c-} - P_{j,t}^{c+}\right] - P_{j,t}^R = -P_{j,t}^L \quad \forall t \quad (7)$$

The peak power at the user POD is calculated with (8), where $\hat{T}_w$ denotes the set of time steps corresponding to the peak power period $w \in W$. Constraint (9) specifies the maximum renewable power dispatch available at every time step for every user; $x_j^{r,U}$ represents the installed capacity of the renewable asset r and $p_{j,t}^{r,U}$ is its specific power production. A_j^R denotes the renewable assets of user j.

$$P_{j,w}^{Umax} \geq \max\left\{P_{j,\hat{t}}^{U+}, P_{j,\hat{t}}^{U-}\right\} \qquad \forall w, \; \hat{t} \in \hat{T}_w \quad (8)$$

$$P_{j,t}^R \leq \sum_{r \in A_j^R} p_{j,t}^{r,U} x_j^{r,U} \qquad \forall t \quad (9)$$

The energy balance of the batteries is modeled using (10), employing cyclical notation ($E_{j,0}^{b,U} = E_{j,|T|}^{b,U}$); equations account for the roundtrip efficiency η_j^b of the battery b, including its corresponding converter $c = c(b) \in A_j^C$, belonging to the set A_j^B. The peak power capacity is ensured by (11), while the maximum and minimum allowed state of charge are taken into account in (12) using coefficients $\beta_j^{b,max}$ and $\beta_j^{b,min}$. The variables x_j^b and $x_j^{c(b),U}$ represent the rated energy capacity of battery b and the power capacity of the corresponding converter, respectively.

$$E_{j,t}^b = E_{j,t-1}^b - \Delta P_{j,t}^{c(b)+} / \sqrt{\eta_j^b} + \Delta P_{j,t}^{c(b)-} \sqrt{\eta_j^b} \quad \forall b, t \quad (10)$$

$$P_{j,t}^{c\pm} \leq x_j^{c,U} \qquad \forall c,t \quad (11)$$
$$x_j^b \beta_j^{b,min} \leq E_{j,t}^b \leq x_j^b \beta_j^{b,max} \qquad \forall b,t \quad (12)$$

D. Energy Community objective and shared energy

In a Cooperative Energy Community, the overall goal is to maximize the so-called social welfare $SW^{CO}(K)$ of the community K, which includes the NPV of each member j and the total reward R_y^{SH} allocated to the community, as detailed in (13). The total reward annually awarded to an EC is formulated in (14), where π_t^{SH} is the regulated unitary reward and P_t^{SH} is the shared energy virtually net-metered. P_t^{SH} is defined as the minimum between the overall production and consumption, as modelled in (15).

$$SW^{CO}(K) = \sum_{j\in K} NPV_j + \sum_{y\in Y} \frac{R_y^{SH}}{(1+r)^y} \quad (13)$$
$$R_y^{SH} = \sum_t \pi_t^{SH} m_t^T P_t^{SH} \quad (14)$$
$$P_t^{SH} = \min\left\{\sum_{j\in K} P_{j,t}^{U+},\ \sum_{j\in K} P_{j,t}^{U-}\right\} \qquad \forall t \quad (15)$$

E. Energy Community problems

1) Coordinated EC problem (CO): In abstract terms, let u_j be the operation ($P_{j,t}^{U\pm}$, $P_{j,w}^{Umax}$, $P_{j,t}^{R}$, $P_{j,t}^{c\pm}$, $E_{j,t}^b$) and investment variables ($x_{j,t}^{a,U}$) of each user j, and s the power shared in an EC. The mathematical problem for the coordinated EC is shown in (16), where matrix M_j and vector b_j denote the constraints in Section II-C, while constants c_j and l_j represent the cost coefficients discussed in Section II-B. The shared power s is constrained to be lower than or equal to the total energy production and consumption, by using matrices $D^{\pm}$, through the identity $P_j^{U\pm} = D^{\pm} u_j$; $\delta > 0$ represents the weighted reward for every unit of shared power.

$$\begin{aligned} SW^{CO}(K) = \quad & \max \quad \textstyle\sum_{j\in K}(c_j^T u_j + l_j) + \delta^T s \\ & \text{s.t.} \quad M_j u_j \leq b_j \qquad \forall j \in K \\ & \qquad\ \ s \leq \textstyle\sum_{j\in K} D^+ u_j \\ & \qquad\ \ s \leq \textstyle\sum_{j\in K} D^- u_j \end{aligned} \quad (16)$$

This formulation turns out to be useful in the discussion of the other EC problems described below.

