From empirical engineering to science-based engineering: the assessment by three mathematicians of the "San Pietro" dome (1742)

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ABSTRACT: The studies on the mechanic behaviour of domes, based on the static analysis of arches and vaults, started in the first half of the XVII century. At the end of 1741, when the Pope Benedetto XIV commissioned an assessment to three mathematicians Roger Joseph Boscovich, François Jacquier e Thomas Le Seur, belonging to the *Repubblica Romana dei dotti*. There where serious worries on the static conditions of the San Pietro vault where important cracks could be appreciated. Some interesting studies were developed on this problem, even followed by learned and heated arguments, that lead to compile treatises that systemized the knowledge on this argument. The historical documents even report the opinion of other experts, for example that one of the mathematician Giovanni Poleni from Padua. On spite of that, the study of the three previous mathematicians was different from the others carried out before because it had an important innovation: the assessment was accomplished, perhaps for the first time, using a scientific criterion in order to interpret the mechanic behaviour of an architectural structure. Its historical importance resides on the scientific theoretical conceptions used to analyze a structural problem that were completely different from the empirical rules, at most of geometric nature, used before. The principle of virtual works was further applied in the assessment. It was used to define the size of the metal rings for the drum of the vault. Some researchers intend this assessment as the moment when the change from the engineering based on artisan traditions to the engineering based on the application of new scientific theories has been accomplished.

1 INTRODUCTION

The present scientific theories aimed at analysing the mechanical behaviour of buildings started to be developed in the second half of the 17th century. Only then were the mechanics of materials taking their first steps with Mariotte and Robert Hooke's research (Timoshenko 1953; Timoshenko 1956). This followed the road which Galileo had opened indicating experimental observation as the basis of scientific knowledge. In his last work Galileo himself had presented the first observations on the *"new science relating to the mechanics"* of structures (Galilei, 1638).

The first applications of the new scientific methods to structural problems started to be enunciated between the end of the 17th century and the early 18th century. It was then, in 1741, that Benedict XIV commissioned three mathematicians Roger Joseph Boscovich, François Jacquier and Thomas Le Seur from the "Repubblica Romana dei Dotti" (Boscovich & Al. 1742) to carry out a historical assessment. Serious concerns had arisen over the static conditions of Saint Peter's dome, where significant cracks had appeared. Interesting studies had already been developed on the subject, accompanied by learned and heated debates. This had led to the compilation of authentic treatises, which also aimed to order the knowledge on the subject. Historical documents also report other experts' opinions on the state of the dome, including that of the well-known mathematician from Venice Giovanni Poleni (Poleni, 1748).

The three Mathematicians' study stood out for its important innovation. It contained an assessment based entirely, perhaps for the first time, on a scientific criterion aimed at interpreting the mechanical behaviour of an architectural building. Its historical importance lies in the fact that, unlike the previous practices, which were based on empirical rules, generally of a geometric nature (see for example Poleni's studies on the statics of arches), theoretical conceptions, this time of a scientific nature, were used and applied to the study of a structural problem. Although not entirely correctly, the PVW (Principle of Virtual Work) was

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adopted in the assessment, and used as an instrument for measuring the metal rings to be applied to the drum of the dome (Capecchi, 1999; Capecchi, 2002). In an attempt to determine an important date, a number of experts (von Halász 1969) regard this assessment as the historical moment when the change took place from engineering based on artisan traditions, of an empirical nature, to engineering based on the application of the new scientific theories; theories, which were just starting to become established.

The three Mathematicians' assessment was presented towards the end of 1742 and printed in 1743. The study method thus introduced could truly represent the historic beginning of modern civil engineering. Unlike the previous practices, which used rules dictated by intuition and experience, a scientific process was applied to assess a building's characteristics of resistance and state of stress. This consequently started a process, which does not yet appear complete (Di Pasquale 1996).

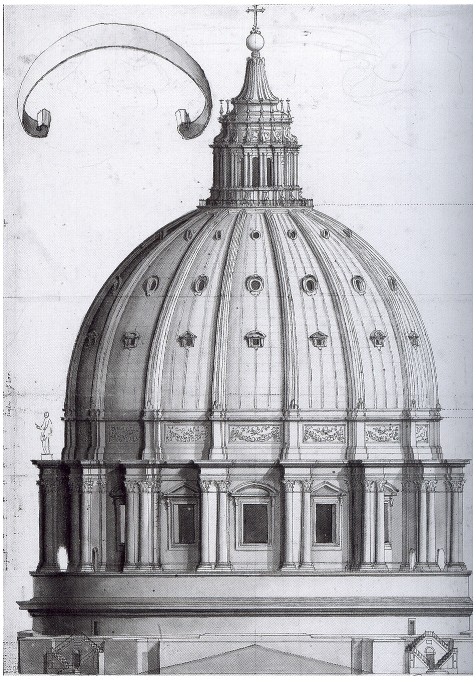

Figure 1. Saint Peter's Dome (Curcio, 2003)

2 "SAINT PETER'S" DOME

The building of "Saint Peter's" Dome was started on 15 July 1588 under Sisto V, however, it was interrupted on 13 May 1590, just before the Pope's death. Following Michelangelo's project, the building work had reached the placing of the drum. It was completed by Giacomo Della Porta at the beginning of the 17th century (Ackerman, 1968). The first cracks were discovered back in 1603, under Clement VIII, just after the building was finished, on completion of the mosaics on the vaults. Subsequent damage was then recorded after 1631, as we can see in Gianlorenzo Bernini's biography written by Filippo Baldinucci. It was suspected that the statics of the dome had been compromised by the insertion of the spiral staircase by Bernini, set within the pillars, under the pontificate of Urban VIII.

The controversy relating to Bernini's work soon quietened down. Baldinucci himself observed that a number of cracks inside the dome on the cornice and on the drum had been found before Bernini. Some had attributed the damage to phenomena of settling of the great dome and to the different working techniques used for its construction. However, the three Mathematicians claimed in their study that the damage described by Baldinucci was not the damage found in 1742. The numerous criticisms raised against Bernini referred to evident conditions of instability which were present already in 1742. Subsequently, the instability had increased and was developing.

"Il Tempio Vaticano" by Carlo Fontana, published in 1694 (Curcio, 2003) made it possible to carry out an in-depth analysis of the phases of construction of Saint Peter's Basilica and its dome, until the complex took on its present arrangement. The work covers the events of the construction of the building from the beginning, when the emperor Constantine wanted the basilica built near the tomb of the apostle Peter, until the end of the 17th century.

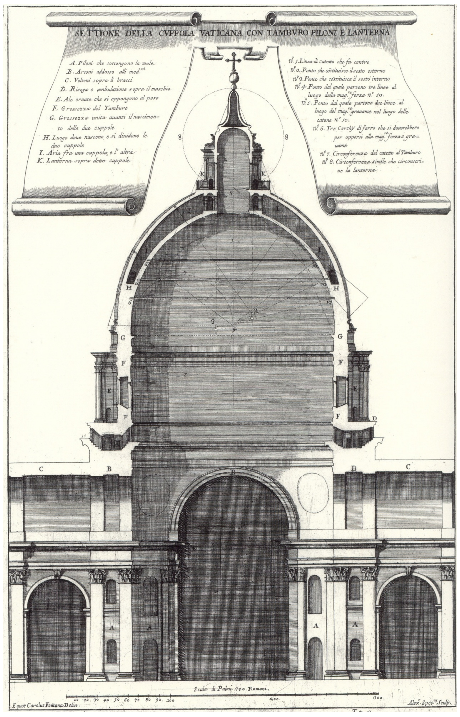

Figure 2. Section of Saint Peter's Dome (Curcio, 2003)

3 THE ASSIGNMENT OF THE ASSESSMENT

Details of the task assigned to the three Mathematicians can be read in the introduction of the assessment. An important point concerns the use of the words *"ristauratione"* (Restoration) and *"conservazione"* (Conservation). This shows a clear wish to orient the proposed interventions at safeguarding the existing building work: an admirable aim, promoted in a time when the philosophy of restoration was not yet clearly defined.