2) Non-Coordinated users problem (NC): As discussed in Section II-B, in this case each user maximizes its own profitability regardless of the others. Let $SW^{NC}(K)$ be the optimal objective function of the optimization of the whole community with no user interaction, as in (17). No shared energy applies and hence no coordination is incentivized.

$$SW^{NC}(K) = \sum_{j\in K} \begin{array}{ll} \max & c_j^T u_j + l_j \\ \text{s.t.} & M_j u_j \leq b_j \end{array} \quad (17)$$

It is worth noticing that the problem in (17) is similar to (16), but no shared energy applies. That indeed leads each user problem to be independent.

3) Aggregated-Non-Coordinated EC problem (ANC): Finally, we consider the so-called Aggregated-Non-Coordinated EC problem, where users create an EC, but no aggregator is present to coordinate the operation of the system, nor to recommend coordinated investments to the users. In this case, the users are expected to behave as in the NC problem, but also benefit from the (probably low) shared energy corresponding to the non-coordinated system operation. Let $\overline{u}_j^{NC}$ be the optimal decision vector of user j in the NC problem, then the overall objective function of the whole community under ANC conditions can be described as in (18):

$$\begin{aligned} SW^{ANC}(K) = SW^{NC}(K) + & \max\ \delta^T s \\ \text{s.t.} \quad & s \leq \textstyle\sum_{j\in K} D^+ \overline{u}_j^{NC} \\ & s \leq \textstyle\sum_{j\in K} D^- \overline{u}_j^{NC} \end{aligned} \quad (18)$$

It is worth noticing that the problem in (18) is similar to (16), but the decision variables u_j are set to the NC optimal solution. Accordingly, users constraints ($M_j \overline{u}_j^{NC} \leq b_j$) are satisfied by definition of $\overline{u}_j^{NC}$, and hence excluded from the optimization.

III. Game-theoretic reward allocation

A. Preliminary definitions: benefit and surplus of a coalition

A cooperative game with transferable utility can be devised to reward the participants in the EC. The set of players I is made by the set J of users that may join the community and the aggregator A. The characteristic function v measures the common benefit of the possible ECs between the players who agree to join it eventually including the aggregator. Each user can always choose its own NC optimal solution, therefore this is considered as the base case configuration. When the Aggregator A participates, the CO optimal solution can be achieved and the corresponding benefit is the difference between the optimal performance of CO and NC configurations; otherwise, no coordination is created and the benefit of the community is limited to the difference between the optimal ANC and NC configurations.

The mathematical expression of the characteristic function for any coalition $K \subseteq I$ is given by

$$v(K) = \begin{cases} SW^{CO}(K_A) - SW^{NC}(K_A) & \text{if } A \in K, \\ SW^{ANC}(K) - SW^{NC}(K) & \text{if } A \notin K, \end{cases} \quad (19)$$

where $K_A = K \setminus \{A\}$.

To identify the improvement of benefit for each user or aggregator in the presence of the community, we consider the set

$$\mathcal{B} = \left\{\Delta \in \mathbb{R}_+^{|I|} :\ \sum_{i\in I} \Delta_i = v(I)\right\} \quad (20)$$

which describes the possible ways the overall improvement $v(I)$ is shared between them. Once an allocation $\Delta \in \mathcal{B}$ is chosen, the improved NPV of each user j with respect to the base case (NC) is given by

$$NPV_j^F = NPV_j^{NC} + \Delta_j. \quad (21)$$

For ease of presentation, we introduce the concept of surplus $\sigma(K,\Delta)$ of a coalition $K \subseteq I$ with respect to allocation Δ as

$$\sigma(K,\Delta) = \sum_{i\in K} \Delta_i - v(K). \quad (22)$$

When $\sigma(K,\Delta)$ is positive, the users are better off within the community rather than being on their own.

B. Core

The *Core* [35] is the set of reward allocations that guarantees that no coalition of the whole community I is worse off within the community than outside, i.e.,

$$\mathcal{C}(I,v) = \{\Delta \in \mathcal{B} : \ \sigma(K,\Delta) \geq 0 \quad \forall K \in \mathcal{P}\}, \qquad (23)$$

where $\mathcal{P} = \{K \subset I : \ K \neq \emptyset\}$ is the set of proper subsets of I. This property ensures the stability of the coalition, in that no user is expected to benefit from leaving the community. As it is defined by a finite number of linear inequalities, $\mathcal{C}(I,v)$ is a polytope and may contain uncountably many allocations.

C. Least Core

The *Least Core* [36] is the set of allocations that maximize the benefit for the least profitable coalition, i.e.,

$$\mathcal{LC}(I,v) = \left\{\Delta \in \mathcal{B} : \ \sigma(K,\Delta) \geq \theta^{LC} \quad \forall K \in \mathcal{P}\right\}, \quad (24)$$

where

$$\begin{aligned} \theta^{LC} = \quad & \max \ \theta \\ & \text{s.t.} \quad \sigma(K,\Delta) \geq \theta \quad \forall K \in \mathcal{P} \\ & \qquad \Delta \in \mathcal{B} \end{aligned} \qquad (25)$$

While the Core might be empty, the Least Core is always nonempty. In particular, if $\theta^{LC} < 0$ then the Core is empty. Otherwise, if $\theta^{LC} > 0$ the Least Core is a proper subset of the Core, while they coincide whenever $\theta^{LC} = 0$. Clearly, the computational burden of $\mathcal{LC}$ is equivalent to the Core.