The innovative aspect of the assessment concerns the application of the method chosen for defining the interventions. Explicit reference was not made to Galileo because the memory of the trial in 1633 was still vivid, and his writings were still banned (they were until 1822). However, the three Mathematicians were nonetheless faced with a problem of static restoration using, perhaps for the first time, a scientific criterion of calculation. They highlighted the importance of acting not only using their *"own visual observations"*, but in particular using a *"good theory based on Mechanics"* for reference.

The process follows a plan that can be divided into the subsequent four phases following a coherent and logical approach. The phases are:

(a) Diagnosis, consisting of a careful observation of the present state to determine the amount and importance of the phenomena;

(b) Aetiology, consisting of an assessment, realized from the previous observations, regarding the identification of the causes, which may have lead to the phenomena;

(c) Prognosis, consisting of an examination of the possible criteria and methodologies available, which could be used to identify and calculate the solutions to be adopted;

(d) Therapy, consisting of a detailed definition of the working methods to be followed for applying the identified solutions.

4 DIAGNOSIS

The first part of the study is dedicated to a detailed description of the dome and the creation of a detailed geometrical survey. This instrument of knowledge is subsequently perfected by superimposing the existing pattern of cracks.

The precise representation aims to lay the basis for analysing the loads and interpreting the relative movements between the various structural parts, which, according to current terminology, could be considered macro-elements. Even the variations in width of the cracks along the development of each of them are evidenced, with the clear intention of representing the kinematic mechanisms of the various relative movements.

The survey consequently becomes an instrument of knowledge and support for thematic close examinations including kinematical analyses of the instability. In the continuation of this part of the assessment the three Mathematicians describe the damage observed, grouping it into three main areas of observation: the drum, the vault and the lantern.

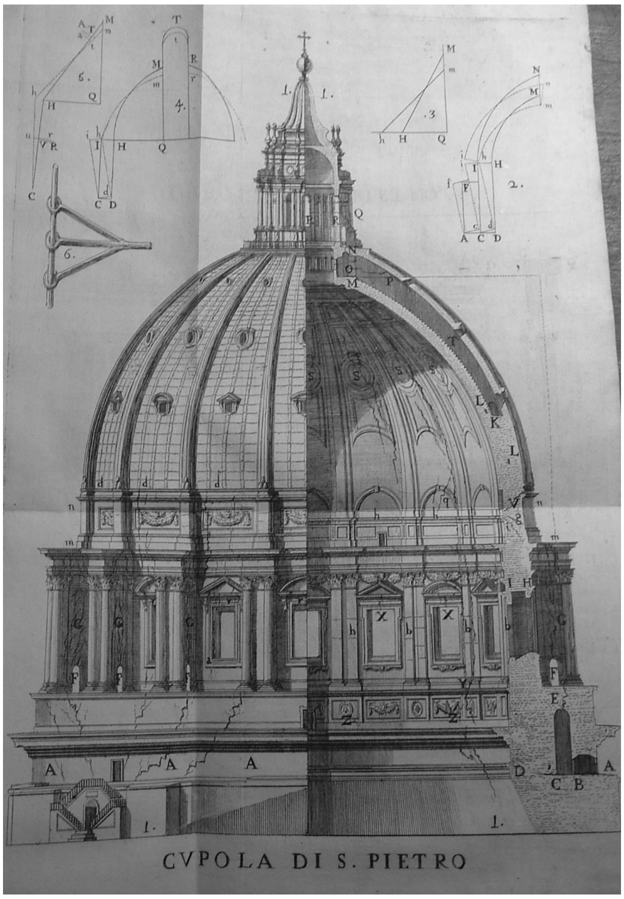

Figure 3. Survey of the Dome (1742 *Parere di Tre Matematici*, Bibliotheca Hertziana, Rome)