D. Nucleolus

Given any allocation Δ, let $\psi(\Delta)$ be the order vector of satisfaction, i.e., the vector of surpluses arranged in non-decreasing order. The Nucleolus [24] is the unique allocation that lexicographically maximizes the vector ψ. In comparison with core and least core, Nucleolus is computationally harder to compute. Indeed, the computation of θ^{LC} is just the first step of the lexicographic maximization of ψ.

E. Shapley Value

The *Shapley Value* is the only allocation that jointly satisfies efficiency, symmetry, dummy, and linearity properties [37]. The allocation of each player $i \in I$ is the weighted average of its marginal contribution to every coalition:

$$\Delta_i^{SV} = \frac{1}{|I|} \sum_{K \subseteq I} \binom{|I|-1}{|K|}^{-1} [\, v(K) - v(K \setminus \{i\})\,]. \quad (26)$$

The Shapley Value may not belong to the Core and it is as computationally intensive as the Core calculation.

F. Variance Core and Variance Least Core

In order to select an allocation in the Core or Least Core, [6] proposed to minimize the squared distance from the uniform allocation. The corresponding unique minima

$$\Delta^{VC} = \arg\min \left\{ \sum_{i \in I} \left[\Delta_i - \frac{v(I)}{|I|} \right]^2 : \Delta \in \mathcal{C}(I,v) \right\} \quad (27)$$

$$\Delta^{VLC} = \arg\min \left\{ \sum_{i \in I} \left[\Delta_i - \frac{v(I)}{|I|} \right]^2 : \Delta \in \mathcal{LC}(I,v) \right\} \quad (28)$$

have been named *Variance Core* (VC) and *Variance Least Core* (VLC). In the authors' opinion this approach appears promising; in the following, we generalize its formulation also including proof of uniqueness.

G. Computational concerns

The common computational issues involved in the reward distributions schemes described above stem from the need of computing the value of $v(K)$ for every subset $K \subseteq I$. This involves solving a number of optimization problems, described in Section II-E, that is exponential in the size $|I|$ of the community. Consequently, these models can hardly be used, with a naïve computation approach, for problems larger than 10-20 users [6]; this has so far limited the use of game theoretical approaches in ECs and in the power systems field.

The complexity issue clears comes from the fact that the above formulations involve a number of variables that grows linearly with the size of the community I, but a number of constraints that is exponentially in the number of coalitions K, i.e., of the order of $2^{|I|}$. Yet, it is well-known that in such a case only a small fraction of the constraints are going to be binding, i.e., that there exists a formulation with a manageable number of constraints—corresponding to a small, well-chosen set of coalitions—that is in fact equivalent to the full one. The issue is that this set is not known in advance: however, *row-generation* approaches have proven able to efficiently solve problems of this type, provided that a proper *separation oracle* can be developed to efficiently identify constraints (coalitions) violated by a given solution. In the following, we show how this can be done for a large class of practical EC models, thereby allowing the actual use of game-theoretic concepts for community of the scale required by practical applications.

IV. Fair Core and Fair Least Core

A. Definition

Different measures of fairness rather than variance can be considered. Therefore, we propose the general *Fair Core* (FC) and *Fair Least Core* (FLC) reward allocation schemes in the same fashion, by maximizing a generic strictly concave function f that measures the fairness of allocation Δ over the Core or Least Core:

$$\Delta^{FC} = \arg\max \{ f(\Delta) : \ \Delta \in \mathcal{C}(I,v) \}, \qquad (29)$$

$$\Delta^{FLC} = \arg\max \{ f(\Delta) : \ \Delta \in \mathcal{LC}(I,v) \}. \qquad (30)$$

B. Uniqueness

When the Core is empty, Δ^{FC} is not even defined. On the contrary, Δ^{FLC} always exists. Moreover, the choice of strict concavity guarantees the uniqueness of the optimal solution of the above problems, see for instance [38], so that (29) and (30) define unique allocations. Note that VC and VLC are special cases of FC and FLC, respectively: minimizing variance is equivalent to maximizing negative variance, that is a (strictly) concave function.

V. The proposed computational algorithm

A. The algorithm

We focus on the solution of problem (30), since it is more complex than (29) and than the computation of just one point in the Core and Least Core.

The algorithm is divided into two consecutive stages. The first aims at computing θ^{LC} together with one point of the Least Core and, once the former is approximately known, the second stage actually solves (30). In order to solve (29) the first stage is not needed, as the Core is nothing else than the Least Core (24) with $\theta^{LC} = 0$. As a consequence, the computation of just a point in the Core can be performed through the second stage with the particular choice of $f = 0$.

Since the problems in both stages involve an exponential number of constraints, we propose the use of a row-generation technique to efficiently deal with them. The overall algorithm is sketched in Fig. 2. Each stage proceeds by iteratively executing a Master Problem (MP) and a Separation Problem (SP). The MP generates candidate reward allocations by considering only the constraints corresponding to a (small) subset Γ of proper coalitions, that is iteratively revised. Given the optimal solution of the MP, the SP seeks to find the coalition with the lowest surplus, that is therefore added to the set Γ.

In the first stage, convergence is reached when the surplus of MP matches the optimal value of SP. In the second stage, it is reached when the coalition found by SP is feasible for the MP, and this happens when the approximated value of θ^{LC} computed at the first stage matches the optimal value of SP.

An important aspect to improve the performance of the algorithm is the initialization of Γ with a well-chosen predefined set of coalitions.

B. Initialization

The aim of the initialization is to populate the set Γ with a pre-set, low number of coalitions, for each of which the quantity $v(K)$ must be computed. While computing $v(K)$ has generally lower computational requirements with respect to the SP, doing so an exponential number of times is prohibitive. Pre-populating Γ has a cost proportional to the chosen size, but on the other hand, a larger Γ can be expected to yield faster convergence. Therefore, a trade-off exists that will be explored in the computational section. Besides the number of coalitions, we will show that their effective choice is crucial.

C. First stage

Given the set Γ, the Master Problem is

$$\omega^M = \max \{ \theta \,:\, \Delta \in \mathcal{B} \;,\; \sigma(K, \Delta) \geq \theta \quad \forall K \in \Gamma \}, \quad (31)$$

which is the relaxation of (25) obtained by only considering the constraints corresponding to the coalitions in Γ. If the set Γ is reasonably small, then (31) can be efficiently solved since it has $|I| + 1$ variables in total (θ and the allocation Δ). This provides an optimal allocation Δ^M and its value ω^M. Since (31) has less constraints than (25), then $\omega^M \geq \theta^{LC}$.

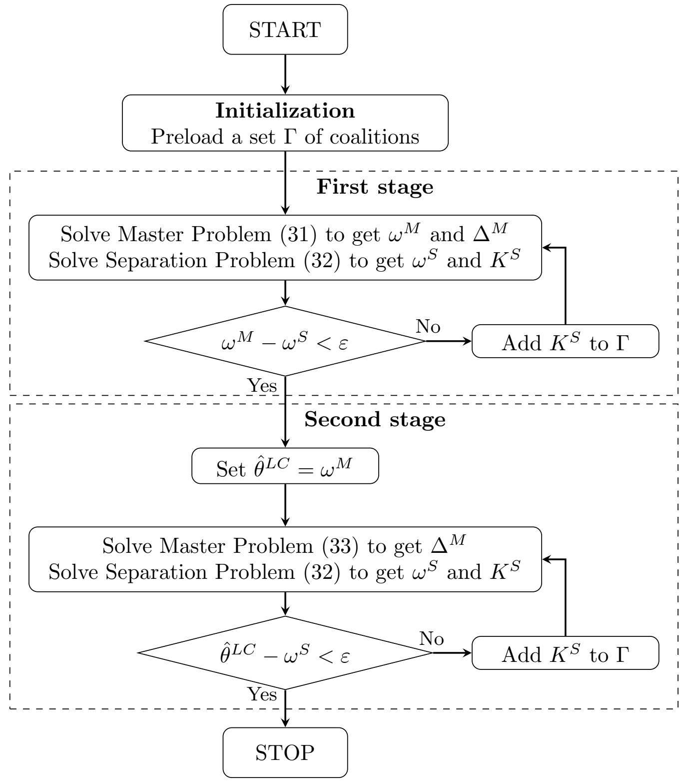

Fig. 2. Proposed solution algorithm for the Fair Least Core.

The Separation Problem checks if Δ^M is actually feasible for (24) by finding the coalition K^S with lowest surplus

$$\omega^S = \min \{ \sigma(K, \Delta^M) : \; K \in \mathcal{P} \} . \quad (32)$$

If $\omega^M = \omega^S$, then $\theta^{LC} = \omega^M$ and Δ^M belongs to the Least Core. To lower the computational burden, the above equality between the optimal values is checked up to some desired precision ε, the approximate value $\hat{\theta}^{LC} = \omega^M$ is exploited in the second stage and the first stage is considered over. Otherwise, K^S is added to Γ and the Master Problem (31) is solved again.