5 AETIOLOGY

The second part of the assessment is dedicated to identifying the causes responsible for the instability. The authors formulate a graphic diagram to show how the movements may have occurred. The cracks are interpreted as hinges around which the parts of stonework, which is not cracked, considered non-deformed, have rotated. The kinematical interpretation of the pattern of cracks identified enables the three Mathematicians to exclude a number of hypotheses formulated by others. In the absence of cracks, which can justify them, they exclude that the causes of the instability can be attributed to the subsiding of the foundations. In substance, they think that the weight of the small dome, the ribs and the double cap have weighed down causing the drum to move outwards. As for the iron rings and possible damage, the three Mathematicians, still with an elasticist mentality, claim that it would not be possible to know if, and to what extent the metal rings are truly effective. This is because they cannot be seen directly as they are inserted into the stonework, nor is it possible to know the tightening tensions. Moreover, the iron could have suffered thermal deformations changing its ring strengths, while some of the rings could even have broken.

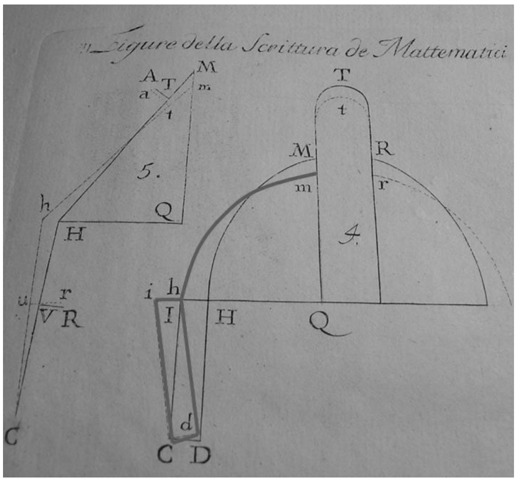

Figure 4. Description of the deformation mechanism (1742, *Parere di Tre Matematici*, Bibliotheca Hertziana, Rome)

6 PROGNOSIS

The assessment continues defining the process that the three Mathematicians intend to use to assess the quantity of actions associated with the kinematical mechanism and the tensional state of the rings whose scarce efficiency appeared to be responsible for the instability.

In terms of scientific innovation this is the topical moment of the assessment because the Three decide to apply a process of calculation based on the PVW. In fact, the instability refers to the excess weight that burdens, above all, the upper parts of the dome, pushing the drum outwards, and to the scarce ringing resistance of the lower buttresses.

At this point the three Mathematicians consider the results obtained by Philippe de la Hire and Couplet (Benvenuto, 1981) concerning the behaviour of arches and vaults. They come to the conclusion that two causes are responsible for pushing the drum outwards: the weight of the small dome and the weight of the ribs including the gores of the dome. Similarly two horizontal forces create resistance: the force of the rings and that of the support made up of the base, drum and buttresses.

To assess the weights of the structures the three Mathematicians weigh a mass of travertine and one of stonework. Thanks also to the geometric data of the single parts surveyed, they succeed in proving that the total weight of the dome is equal to about 56·000 tons.

Whereas, to asses the force corresponding to the iron rings the three Mathematicians appeal to the treatise *Coesione de' corpi solidi* by Pietro Ban Musschenbroek deducing that the traction resistance of the first ring corresponds to 114 t and that of the second to 95.

After observing that the variation in the length of a circular chain increases in the proportion of 2π in relation to the variation of the radius, the Three apply the PVW equalling all of the positive and negative works made by the elements in play. The positive work is produced by the weights of the macro-elements, which represent the damaged parts of the dome and small dome. The negative work is determined both by the resistance opposed by the drum in contrasting the deformations outwards, and by the resistance available in the rings on the various levels. This latter work, however, is assessed incorrectly: as the concept of potential elastic energy was not yet clear, it is calculated directly considering the resistance on breaking.

After obtaining the values of the forces (thrust and resistance), the problem of balance, however, is handled strictly in terms of energy.

The process adopted by the three Mathematicians to solve the problem, despite containing a number of imperfections, is daring and modern. The Mathematicians renounce the use of processes based on the polygons of the forces, and refer to a method, the PVW, mentioned previously by *René Descartes* in his principal work published in 1637 (Descartes, 1637) and subsequently perfected by *J. Bernoulli*.

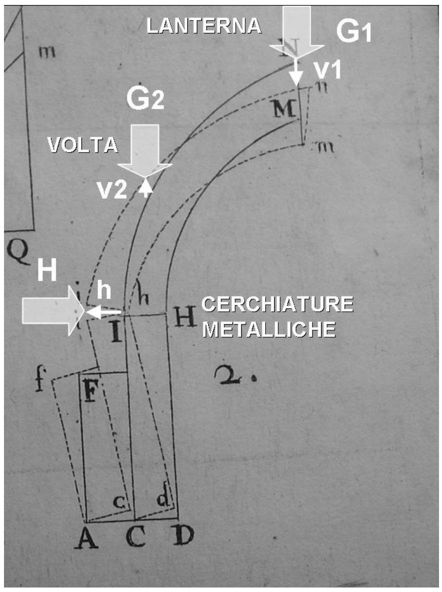

Figure 5. Analysis of the balance dealt with by applying the PVW

In this way the three Mathematicians succeed in proving that the weight of the small dome and the dome exert a force **H** on the impost ring of the dome distributed as internal pressure **p** (**H=2πR×p**), whose total value results from the following relation, obtained equalling the virtual works considered (Fig. 5):

$$\mathbf{H} \times \mathbf{h} = \Sigma\, (\mathbf{G} \times \mathbf{v})$$

where **G** indicates the weights of the small dome and the portions of the dome, where **v** indicates the lowerings of their centres of gravity and where **h** indicates the horizontal opening of the impost.

The resistance **W**, which contrasts the thrust **H** is made up partly of the resistance exerted by the rings (hence the need to calculate the force of these exerted as radial pressure **p**) and partly of the horizontal resistances with which the drum and the buttresses resist overturning. The state of balance between **W** and **H** is thus calculated by the three Mathematicians applying the new principles of Mechanics. Thanks to this process they succeed in calculating a missing thrust equal in total to about three million pounds, in other words to approximately 1000 tons.

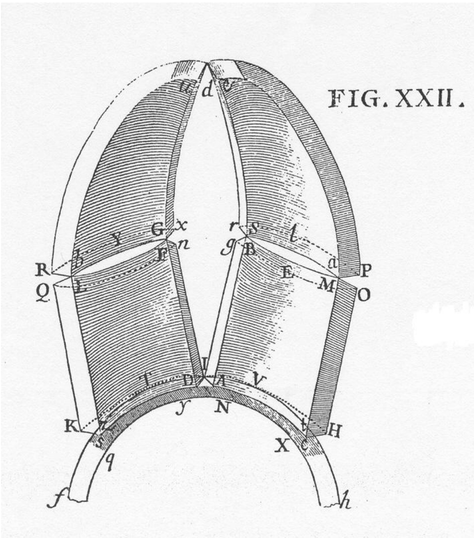

Figure 6. Kinematical diagram proposed by Giovanni Poleni

Consequently, the cause of the instability is attributed to this imbalance. The rings, which had been laid during the construction are therefore unable to contrast the pushing action of the structures. In substance, the three Mathematicians conclude observing that the upper part tends to move inwards under the action of the loads of the small dome, while the lower part tends to move outwards developing traction tensions in the rings.