D. Second stage

Given $\hat{\theta}^{LC}$ and the set Γ provided by the first stage, the Master Problem in the second stage is

$$\max \{ f(\Delta) \,:\, \Delta \in \mathcal{B} \;,\; \sigma(K, \Delta) \geq \hat{\theta}^{LC} \quad \forall K \in \Gamma \}, \quad (33)$$

which is an approximation of (30) since $\hat{\theta}^{LC}$ is kept fixed. When $\hat{\theta}^{LC} = \theta^{LC}$, any optimal allocation Δ^M of (33) provides an upper bound $f(\Delta^M)$ of the optimal value of (30) and it is optimal if it is feasible for (30). Therefore, we stop the second stage whenever Δ^M is feasible in any case since $\hat{\theta}^{LC}$ is always expected to be very close to the true value θ^{LC}. Feasibility can be checked by solving the Separation Problem (32) and comparing ω^S with $\hat{\theta}^{LC}$. If they are (approximately) equal, then Δ^M is feasible, otherwise the optimal coalition K^S is added to the set Γ and a new iteration is performed.

VI. The Separation Problem

While the MP is a continuous optimization problem, the SP is combinatorial. Yet, by exploiting the mathematical formulation for the EC problems of Section II-E, it can be recast as a MILP, and therefore solved efficiently for communities of practical size, as shown in the following.

A. Mapping a generic coalition

The crucial challenge is to develop a proper row-generation algorithm to efficiently solve the Separation Problem (32). This requires in particular to describe the surplus function defined in (22) for a generic coalition $K \subseteq I$. The fundamental modelling trick we exploit is to augment the models of Section II-E with membership binary variables $z \in \{0,1\}^{|I|}$ to represent the chosen coalition; that is, z_i equals 1 when member i belongs to the coalition K, and 0 otherwise.

B. Benefit of a coalition

We now describe how to model the benefit $v(K)$ of a coalition, defined in (19), for any coalition K represented by the variable z. Since the presence of the aggregator significantly changes the structure of the mathematical problem in (19) that must be solved, we separate the function v into the two components, v^W and $v^{W/O}$, which represent the case with and without the aggregator, respectively; that is,

$$v(K) = v(z) = \begin{cases} v^W(z) & \text{if } z_A = 1, \\ v^{W/O}(z) & \text{if } z_A = 0, \end{cases} \tag{34}$$

where z_A is the membership variable of the aggregator.

1) Coalition with the aggregator: The benefit $v^W(z)$ represents the difference between (16) and (17), namely

$$\begin{aligned} v^W(z) = \quad \max_{u,s} \quad & \textstyle\sum_{j\in J}(c_j^T u_j - c_j^T \bar{u}_j^{NC} z_j) + \delta^T s \\ \text{s.t.} \quad & M_j u_j \le b_j z_j \qquad \forall j \in J \\ & s \le \textstyle\sum_{j\in J} D^+ u_j \\ & s \le \textstyle\sum_{j\in J} D^- u_j \end{aligned} \tag{35}$$

where the variables u of all members are formally included together with the energy exchange s. Anyway their actual occurrence is driven by the choice of the coalition addressed by z. In fact, since $M_j u_j \le b_j$ includes box constraints, zeroing the right-hand-side forces all variables u_j to be zero, as $\{u_j : M_j u_j \le 0\} = \{0\}$. This suggest to replace b_j with $b_j z_j$. Indeed, choosing $z_j = 0$ implies $u_j = 0$: member j "disappears" from the problem and cannot contribute to the energy exchange variables s and their reward, while on the other hand not incurring in any cost. Conversely, when $z_j = 1$ the constraint reads $M_j u_j \le b_j$ and member j "operates normally", thereby contributing to the community but having to pay its normal costs.

2) Coalition without the aggregator: The benefit $v^{W/O}(z)$ is the difference between (18) and (17), which corresponds to

$$\begin{aligned} v^{W/O}(z) = \quad \max_{s} \quad & \delta^T s \\ \text{s.t.} \quad & s \le \textstyle\sum_{j\in J} D^+ \bar{u}_j^{NC} z_j \\ & s \le \textstyle\sum_{j\in J} D^- \bar{u}_j^{NC} z_j \end{aligned} \tag{36}$$

The summation in the constraints is extended to the whole set of users J, but each term is multiplied by the membership attribution z_j to ensure no contribution to the shared energy when the member does not belong to the community. The above optimization problem has only the shared energy variables s, hence it is significantly smaller than (35) and this is exploited in the subsequent decomposition.

C. Procedure for SP decomposition

The Separation Problem (32) can be formulated by exploiting the membership variables as the following MILP

$$\begin{aligned} \min_{z} \quad & \textstyle\sum_{i\in I} \Delta_i^M z_i - v(z) \\ \text{s.t.} \quad & 1 \le \textstyle\sum_{i\in I} z_i \le |I| - 1 \\ & z \in \{0,1\}^{|I|} \end{aligned} \tag{37}$$

The objective function involves an inner maximization problem so that a min-max structure seems to appear. Since $v(z)$ compares with minus sign, the problem is actually a standard minimization problem.

To further increase the efficiency of the algorithm, since the computation of $v^{W/O}(z)$ involves significantly less variables than $v^W(z)$, the restriction of (37) without the aggregator fixing $z_A = 0$ is solved first. If the optimal value is enough to identify a coalition to add to Γ, then it is added without fully solving (37). Otherwise, also the case with the aggregator ($z_A = 1$) is analyzed. The following steps summarize the above procedure:

1) solve (37) with the additional constraint $z_A = 0$ to get the optimal value ω_0^S and the corresponding optimal solution K_0^S;
2) add K_0^S to Γ in the first stage if $\omega^M - \omega_0^S \ge \varepsilon$, in the second if $\hat{\theta}^{LC} - \omega_0^S \ge \varepsilon$;
3) otherwise, solve (37) with the additional constraint $z_A = 1$ to get ω^S.

VII. Case Study

A. Description

To validate the methodology, we apply the proposed approach to a realistic case study that describes ECs of various sizes (10-50) for a peri-urban area in Italy; yet, the approach does not depend on specifics of the Italian case. The demand data have been adapted from the dataset measured from a Portuguese substation [39], whose consumption patterns are similar to the Italian ones, with average peak demand in the range 12-40 kW. Given their abundance, solar and wind resources have been considered, and their time series have been obtained from [40]. To avoid market distortion, the market prices of 2019 have been selected.

B. Users composition

In this study, we considered the four EC sizes of 10, 20, 30 and 50 members, which aligns to expected values in the Italian context. To keep results comparable, the ECs with size larger than 10 have been obtained by replicating the composition of the 10-user EC. For instance, in the 30-user EC, members 11 and 21 perfectly match user 1. This is justified by the observation that typical consumers in the power grid do have similar habits and, consequently, similar demand patterns.

TABLE I
PERCENTAGE DIFFERENCE BETWEEN FINAL SURPLUS ω^S AND TRUE VALUE.

EC size	Precoal.	Core	LC	VC	VLC
10	All	0.0*	0.0*	0.0*	0.0*
20	[1, 20]	0.0*	0.0*	0.0*	0.0*

*below 10^{-3}

TABLE II
MAXIMUM PERCENTAGE DIFFERENCE OF USERS' ALLOCATION BETWEEN THE ITERATIVE AND ENUMERATIVE APPROACHES.

EC size	Precoal.	Core	LC	VC	VLC
10	[1]	100.00	50.26	0.0*	0.27
10	[1, 2]	75.04	50.26	0.0*	0.27
10	[1, 10]	142.73	50.22	0.0*	0.0*
10	[1, 2, 3]	116.02	36.29	0.0*	0.27
10	[1, 9, 10]	100.00	95.51	0.0*	0.0*
20	[1, 20]	> 1000	38.66	0.0*	0.95

*below 10^{-3}

C. Main techno-economic parameters

The cost of installing photovoltaic (PV) systems is between 1.4 and 1.7 k€/kWp, with a space limitation up to 100 kWp. Wind turbines cost 3 k€/kW. Lithium batteries cost 400 €/kWh plus 200 €/kW (converter) and have a round-trip efficiency of 92%. The lifetime of PV is 25 years, while wind turbines, batteries, and converters have a lifespan of 20, 15, and 10 years, respectively. Yearly maintenance charges have been estimated between 1 and 2% of the initial investment. Purchase and selling prices of 18 c€/kWh, including taxes, and 5 c€/kWh, respectively, have been assumed, with monthly peak power charges of 3 €/kW/month [33].

D. Testing procedure

To validate the proposed framework, we used Shapley Value, Nucleolus, Core, Least Core, Variance Core and Variance Least Core as reward allocation functions for the considered ECs. We applied the approach described in Section V and Section VI to the latter four allocation functions for all EC configuration. In order to compare the effectiveness of our approach, we also performed the complete enumeration of all coalitions. Due to obvious computational limitations, this has been done only for the cases of 10 and 20 users. For our approach we performed a sensitivity analysis on the pre-coalition set Γ, considering up to 6 configurations. The notation of the pre-loading is as follows: $[1]$ denotes that Γ is pre-loaded with all the coalitions with up to 1 member, $[1, |J|]$ denotes the coalitions with 1 or $|J|$ members, and so on.

In the following section, we first compare the enumerative approach with the iterative one, to show the equivalence of their results but the far superior performances of the latter, which makes it usable for large ECs. We then perform a sensitivity analysis with respect to the size of the community, to suggest guidelines for fair stable reward allocations.

The methodology has been solved using Gurobi 9 and 10 threads on a 72-core Xeon computer with 1.2TB RAM. A relative tolerance of 1% or an absolute tolerance of 1€ have been used as stopping criterion of the algorithm.