In the light of the knowledge relating to the mechanics of structures, developed later on, the adopted process does not appear without imperfections. First of all, the work carried out to dilate the rings, calculated as work of extending an equivalent straight rod, is assessed considering a constant force applied from the beginning of the elastic deformations, unduly associating a size growing elastically with a static one. Moreover, no reference is made to the work of elastic deformation of the “macro-elements” according to which the kinematism of the instability was examined, nor of the anelastic deformations of the unbalanced areas. On the other hand, as we have already seen, a mechanical theory for structures had not yet been developed to the point of being able to assess these aspects correctly. Therefore, the daring choice of a process of theoretical calculation applied for interpreting mechanical phenomena remains significant.

7 THERAPY

Again in the light of the PVW, the need is confirmed to find a solution so the thrusts acting outwards are rebalanced by thrusts acting in the opposite direction to guarantee the equilibrium of the whole structure. To do this, the three Mathematicians suggest placing additional rings, considering a safety coefficient equal to two, justifying the reason and consequently showing a typical engineering approach.

The possible solutions examined are divided into three groups, according to which different alternative solutions are proposed: placing iron *“struts”* and chains; walling up the spaces, which are currently open, to strengthen the buttresses; eliminating the structural loads where they are not needed. Of the three groups the three Mathematicians prefer the first, or rather the placing of new rings. To do this they refer to the data analysed and to the numerical results obtained again applying the PVW. This enables them to quantify the number of interventions and optimise the position of the reinforcing rings.

In response to the criticism raised by various experts on the spiral staircase made in the four pillars by Bernini, which allegedly also affected the statics of the dome, on the strength of the results obtained with a scientific process, the three Mathematicians claim that it is not necessary to fill them in since the pushing action of the drum is clearly less than that of the small dome; therefore they can be preserved in the state they still are today. However, they suggest filling in a number of spaces, which support the vaults and indicate other rings to be inserted level with the drum.

Lastly, the three Mathematicians claim that all of the other solutions they had heard, and which had been suggested to them, were superfluous for resolving the static problem of the dome. The six rings alone and the various careful interventions

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planned would have undoubtedly contributed to improving the situation. In fact, the other solutions would have given a load of approximately 950 tons in relation to a total weight of the dome of over 56˙000 tons, so with rather insignificant added value.

The indicated project solution applying a scientific process made it possible to propose work that was not invasive, respectful of the Michelangelesque building and its decorations and aesthetics. The assessment generated great controversy from well-known scholars including Poleni himself, who had also been commissioned to make an analysis of the same problem (Poleni, 1748; Baggio & Da Gai, 2000). Boscovich's work was later praised by C.L. Navier (Navier, 1829) who recognised its originality.

The reinforcement work was carried out under the supervision of Luigi Vanvitelli (Buccaro, 1988) who applied the three Mathematicians' proposal, and Master Nicola Zagaglia was responsible for the organisation of the site (Cosatti, 1743; Zander 1991).

The assessment described in this memorial is illustrated in detail in the book *Dall'ingegneria empirica verso l'ingegneria della scienza* (Niglio, 2007).

Figure 7. Master Nicola Zabaglia

8 CONCLUSIONS

The results of the assessment were presented by Boscovich on 20 January 1743 and published in the same year. The incident involving Saint Peter's Dome had repercussions on other situations, for example the interventions carried out on the spire of Milan Cathedral.

This marked the start of a debate on the relationship between Architecture and Mechanics, between consolidated humanistic knowledge and new science, which was destined to revolutionise the future of building practices.

It was an important and daring step for the Three Mathematicians, taken at a historical time in which no other information of equal importance emerges. It involved basing a whole expert analysis and the consequent project proposals on the use of a scientific principle of a purely theoretical nature, which was completely innovative and not yet used in other real situations.

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