VIII. RESULTS

A. Validation of results

Table I and Table II validate the iterative technique described in Section V-VI by reporting the percentage difference between its results and those of the traditional enumerative computation. Table I confirms that the iterative approaches successfully capture the true surplus value, computed by complete enumeration, with differences compatible with the target tolerance. Table II rather shows the maximum percentage difference across users in reward allocation. The results show that the VC and VLC allocations have negligible differences with respect to the exact solutions, which confirms reproducibility in agreement with the theory. On the contrary, the computations of allocations in Core and Least Core are merely feasibility problems. Therefore, it is natural that different procedures point to allocations that are far from each other, although having comparable surplus. This is in agreement with the theory and further confirms the importance of finding allocations that are uniquely defined, such as the F[L]C proposed in Section IV.

B. Convergence characteristics

Fig. 3 and Fig. 5 highlight the computational time and the convergence characteristics of the proposed method for the 10- and 20-member ECs. Fig. 3 clearly confirms that the iterative algorithm can dramatically reduce computational requirements by 20x even for the 10-member EC, and beyond 16000x for the 20-member EC. As the enumerative technique required longer than 2 months to compute, and the computational requirements grew exponentially, no validation was possible for larger ECs. A proper pre-loading can have a significant impact on the iterative algorithm, as the $[1, |J|]$ choice reduced the computational cost by about 64% with respect to $[1]$; this is why it is selected as the reference case for the subsequent investigations.

The efficiency of the algorithm is confirmed in Fig. 4, which shows that the computational cost scale relatively proportional to the size of the community. This is a significant improvement with respect to the exponential requirements of traditional techniques illustrated in Fig. 3. Moreover, in Fig. 5 we plot the difference $\omega^M - \omega^S$ in the first stage and $\hat{\theta}^{LC} - \omega^S$ in the second stage that is used as convergence criterion of the proposed algorithm (Section V); the picture shows that the algorithm generally converges fairly quickly in a limited number of iterations.

Overall, these results confirm the ability of the proposed algorithm to scale in size and make game-theoretical allocation approaches feasible for large ECs.

C. Benefit and reward allocation by size of community

Finally, we show in Fig. 6 and Fig. 7 the effect of EC size on the community surplus and reward allocation by user, respectively. Fig. 6 interestingly shows that the surplus decreases the larger the EC size. Indeed, the larger the EC, the lower each user's market power within the community, which in turn decreases the LC value. However, the marginal

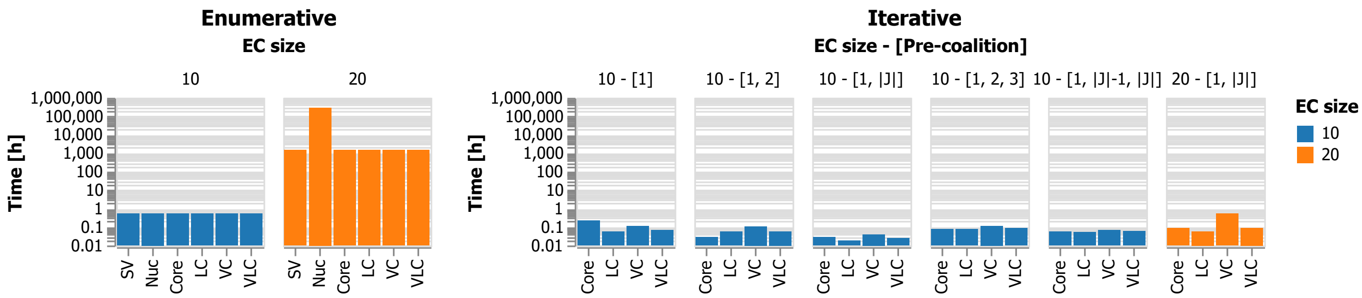

Fig. 3. Comparison of execution time for selected enumerative and iterative methods.

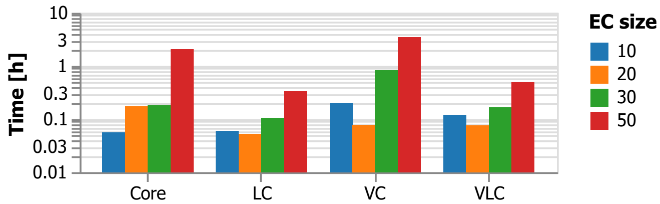

Fig. 4. Performances at increasing size of the community of the iterative techniques - precoalitions $[1, |J|]$.

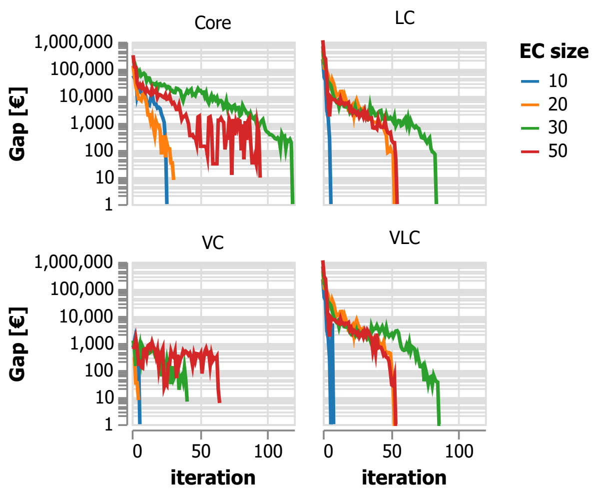

Fig. 5. Convergence characteristics of the iterative techniques - precoalitions $[1, |J|]$; tolerance is 1-10€ (about 1%).

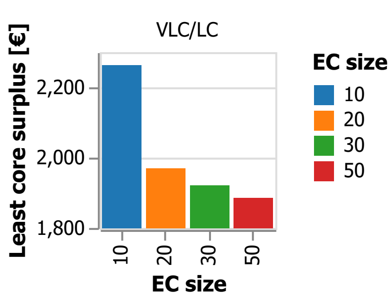

Fig. 6. Community surplus (θ^{LC}) by EC size - precoalitions $[1, |J|]$.

reduction decreases the larger the community. As the LC changes, the users' relative reward allocation changes, to reflect the different market power within the community.

Fig. 7 shows the expected user benefit in terms of Δ by member type and reward allocation. For simplicity, as for 20-member EC or larger the users are identically replicated, error bars depict the maximum and minimum benefit allocation between the same member types. First, it is worth noting that error bars are negligible, which means that each member type is remunerated in the same way. For example, in the 50-member EC, there are 5 instances of member types "user1" that are all remunerated with the same value, which goes in favor of stability and fairness.

IX. Conclusion

Based on the state-of-the-art on game-theoretic allocations, this paper proposes and discusses the novel fair stable reward allocations Fair Core and Fair Least Core for Energy Communities. These successfully maximize fairness of benefit allocation, while enforcing stability by ensuring that no member is worse off within the community than outside (by the largest possible margin in the Least Core variant). The new allocations guarantee uniqueness and reproducibility, which go in favor of the practical use of the methodology.

Crucially, the work also proposes a row-generation algorithm to reduce the hitherto staggering computational requirements for game-theoretic benefit allocations. The new algorithm is extensively validated on communities up to 50 members, which was before impossible. The results suggest that the methodology is a breakthrough that makes game-theoretic allocations practical for large coalitions while ensuring uniqueness, reproducibility, and stability. As an example of the managerial insights that the methodology offers, our case study shows that the larger the community, the higher the influence of market power within the community, which has impact on reward allocation.

This paper lays the foundations for reproducible, fair, and stable reward allocations, and it can be expected to steer research in the design of incentive schemes for Energy Communities, power systems, and beyond.

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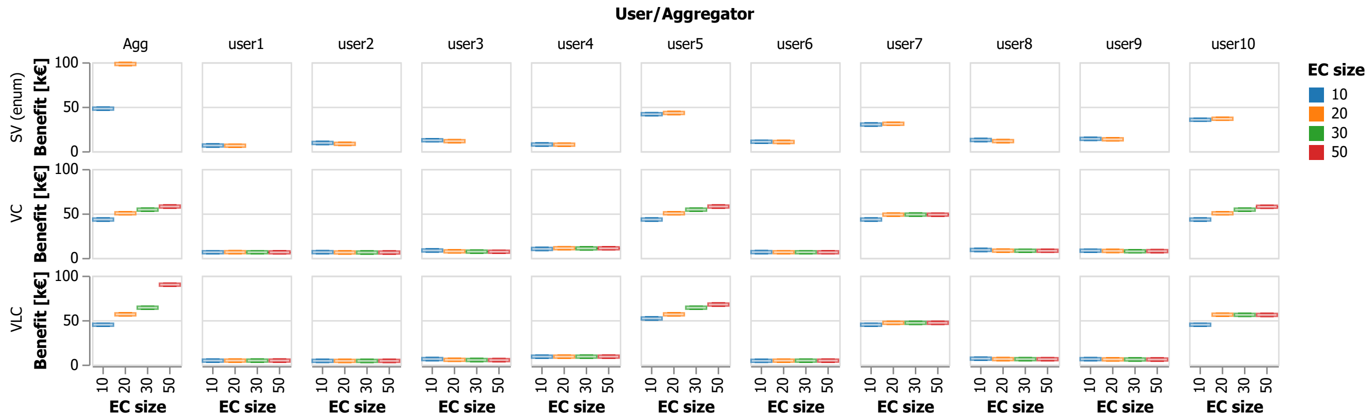

Fig. 7. Benefit (Δ) by user and aggregator: bars represents the average value by user type and error bars highlight the variation.

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